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I Introduction

Thermodynamics and phase structure of QCD are fundamental issues that test our understanding of gauge theories.

They are of intense present interest because of experimental results from RHIC.

⟹ High T is NOT a weakly interacting quark-gluon plasma.

For simplicity I discuss pure gluodynamics.

$\mu = 0$.

II Thermodynamics

$$Z = \mathrm{tr}\, e^{-\beta H} = \int d\Phi \, e^{-S_E}$$

$$\Phi = (A_\mu, c, \bar{c}, b)$$

$$S_E = \int d^4x \, \mathcal{L}_E$$

$$0 \le x_0 \le \beta = 1/T$$

free energy density $\quad w = \frac{1}{V} \ln Z$

pressure $\quad p = w/\beta$

energy density $\quad e = -\partial w/\partial\beta$

anomaly $\quad a \equiv e - 3p$

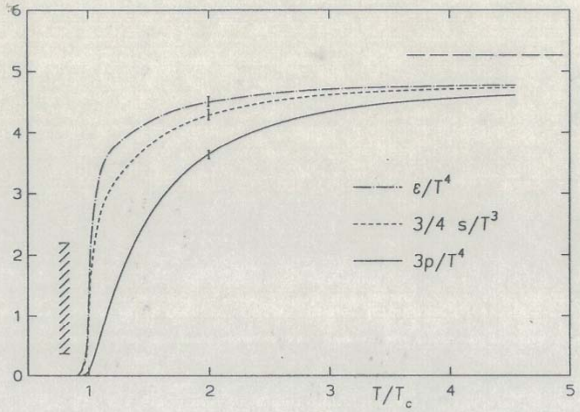


Figure 7: Extrapolation to the continuum limit for the energy density, entropy density and pressure versus T/T_c. The dashed horizontal line shows the ideal gas limit. The hatched vertical band indicates the size of the discontinuity in ϵ/T^4 (latent heat) at T_c [15]. Typical error bars are shown for all curves.

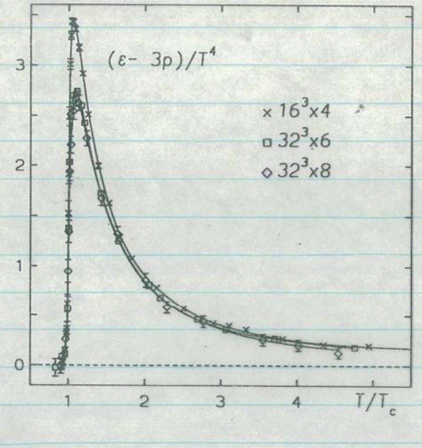


Figure 5: The difference $(\epsilon - 3p)/T^4$.

Evidence of phase transition

$$T_c = 0.629\,\sigma^{1/2} = 0.629 \times 440\,\mathrm{MeV}$$

$$= 276\ \mathrm{MeV}$$

Latent heat per unit volume

$$\approx 3 \times T_c^4$$

IV High T limit and corrections

At strict high-T limit phase approaches Stefan-Boltzmann limit. This suggests that we should be able to calculate the equation of state at high T as a weakly interacting gas of gluons especially because at high T

$$\lim_{T \to \infty} g(T) \to 0$$

by asymptotic freedom.

Let's try and see what happens!

$$\frac{w}{T^3} = \frac{p}{T^4}$$

$$= (N^2-1)\frac{\pi^2}{45}\Big[1 + a_2 g^2 + a_3 g^3$$

$$+ a_4 g^4 + a_5 g^5$$

$$+ (\cdots \ln^2 \cdots + \cdots \ln \cdots + c) g^6 + \cdots\Big]$$

$$g^2 = g^2(T) = \frac{24\pi^2}{11 N \ln\left(\frac{\mu(T)}{\Lambda_{\overline{MS}}}\right)} \longrightarrow 0$$

$$\mu(T) \longrightarrow 2\pi T$$

Terms non-analytic in g^2 come from resummation of perturbation theory.

But disaster: c is not calculable!!! A. Linde (1980)

I High-T disaster!

Due to Failure of perturbation theory at high T.

At finite T get sum over Matsubara frequencies

$$\int \frac{d^4k}{(2\pi)^4} \longrightarrow T \sum_n \int \frac{d^3k}{(2\pi)^3}$$

$$\text{gluon propagator} = \frac{1}{(2\pi T n)^2 + \vec{k}^2}$$

because $0 \leq x_0 \leq \beta = \frac{1}{T}$.

