PROPERTIES OF STRONGLY COUPLED QCD FROM RENORMALIZATION FLOWS

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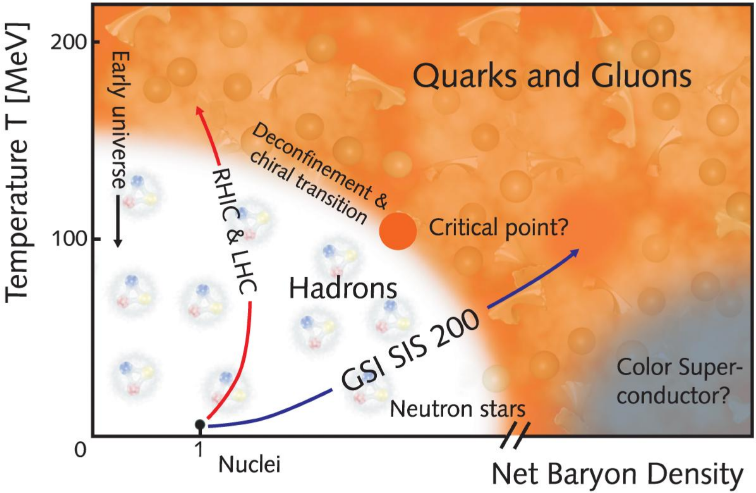
QCD Phase Diagram



[FAIR@www.gsi.de]

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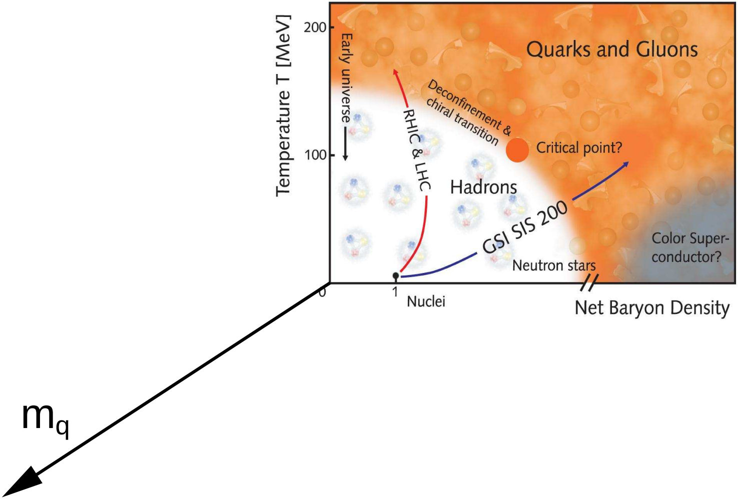

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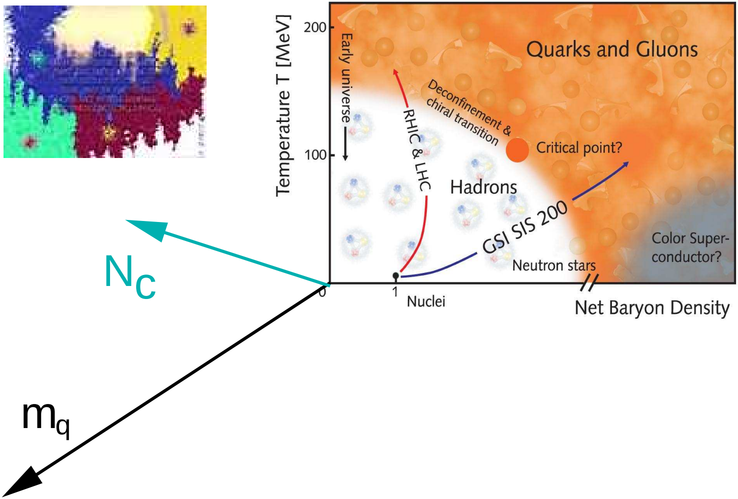

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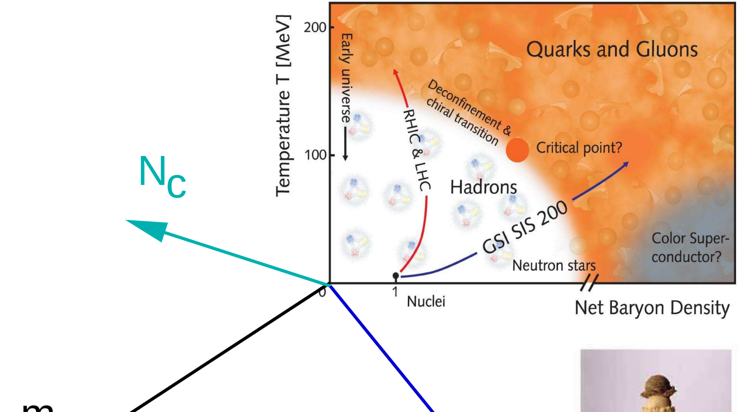

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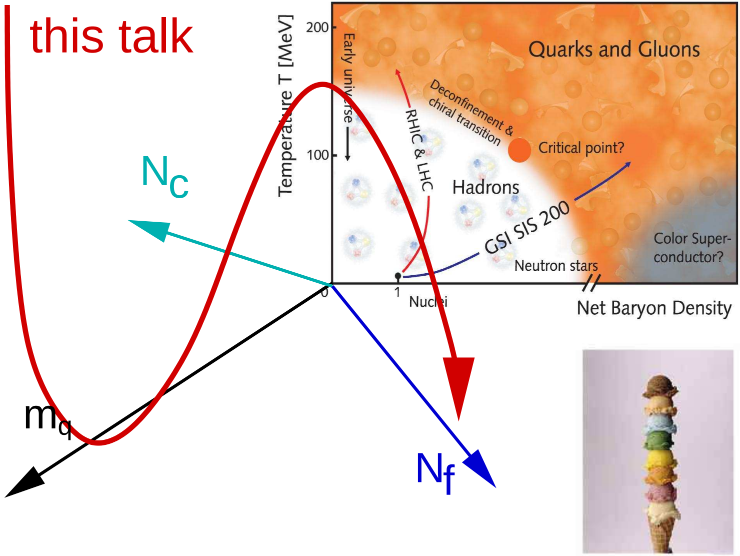
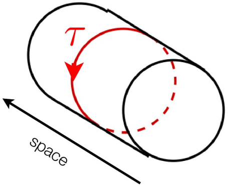


(De-)Confinement in Gauge Theories

(De-)Confinement Order Parameter

▷ order parameter: Polyakov loop (Polyakov'78; Susskind'79)

$$L(\mathbf{x}) = \frac{1}{N_c} \mathcal{P} \exp\left(i \int_0^\beta \tau \, A_0(\mathbf{x}) \right)$$