Matsubara frequency $n=0$ has infrared divergences of 3D gauge theory! ? ? ?

? This is a confining, non-perturbative ? gauge theory. ? ?

Why is disaster of order g^6 ?

$$S_E = \int d^3x \int^{1/T} dx_0 \, \frac{1}{g^2} \frac{1}{4} F_{\mu\nu}^2$$

$$\rightsquigarrow \qquad (T \to \infty)$$

$$\int d^3x \, \frac{1}{g^2 T} \frac{1}{4} F_{ij}^2 + \cdots$$

$$g_3^2 = g^2 T \qquad\qquad [g_3^2] = 1$$

$$e^{W(T)} = \int dA \, e^{-S_E}$$

$$\rightsquigarrow \int dA \, e^{-\int d^3x \frac{1}{g_3^2} F_{ij}^2 \frac{1}{4}}$$

$$= e^{W_3}$$

$$W(T) \sim W_3 = \text{const} \left(g_3^2\right)^3 V_3$$

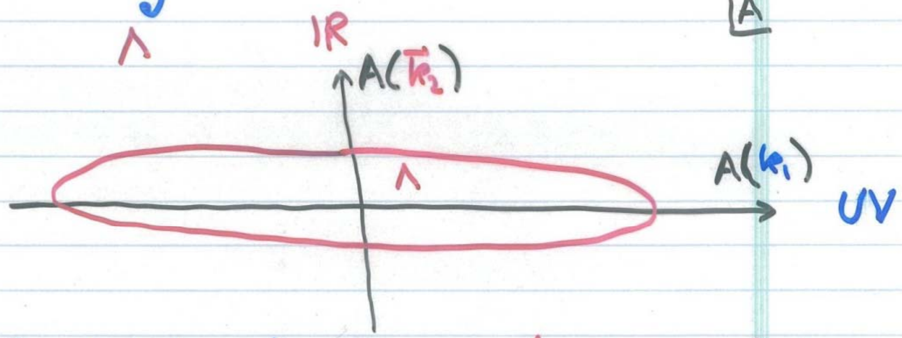
by dimensional analysis

$$= \text{const} \, (g^2 T)^3$$

const NOT perturbatively calculable.

VI Gribov to the Rescue!

$$e^{W} = \int_{\Lambda} dA\, e^{-S_E}$$



Must integrate only over fundamental modular region Λ. (Region free of Gribov copies)

Proximity of boundary of Λ in infrared (IR) direction suppresses infrared components $A(\vec{k}_2)$.

We are rescued from Linde disaster!!!

The Gribov region is the domain in A-space where the Faddeev-Popov operator

$$M^{ac}(A) = -\partial_i^2 \delta^{ac} - g f^{abc} A_i^b \partial_i$$

has all eigenvalues positive.

It is characterized by the positivity of the horizon function

$$H(A) = \int d^{D-1}x \; h(x)$$

$$h(x) = g f^{abc} A_i^b \left[(M^{-1})^{ad} g f^{dec} A_i^e \right] - (N^2-1)(D-1)$$

VI Gribov to the Rescue! (cont.)

Cut-off of functional integral at Gribov horizon suppresses infrared components of gauge field.

Gribov (1978) gluon dispersion relation

$$E(\vec{k}) = \sqrt{\vec{k}^2 + \frac{m^4}{\vec{k}^2}}$$

What is m? Answer:

$$m(T) = \frac{N}{2^{3/2}\, 3\pi} \underbrace{g^2(T)\, T}_{\text{magnetic mass}}$$

D. Z. hep-ph/0610021 v3

VII Cut-off of functional integral at Gribov horizon is implemented by local, renormalizable action and gap eq.

$$e^{W} = \int dA \, e^{-S_E}$$

$\wedge$

$$= \int dA \, \Theta(-H) \, e^{-S_E} \qquad H = H(A)$$

H is non-local "horizon function"

$$= \int dA \, e^{-S_E - m^4 H}$$

Gap equation

$$\frac{\partial W}{\partial m} = 0 \iff \langle H \rangle = 0$$

$$e^{W} = \int dA \, d\varphi \, d\bar{\varphi} \ldots \, e^{-S_{local}}$$

$$S_{local} = S_E(A) + S_H(A, \varphi, \bar{\varphi} \ldots ; m)$$

Once this action is given <u>nothing</u> is ad hoc!