▷ heavy-quark free energy

$$\left\langle \mathrm{tr}_F L(\mathbf{x}) \right\rangle \sim e^{-\beta \mathcal{F}}$$

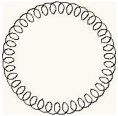
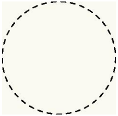
confinement:	$\mathcal{F} \to \infty$	$\Longleftrightarrow$	$\langle L \rangle = 0$	SYM Z_{N_c}
deconfinement:	$\mathcal{F} < \infty$	$\Longleftrightarrow$	$\langle L \rangle \neq 0$	SSB Z_{N_c}

[Talk by F. Marhauser]

Perturbative Order-Parameter Potential

- e.g., perturbation theory in background-field gauge:

$$V_{\text{pert}}(A_0) = \frac{1}{2}\,\text{Tr}_{\text{cL}x} \ln G^{-1}_{\text{pert,gluon}}[A_0] - \text{Tr}_{\text{c}x} \ln G^{-1}_{\text{pert,ghost}}[A_0]$$

$$= \frac{1}{2}$$  $-$ 

- perturbative propagators

$$G^{-1}_{\text{pert,gluon}}, G^{-1}_{\text{pert,ghost}} \sim p^2$$

- A_0 background

$$\Longrightarrow \quad G^{-1}_{\text{pert,gluon}}(A_0), G^{-1}_{\text{pert,ghost}}(A_0) \sim -D^2[A_0]$$

Perturbative Order-Parameter Potential

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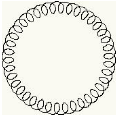
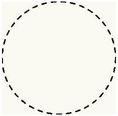
$$= \frac{1}{2}$$  $-$ 

▷ background field in Polyakov gauge

$$A_0 = A_0(\mathbf{x}) \in \text{Cartan}, \quad \text{e.g. for SU(2):} \quad A_0^a = A_0 \delta^{a3}$$

▷ e.g., SU(2) order parameter

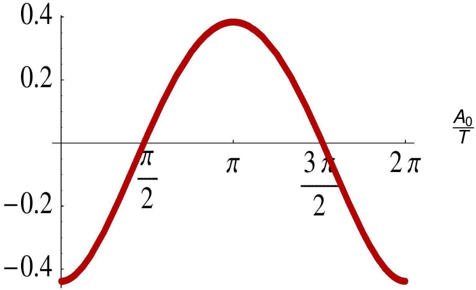
$$\mathrm{tr}_F L(\mathbf{x}) = \cos\frac{A_0(\mathbf{x})}{2T} \qquad \underset{\Longrightarrow}{L[\langle A_0\rangle] \geq \langle L[A_0]\rangle} \qquad \text{confinement:} \quad \left\langle \frac{A_0}{T} \right\rangle = \pi$$

PERTURBATIVE ORDER-PARAMETER POTENTIAL

▷ e.g., perturbation theory in background-field gauge for $A_0 =$const.

$$V_{\text{pert}}(A_0) = -\frac{4}{\pi^2} T^4 \sum_{n=1}^{\infty} \frac{\cos\left(2\pi n \frac{A_0}{2\pi T}\right)}{n^4}$$

(WEISS'81)



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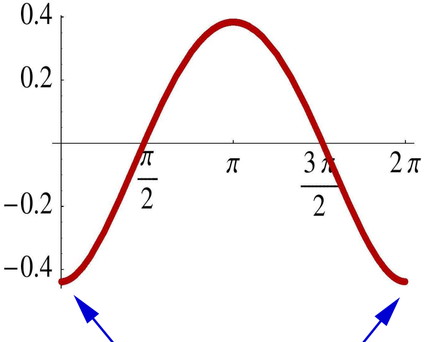
(Weiss'81)

$V_{\rm pert}$

deconfining



deconfinement

PERTURBATIVE ORDER-PARAMETER POTENTIAL

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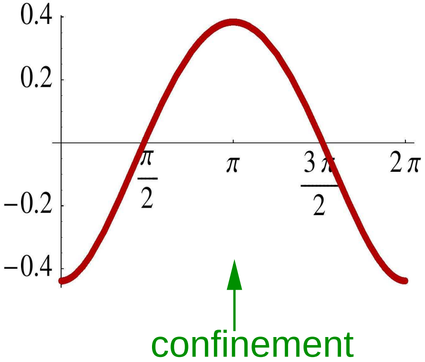

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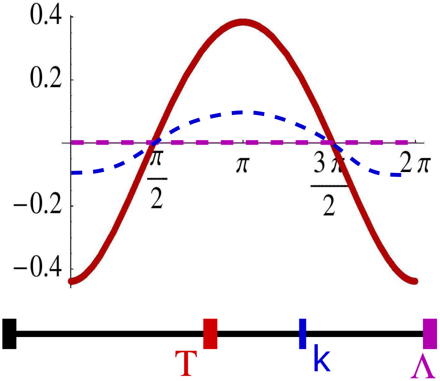
$V_{\rm pert}$

dominated by

$k \sim 2\pi T$



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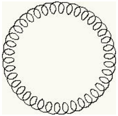
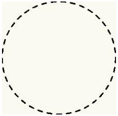
$$= \frac{1}{2}$$  $-$ 

Perturbative Order-Parameter Potential

- ▷ "improved" potential

$$V_{\text{impr}}(A_0) = \frac{1}{2}\,\text{Tr}_{\text{cL}x}\,\ln\,G^{-1}_{\text{gluon}}[A_0] - \text{Tr}_{\text{c}x}\,\ln\,G^{-1}_{\text{ghost}}[A_0]$$

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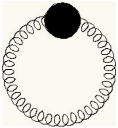

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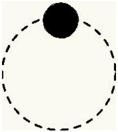

Perturbative Order-Parameter Potential

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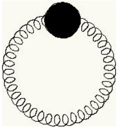

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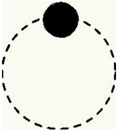


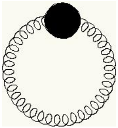
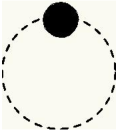
- ▷ $V_{\text{impr}}(A_0)$ dominated by modes $k \sim T$

$$\text{UV:}\quad G^{-1}_{\text{pert,gluon}},\, G^{-1}_{\text{pert,ghost}} \sim p^2$$

Perturbative Order-Parameter Potential

▷ “improved” potential

$$V_{\mathrm{impr}}(A_0) = \frac{1}{2}\,\mathrm{Tr}_{\mathrm{cL}x}\,\ln\,G^{-1}_{\mathrm{gluon}}[A_0] - \mathrm{Tr}_{\mathrm{c}x}\,\ln\,G^{-1}_{\mathrm{ghost}}[A_0]$$