Form of local action + gap equation

$$\mathcal{L} = \frac{1}{4} F_{\mu\nu}^2 + \partial_i \bar{c} D_i c + i \partial_i b A_i$$

$$+ \partial_i \bar{\varphi}_j D_i \varphi_j + \cdots$$ (auxiliary bose ghosts + fermi ghosts)

$$+ m^2 A_i (\bar{\varphi}_i - \varphi_i)$$ mixing term, m = Gribov mass

$$- \frac{m^4}{2Ng^2} (D-1)(N^2-1)$$ const.

$$S_{loc} = \int d^3x \int_0^{1/T} dx_0 \, \mathcal{L}$$

$$e^{W(T,m)} = \int dA \, e^{-S_{loc}}$$

Gap equation

$$\frac{\partial W(T,m)}{\partial m} = 0$$

$\underset{\text{high } T}{\Longrightarrow}$ $$m(T) = \frac{N}{2^{1/2} \, 3\pi} g^2(T) \, T$$

Gluon propagator

Because of mixing term

$$m^2 A_i (\varphi_i - \bar{\varphi}_i)$$

gluon propagator is given by

$$D(k) = \langle A(k) A(-k) \rangle$$

$$= \frac{1}{E^2(\vec{k}) + k_0^2} = \frac{1}{\vec{k}^2 + \frac{m^4}{\vec{k}^2} + k_0^2}$$

$$= \frac{\vec{k}^2}{(\vec{k}^2)^2 + m^4 + (2\pi T n)^2 \vec{k}^2}$$

Infrared suppression ($\vec{k}^2$ in numerator) results from proximity of Gribov horizon in infrared directions.

Gribov mass provides infrared cut-off for 0 Matsubara frequency $n=0$.

VIII Gap equation

$$\frac{\partial W}{\partial m} = \frac{\partial \Gamma}{\partial m} = 0$$

$$\langle A(\varphi - \bar{\varphi}) \rangle = \frac{m^2}{Ng^2}(D-1)(N^2-1)$$

$$\int \frac{d^3k}{(2\pi)^3}\, T \sum_{k_0} \frac{2}{(\vec{k}^2 + k_0^2)\,\vec{k}^2 + m^4} = \frac{3}{Ng^2}$$