$= \frac{1}{2}$  $-$ 

▷ $V_{\mathrm{impr}}(A_0)$ dominated by modes $k \sim T$

$$\text{IR}\ (k \ll \Lambda_{\mathrm{QCD}}): \quad G^{-1}_{\mathrm{gluon}} \sim (p^2)^{1+\kappa_A}, \quad G^{-1}_{\mathrm{ghost}} \sim (p^2)^{1+\kappa_C}$$

Perturbative Order-Parameter Potential

▷ “improved” potential

$$V_{\text{impr}}(A_0) = \frac{1}{2}\,\text{Tr}_{\text{cL}x}\ln\,G^{-1}_{\text{gluon}}[A_0] - \text{Tr}_{\text{c}x}\ln\,G^{-1}_{\text{ghost}}[A_0]$$

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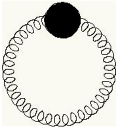

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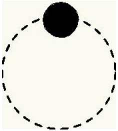

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▷ A_0 background

$$\ln(-D^2[A_0])^{1+\kappa} = (1+\kappa)\ln(-D^2[A_0])$$

Order-Parameter Potential

▷ low-energy effective potential (Braun,HG,Pawlowski'07)

$$V_{\mathrm{IR}}(A_0) \simeq \left\{ \frac{d-1}{2}(1+\kappa_A) + \frac{1}{2} - (1+\kappa_C) \right\} \frac{1}{\Omega} \mathrm{Tr} \ln \left(-D^2[A_0] \right)$$

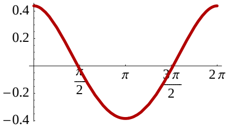
ORDER-PARAMETER POTENTIAL

▷ low-energy effective potential (BRAUN,HG,PAWLOWSKI'07)

$$V_{\mathrm{IR}}(A_0) = \left\{ \underbrace{\frac{d-1}{2}(1+\kappa_A)}_{\text{transv. gluons}} + \underbrace{\frac{1}{2}}_{\text{long. gluons}} \underbrace{-(1+\kappa_C)}_{\text{ghosts}} \right\} \underbrace{\frac{1}{\Omega} \mathrm{Tr} \ln \left(- D^2[A_0] \right)}_{\sim V_{\mathrm{pert}}}$$

▷ confinement criterion (Landau gauge)

$$d - 2 + (d-1)\kappa_A - 2\kappa_C < 0$$



▷ $d = 4$:

$$3\kappa_A - 2\kappa_C < -2$$

▷ quark confinement induced by:

IR gluon suppression and/or ghost enhancement

Confinement Criterion

(Taylor'71)

(Zwanziger'02; Lerche, von Smekal'02)

(Schleifenbaum, Maas, Wambach, Alkofer'05)

(Cucchieri, Maas, Mendes'08)

▷ Landau-gauge sum rule

$$0 = \kappa_A + 2\kappa_C - \frac{d-4}{2}$$

▷ quark confinement (Braun, HG, Pawlowski'07)

$$\kappa \equiv \kappa_C > \frac{d-3}{4}$$

▷ $d = 4$:

$$\Longrightarrow \quad \kappa > \frac{1}{4}$$

Confinement Criterion ($d = 4$)

▷ quark confinement (Braun,HG,Pawlowski'07)

$$\kappa > \frac{1}{4}$$

▷ Kugo-Ojima color confinement (Kugo,Ojima'79)

$$\kappa > 0$$

▷ Gribov-Zwanziger color confinement (Gribov'78, Zwanziger'94,'04)

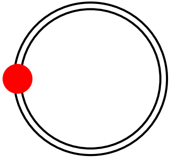
$$\kappa > \frac{1}{2} \quad \text{(horizon condition)}$$

FUNCTIONAL RG

Functional RG Flow Equation

$$\partial_t \Gamma_k \equiv k \partial_k \Gamma_k \;=\; \frac{1}{2}\,\mathrm{Tr}\,\frac{1}{\Gamma_k^{(2)} + R_k}\,\partial_t R_k$$

$$=$$

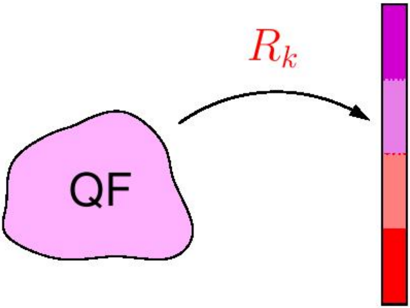


(Wilson'71; Wegner&Houghton'73; Polchinski'84; Wetterich'93)

Functional RG Flow Equation

$$\partial_t \Gamma_k = \frac{1}{2} \mathrm{Tr} \frac{1}{\Gamma_k^{(2)} + R_k} \partial_t R_k$$

▷ quantum fluctuations:



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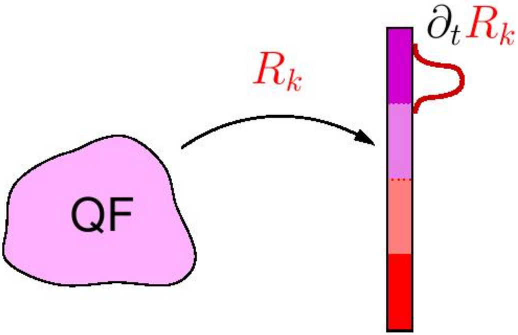

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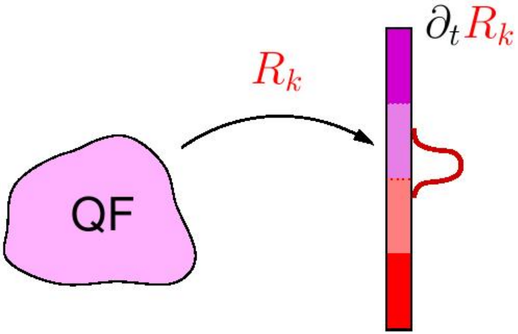

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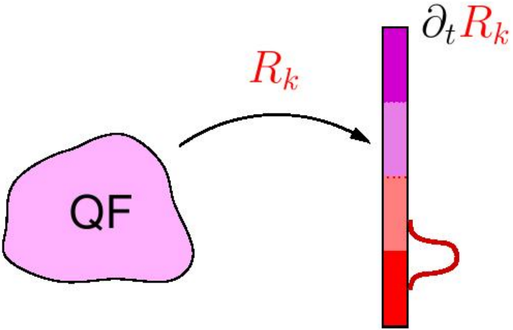
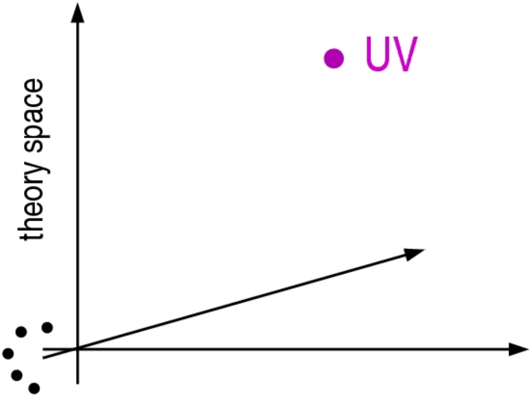


Functional RG Flow Equation

$$\partial_t \Gamma_k = \frac{1}{2}\,\mathrm{Tr}\,\frac{1}{\Gamma_k^{(2)} + R_k}\,\partial_t R_k$$

- RG trajectory: $\Gamma_{k=\Lambda} = S_{\text{bare}} = \int \frac{1}{4} F^z_{\mu\nu} F^z_{\mu\nu} + \bar{\psi}\,(\mathrm{i}\partial\!\!\!/ + gA\!\!\!/)\,\psi$



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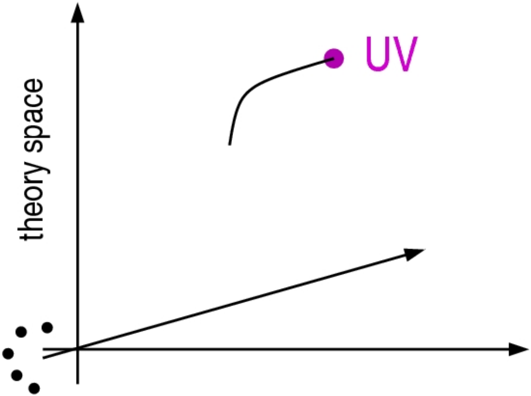

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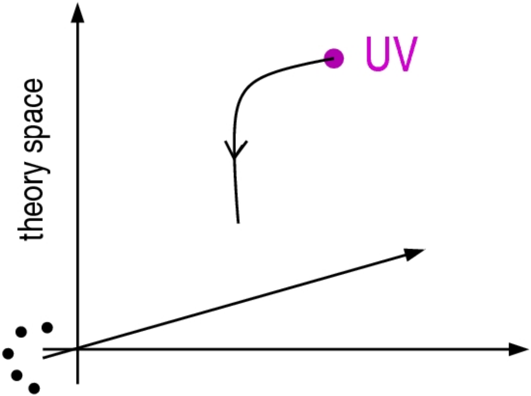

Functional RG Flow Equation

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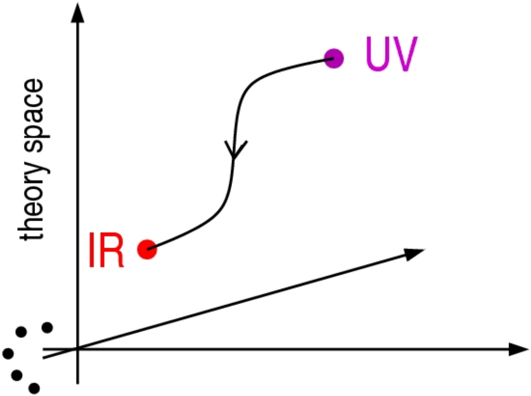
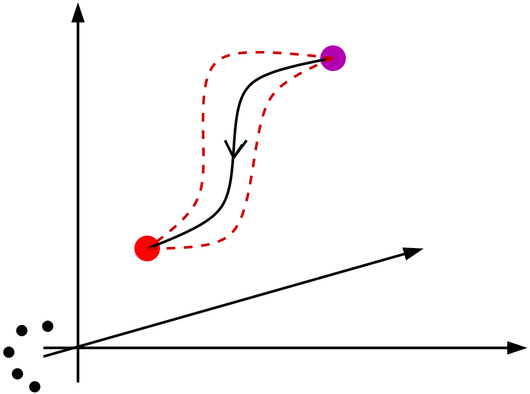
▷ RG trajectory: $\Gamma_{k \to 0} = \Gamma$



Functional RG Flow Equation

$$\partial_t \Gamma_k = \frac{1}{2} \, \mathrm{Tr} \, \frac{1}{\Gamma_k^{(2)} + R_k} \, \partial_t R_k$$

▷ RG trajectory: R_k scheme independence



Functional RG Flow Equation

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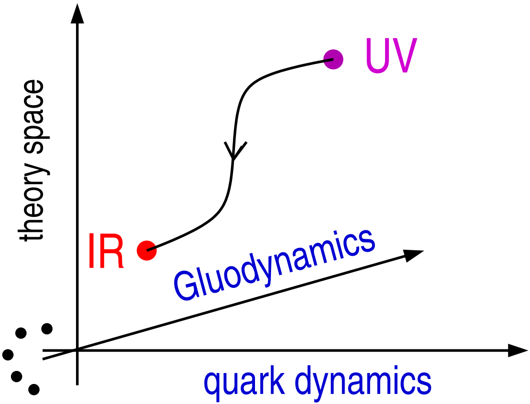
▷ RG trajectory: truncation



Functional RG Flow Equation

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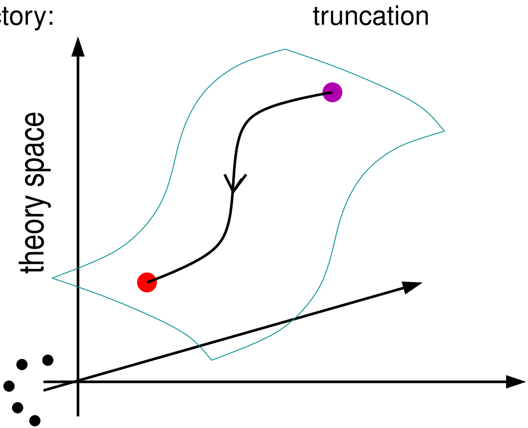
- RG trajectory:



ORDER PARAMETER POTENTIAL FROM FUNCTIONAL RG

- flow equation

$$\partial_t \Gamma_k = \frac{1}{2}\,\mathrm{Tr}\;\partial_t R_k [\Gamma_k^{(2)} + R_k]^{-1}$$

$$\Longrightarrow \quad \Gamma = \frac{1}{2}\mathrm{Tr}\ln\Gamma^{(2)} - \frac{1}{2}\int_0^\infty \frac{dk}{k}\,\mathrm{Tr}\,\frac{1}{\Gamma_k^{(2)} + R_k}\,\partial_t\Gamma_k^{(2)} + \text{c.t.}$$

- A_0 potential:

$$\Gamma[A_0] = \int d^d x\; V(A_0) + Z(A_0)\partial_\mu A_0 \partial_\mu A_0 \ldots$$