$$k_0 = 2\pi T n$$

Solve for $m = m(T)$

$$g = g\left(\frac{\mu(T)}{\Lambda_{\overline{MS}}}\right)$$

IX Free energy W

$$e^{W} = \int d\Phi \; e^{-S_{-2}+S_0} = e^{W_{-2}+W_0}$$

$$W_{-2} = -S_{-2} = \frac{3m^4}{2Ng^2}(N^2-1)\frac{V}{T}$$

$$e^{W_0} = \int d\Phi \; e^{-S_0}$$

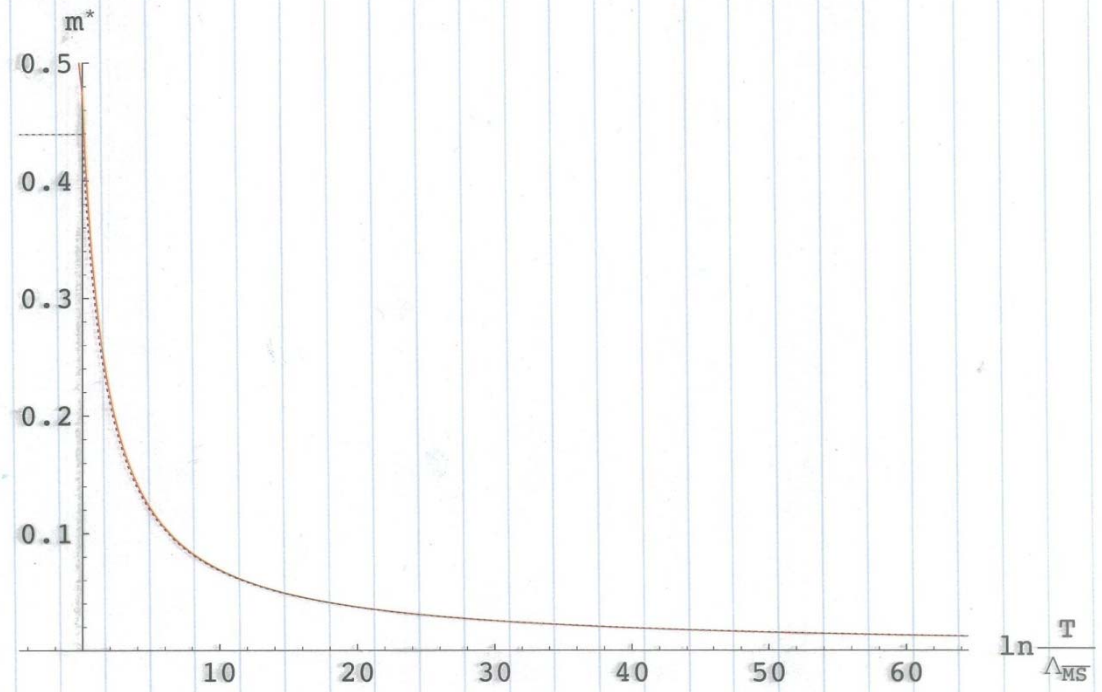
Planck free energy with

$$E(\bar{k}) = \sqrt{\bar{k}^2 + \frac{m(T)}{\bar{k}^2}}$$

free energy per unit volume

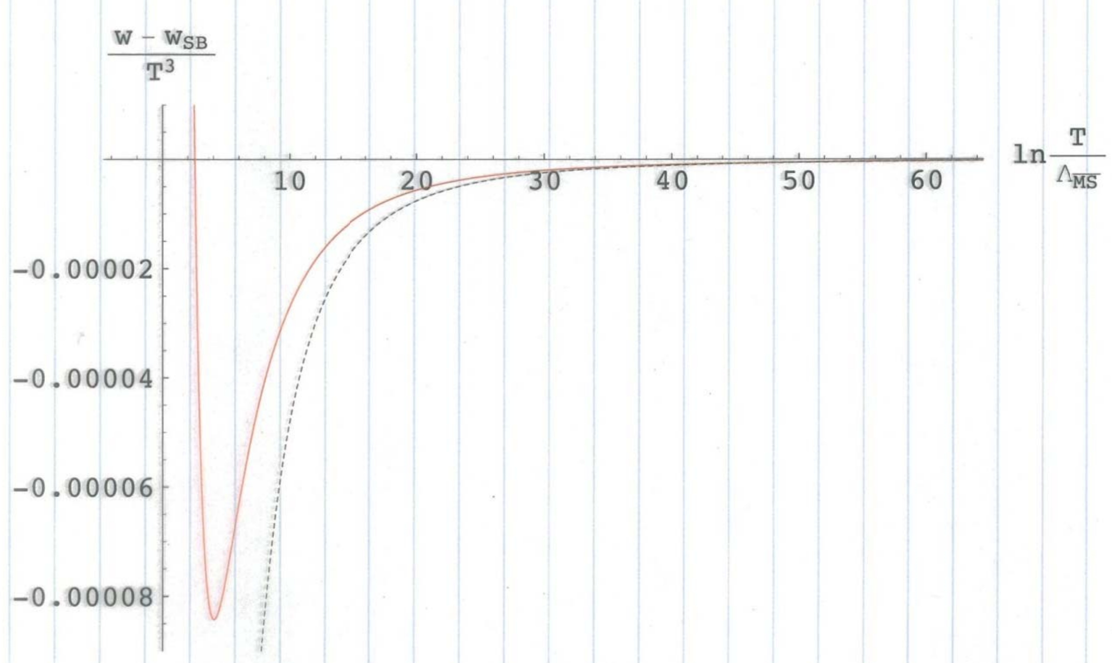
$$w = (N^2-1)\beta\left\{\frac{3m^4}{2Ng^2} + \frac{1}{3\pi^2}\int_0^\infty \frac{dk\,(k^4-m^4)}{E\left[\exp(E\beta)-1\right]}\right\}.$$

Gribov mass:



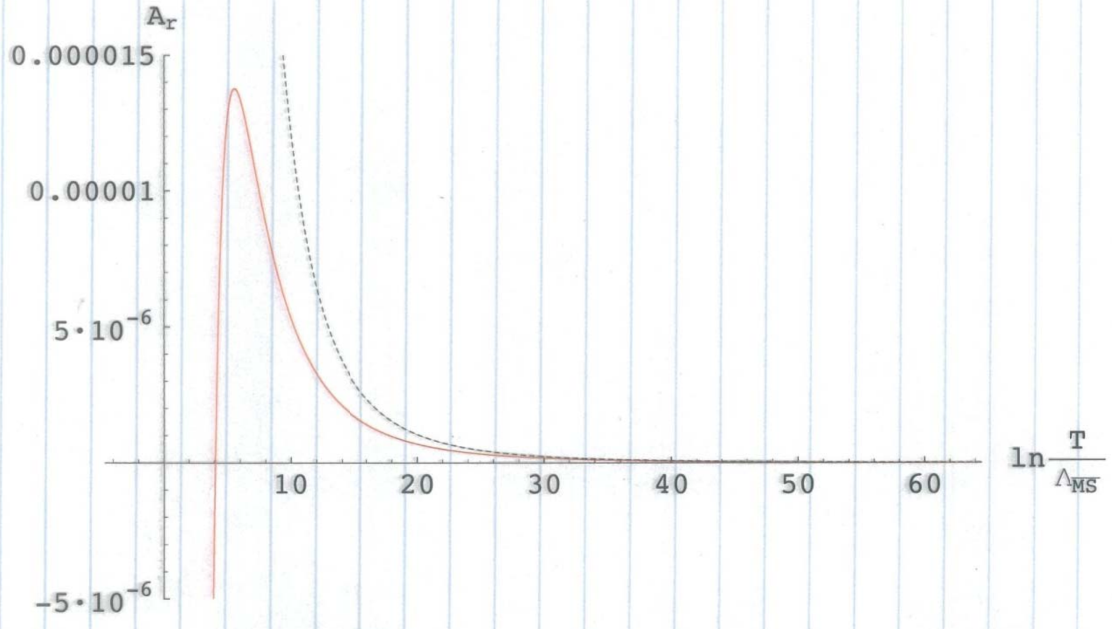
(numerical solution and asymptotic form $\sim g^2$)

rescaled free energy:



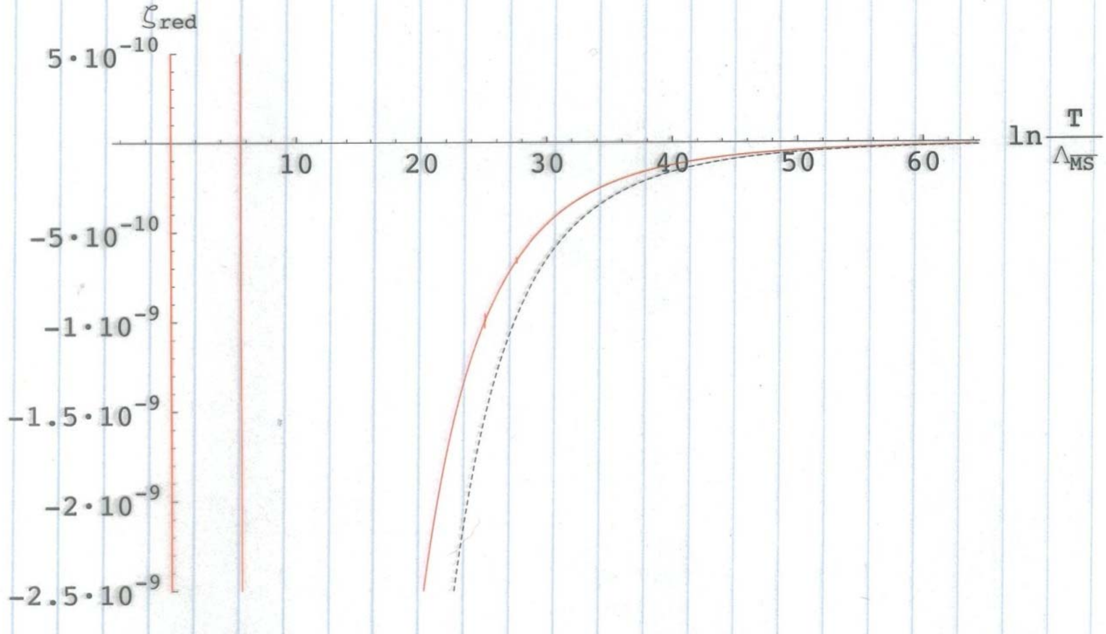
(numerical solution and asymptotic form $\sim g^6$)

rescaled anomaly:



(numerical solution and asymptotic form $\sim g^8$)

bulk viscosity:



(numerical solution and asymptotic form $\sim g^{10}$)