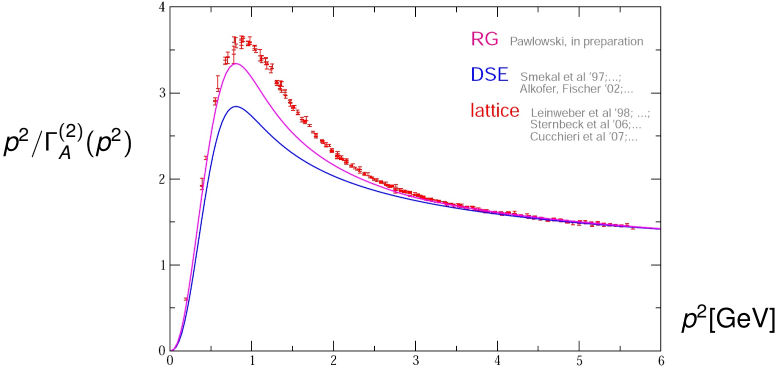
- $G \cdot \Gamma^{(2)} \equiv \mathbb{1}$

$$V(A_0) = \frac{1}{2\Omega}\,\mathrm{Tr}\;\ln\;\; G^{-1} + \mathcal{O}(\partial_t \Gamma_k^{(2)})$$

Deconfinement Phase Transition

▷ INPUT:

$$\text{Landau-gauge } \Gamma^{(2)} \equiv G^{-1}\Big|_{T=0}$$



[Pawlowski@DeltaMeeting'07]

Deconfinement Phase Transition

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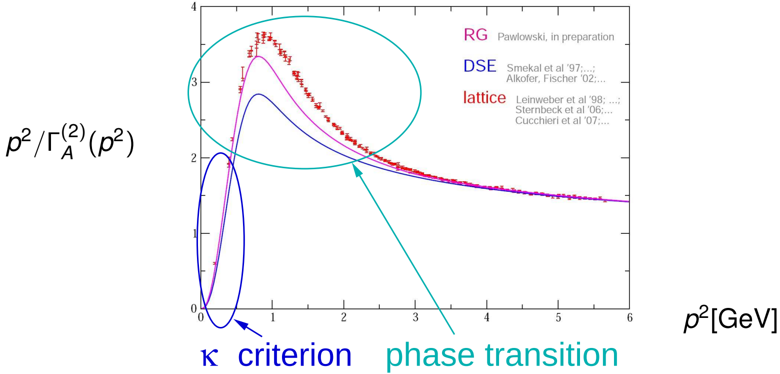
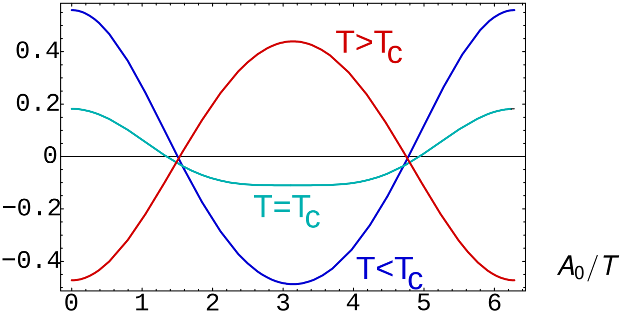


$\Longrightarrow$ mid-momentum regime is decisive

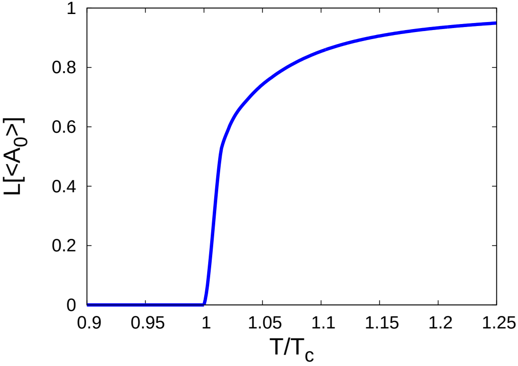
Deconfinement Phase Transition

▷ SU(2) A_0 potential (Braun,HG,Pawlowski'07)



Deconfinement Phase Transition

▷ SU(2): 2nd order phase transition (Braun,HG,Pawlowski'07)

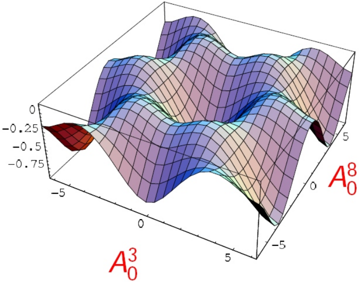


$T_c/\sqrt{\sigma} = 0.614 \pm 0.023$, cf. lattice: $T_c/\sqrt{\sigma} \simeq 0.709$

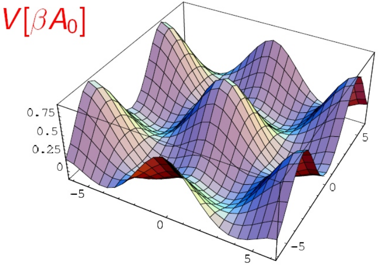
(Kaczmarek et al.'02)

DECONFINEMENT PHASE TRANSITION

▷ SU(3) A_0 potential (BRAUN,HG,PAWLOWSKI'07)



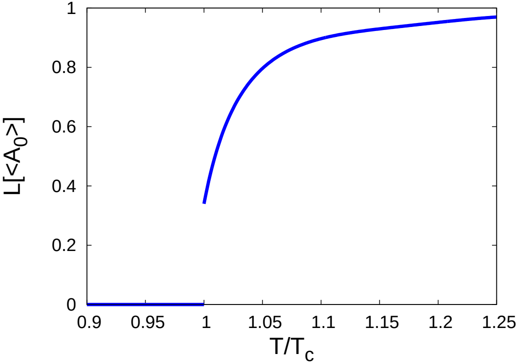
$T > T_c$



$T < T_c$

Deconfinement Phase Transition

▷ SU(3): 1st order phase transition (Braun,HG,Pawlowski'07)



$T_c/\sqrt{\sigma} = 0.646 \pm 0.023 \quad \Longrightarrow \quad T_c \simeq 284\text{MeV}, \qquad$ cf. Lattice: $T_c/\sqrt{\sigma} \simeq 0.646$

(Kaczmarek et al.'02)

Error Estimate

$$\Gamma = \frac{1}{2}\mathrm{Tr}\ln\Gamma^{(2)} - \frac{1}{2}\int_0^\infty \frac{dk}{k}\,\mathrm{Tr}\,\frac{1}{\Gamma_k^{(2)} + R_k}\,\partial_t\Gamma_k^{(2)}$$

lattice (mid momentum!)
@ T=0

RG flow

▷ A_0 fluctuations neglected: $\Gamma_k^{(2)} \sim -\partial^2 + V_k''(A_0)$

relevant → SU(2): Ising universality class

less relevant → SU(3): 1st order

RG Flow towards the Chiral Transition

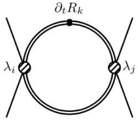
RG Flow of the Chiral Sector

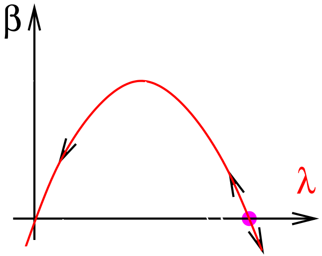
▷ effective action:

$$\Gamma_k = \int \frac{1}{2}\frac{\lambda_\sigma}{k^2}\left[(\bar\psi^a\psi^b)^2 - (\bar\psi^a\gamma_5\psi^b)^2\right]$$

▷ RG flow

$$\partial_t\lambda_\sigma = 2\lambda_\sigma - \frac{N_c}{4\pi^2}\,{\lambda_\sigma}^2$$





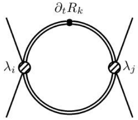
RG Flow of the Chiral Sector

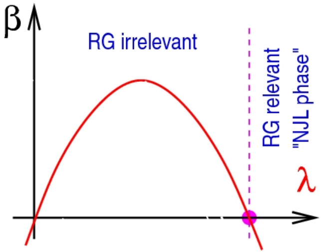
▷ effective action:

$$\Gamma_k = \int \frac{1}{2} \frac{\lambda_\sigma}{k^2} \left[(\bar{\psi}^a \psi^b)^2 - (\bar{\psi}^a \gamma_5 \psi^b)^2 \right]$$

▷ RG flow

$$\partial_t \lambda_\sigma = 2\lambda_\sigma - \frac{N_c}{4\pi^2} \lambda_\sigma{}^2$$





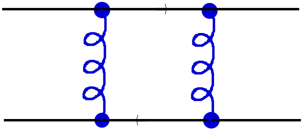
RG Flow of the Chiral Sector

▷ effective action:

$$\Gamma_k = \int \frac{1}{4} F^z_{\mu\nu} F^z_{\mu\nu} + \ldots + \bar{\psi}\,(\mathrm{i}\partial\!\!\!/ + \bar{g}A\!\!\!/)\,\psi + \frac{1}{2}\frac{\lambda_\sigma}{k^2}\left[(\bar{\psi}^a\psi^b)^2 - (\bar{\psi}^a\gamma_5\psi^b)^2\right]$$

▷ RG flow

$$\partial_t \lambda_\sigma = 2\lambda_\sigma - \frac{N_c}{4\pi^2}\,{\lambda_\sigma}^2 - \frac{3}{8\pi^2}\,\frac{{N_c}^2 - 1}{N_c}\,g^2\lambda_\sigma - \frac{9}{256\pi^2}\,\frac{3{N_c}^2 - 8}{N_c}\,g^4$$



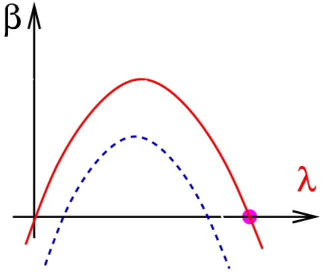
RG Flow of the Chiral Sector

▷ effective action:

$$\Gamma_k = \int \frac{1}{4} F^z_{\mu\nu} F^z_{\mu\nu} + \ldots + \bar\psi\,(\mathrm{i}\partial\!\!\!/ + \bar g A\!\!\!/)\,\psi + \frac{1}{2}\frac{\lambda_\sigma}{k^2}\left[(\bar\psi^a\psi^b)^2 - (\bar\psi^a\gamma_5\psi^b)^2\right]$$

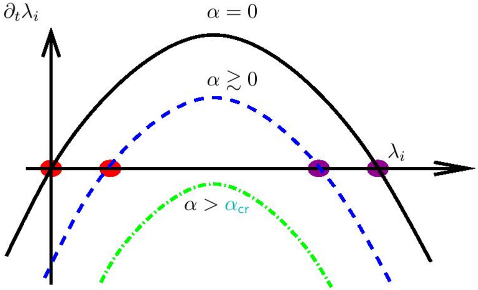
▷ RG flow

$$\partial_t \lambda_\sigma = 2\lambda_\sigma - \frac{N_c}{4\pi^2}\lambda_\sigma{}^2 - \frac{3}{8\pi^2}\frac{N_c{}^2-1}{N_c}g^2\lambda_\sigma - \frac{9}{256\pi^2}\frac{3N_c{}^2-8}{N_c}g^4$$



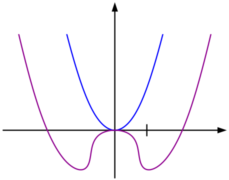
CHIRAL CRITICALITY

▷ critical gauge coupling α_{cr}:



$\Longrightarrow$ bosonization $\rightarrow \chi$SB:

if $\alpha > \alpha_{\text{cr}}$: $\lambda \sim \dfrac{1}{m_\phi^2} \rightarrow \infty$



[TALK BY B.-J. SCHAEFER]

RG Flow of the Chiral Sector

▷ effective action: SU(N_c), $\mathrm{SU}(N_f)_L \times \mathrm{SU}(N_f)_R$

$$\Gamma_k = \int \frac{Z_F}{4} F^z_{\mu\nu} F^z_{\mu\nu} + \ldots + \bar\psi \left(\mathrm{i} Z_\psi \partial\!\!\!/ + Z_1 \bar g A\!\!\!/ \right) \psi$$
$$+ \frac{1}{2} \frac{\lambda_\sigma}{k^2} (\text{S-P}) + \frac{1}{2} \frac{\lambda_{VA}}{k^2} \left[2(\text{V-A})^{\text{adj.}} + (1/N_c)(\text{V-A}) \right]$$
$$+ \frac{1}{2} \frac{\lambda_+}{k^2} (\text{V+A}) + \frac{1}{2} \frac{\lambda_-}{k^2} (\text{V-A})$$

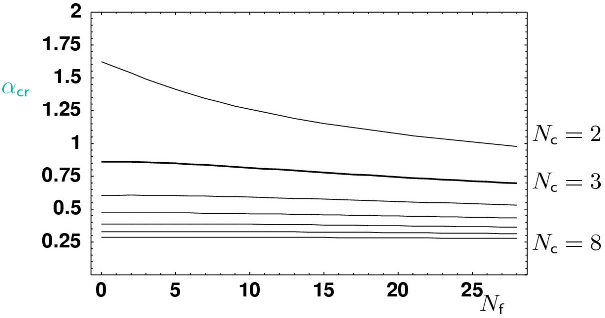
▷ RG flow, e.g.,

$$\partial_t \lambda_\sigma = 2\lambda_\sigma - \frac{1}{4\pi^2} l_1^{(F)}[R_k] \left\{ 2N_c \lambda_\sigma{}^2 - 2\lambda_- \lambda_\sigma - 2N_f \lambda_\sigma \lambda_{VA} - 6\lambda_+ \lambda_\sigma \right\}$$
$$- \frac{1}{8\pi^2} l_{1,1}^{(FB)}[R_k] \left[3 \frac{N_c{}^2 - 1}{N_c} g^2 \lambda_\sigma - 6 g^2 \lambda_+ \right]$$
$$- \frac{3}{128\pi^2} l_{1,2}^{(FB)}[R_k] \frac{3N_c{}^2 - 8}{N_c} g^4$$

(HG, Jaeckel, Wetterich'04)

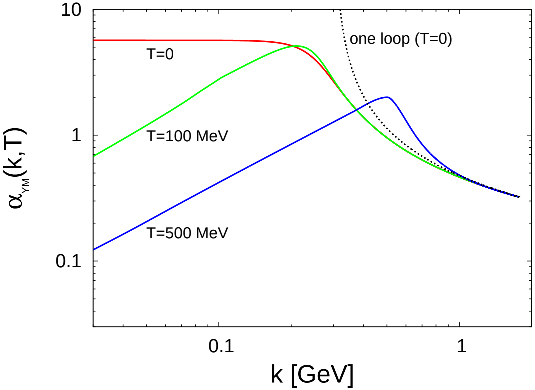
χSB Critical Coupling

(HG, Jaeckel'05)



e.g., for $N_c = 3 = N_f$: $\alpha_{cr} \simeq 0.85$

Running Gauge Coupling at finite T



Background gauge:

(HG'02)

(Braun,HG'05)

cf. Landau gauge:

(v.Smekal,Alkofer,Hauck'97)

(Leinweber et al'98)

(Lerche,v.Smekal'02)

(Fischer,Alkofer'02)

(Zwanziger'02)

(Pawlowski,Litim,Nedelko,v.Smekal'03)

(Fischer, HG'04)

(Oliveira,Silva'04)

(Bloch,Cucchieri,Langfeld,Mendes'04)

(Sternbeck et al.'06)

(Maas'07)

(Cucchieri,Mendes,Oliveira,Silva'07)

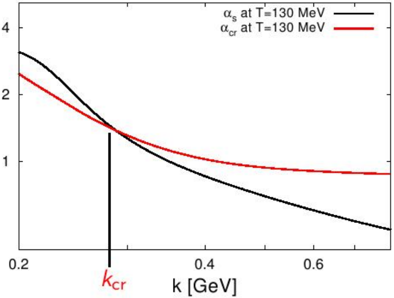
▷ $T/k \to \infty$: strongly interacting 3D theory

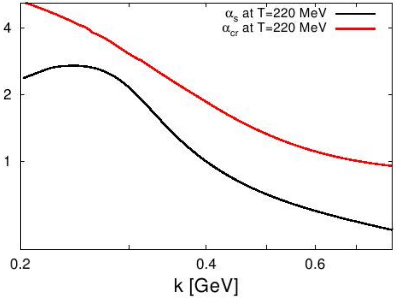
$$\alpha \to \frac{k}{T}\,\alpha_{3D}, \quad \alpha_{3D} \to \alpha_{3D,*} \simeq 2.7$$

cf. lattice: (Cucchieri,Maas,Mendes'07)

CHIRAL PHASE TRANSITION

▷ $\alpha(k, T)$ vs. $\alpha_{cr}(T/k)$





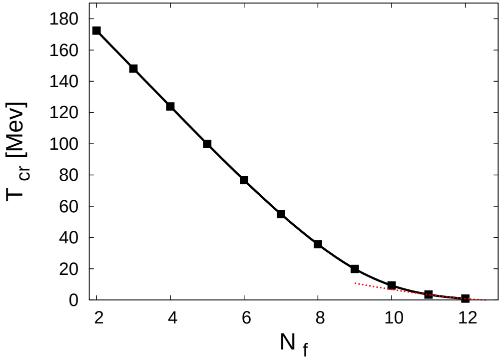
$\Longrightarrow$ χSB triggered by α_s

single input: $\alpha_s(m_\tau) = 0.322$

T_c [MeV]	RG (BRAUN,HG'05)
N_f=2	172 ± 37
N_f=3	148 ± 32

T_c [MeV]	Lattice (BI) (CHEN ET AL.'06)	Lattice (W) (AOKI ET AL.'06)
N_f=2+1	192(7)(4)	151(3)(3)

CHIRAL PHASE BOUNDARY $T - N_F$

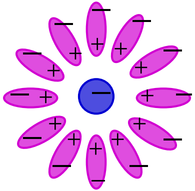


(BRAUN,HG'05,'06)

▷ small N_f: fermionic screening, $\beta_{\text{quark}} \simeq \frac{2}{3} N_f \frac{g^4}{8\pi^2}$

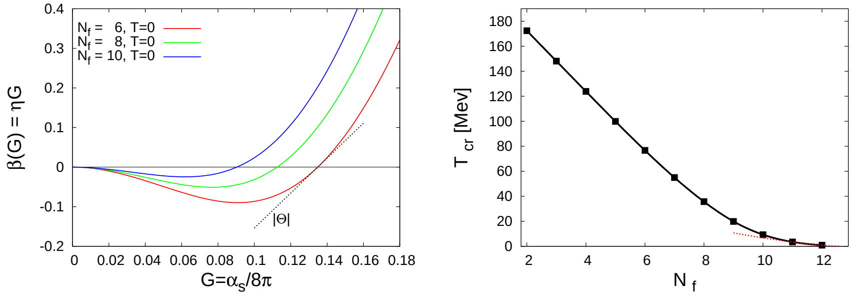
▷ critical flavor number:

$$N_f^{cr} \simeq 12$$



(CF. APPELQUIST ET AL.'96; MIRANSKI,YAMAWAKI'96; HG,JAECKEL'05)

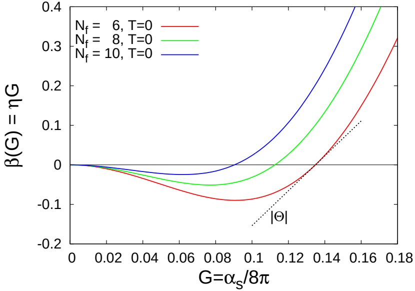
CHIRAL PHASE BOUNDARY $T - N_F$

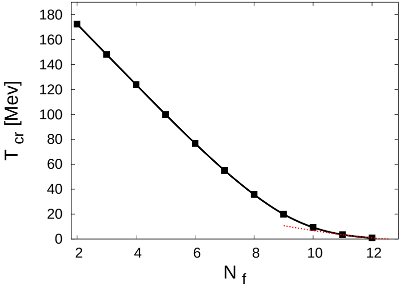


▷ fixed-point regime: critical exponent Θ

$$\beta_{g^2} \simeq -\Theta\,(g^2 - g_*^2)$$

CHIRAL PHASE BOUNDARY $T - N_F$





▷ fixed-point regime: critical exponent Θ

$$\beta_{g^2} \simeq -\Theta\,(g^2 - g_*^2)$$

▷ shape of the phase boundary for $N_f \simeq N_f^{cr}$: (BRAUN,HG'05,'06)

$$T_{cr} \sim k_0\,|N_f - N_f^{cr}|^{\frac{1}{|\Theta|}}, \quad \Theta \simeq -0.71$$

Conclusions

▷ A_0 potential: quark confinement from color confinement

$$3\kappa_A - 2\kappa_C < -2$$

quark confinement from IR gluon suppression / ghost enhancement

▷ functional RG for $\Gamma[\phi]$

- systematic and consistent expansion schemes for QCD
- chiral symmetry ✓
- calculations "from first principles"

▷ Many-flavor QCD: relation among universal aspects:

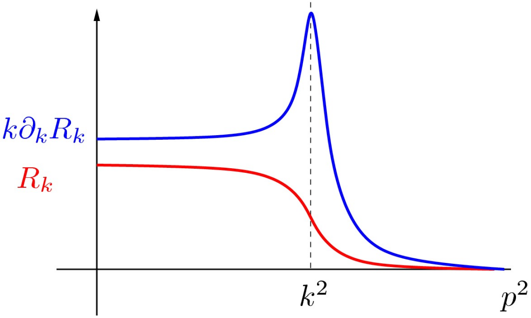
shape of the phase boundary $\Longleftrightarrow$ IR critical exponent

Appendix

Functional RG Flow Equation

$$\partial_t \Gamma_k = \frac{1}{2} \, \mathrm{Tr} \, \frac{1}{\Gamma_k^{(2)} + R_k} \, \partial_t R_k$$

- regulator

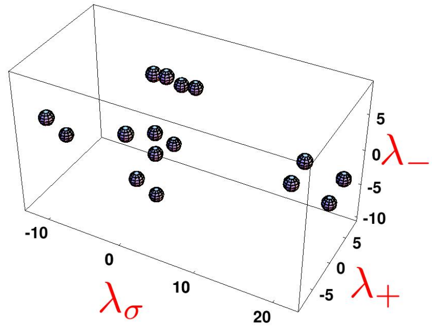


RG Flow of the Chiral Sector

▷ 2 fixed points per $\partial_t \lambda$

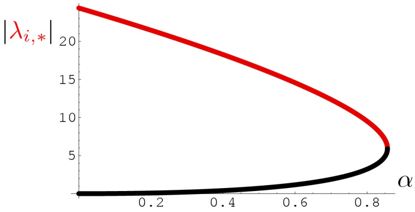
$\Longrightarrow$ $2^4 = 16$ fixed points

▷ in general: 2^n FP's
for $n = \#$ of λ's



▷ fixed-point annihilation

e.g., $N_c = N_f = 3$



CHIRAL CRITICALITY AT FINITE TEMPERATURE

▷ quark modes:

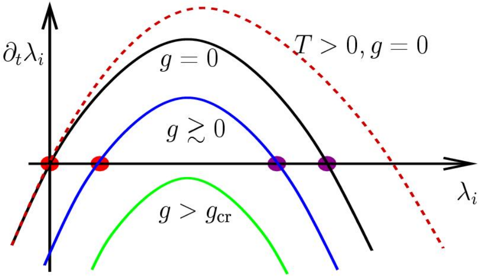
$$m_T^2 = m_f^2 + (2\pi T(n + \frac{1}{2}))^2$$

$\Longrightarrow$ T-dependent

critical coupling:

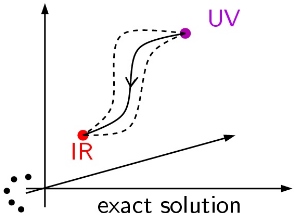
$$\alpha_{cr}(T) \gtrsim \alpha_{cr} \simeq 0.85$$

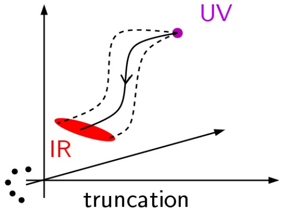
(BRAUN,HG'05)



ERROR ESTIMATE

▷ regulator dependence





▷ fermion sector: "optimized" regulator vs. "sharp cutoff" (LITIM'01)

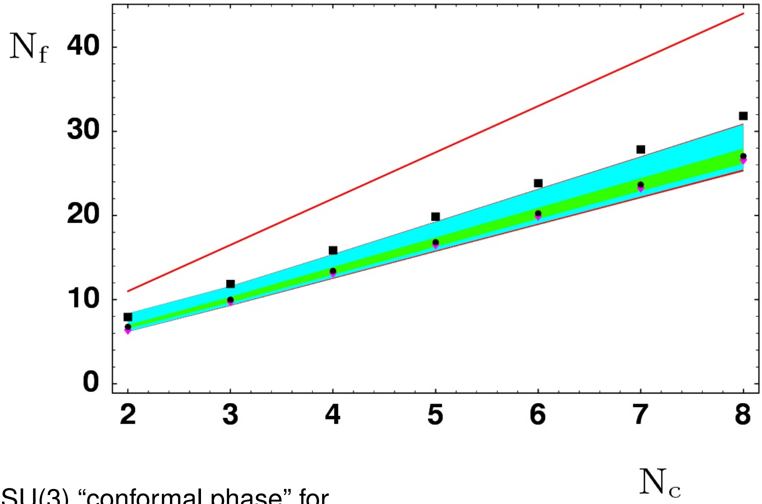
$$l_1^{(\mathrm{F}),4} = \frac{1}{2},\ l_{1,1}^{(\mathrm{FB}),4} = 1,\ l_{1,2}^{(\mathrm{FB}),4} = \frac{3}{2} \qquad \text{vs.} \qquad l_1^{(\mathrm{F}),4} = l_{1,1}^{(\mathrm{FB}),4} = l_{1,2}^{(\mathrm{FB}),4} = 1$$

▷ anomalous dimensions, momentum dependencies, higher-order operators $\sim \psi^8$, etc. ...

▷ gauge sector: 2-loop, 3-loop, 4-loop β function

$\overline{\mathrm{MS}}$ scheme vs. RG scheme (~10, 30, 50 % variation (?))

χSB Critical Coupling



▷ SU(3) "conformal phase" for

$$N_{f,cr} = 10.0 \pm 0.29(\text{fermion})\,^{+1.55}_{-0.63}(\text{gluon}) \lesssim N_f < 16.5$$

(HG,Jaeckel'05)

Lessons to be learned for “real QCD”

- fermionic screening is rather weak
- fermionic truncation (surprisingly) stable in χ symmetric phase
- phase boundary detectable with fermionic “derivative expansion”
- “real QCD” requires nonperturbative estimate of β_{g^2}