Getting an IR finite ghost propagator in the Feynman gauge

Arlene C. Aguilar

University of Valencia, Spain

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Based on:
A. C. Aguilar and J. Papavassiliou, arXiv:0712.0780 [hep-ph].
A. C. Aguilar and J. Papavassiliou, EPJA **35**, 189-205 (2008). arXiv:0708.4320 [hep-ph].

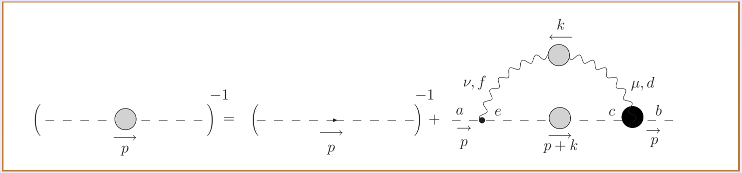
Motivation

- Nowadays lattice simulations give clean predictions about the non-perturbative behavior of QCD Green's functions.
- At the same time, various recent developments allow for a fruitful comparison between theory and lattice experiments.

A. C. Aguilar, D. Binosi, J. Papavassiliou, arXiv:0802.1870 [hep-ph]
A. C. Aguilar and J. Papavassiliou, JHEP **0612**, 012 (2006)
A. C. Aguilar and J. Papavassiliou, EPJA **35**, 189-205 (2008)
D. Dudal, S. P. Sorella, N. Vandersickel and H. Verschelde, 0711.4496 [hep-th].
M. A. L. Capri, V. E. R. Lemes, R. F. Sobreiro, S. P. Sorella, R. Thibes, 0801.0566 [hep-th]

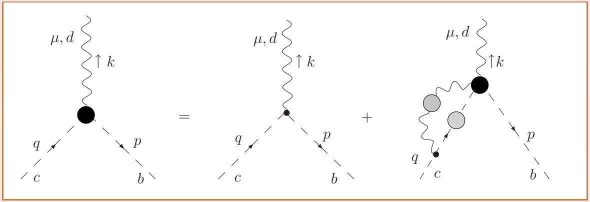
- It is essential to attack this problem from all possible angles.
- The ghost propapagator, one of basic quantities of the theory, has been extensively studied in Landau gauge.
- Study the IR behavior predicted by the Schwinger-Dyson equations for the ghost propagator in the Feynman gauge.

The ghost equation



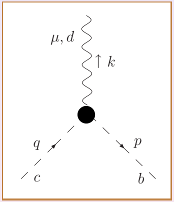
$$D^{-1}(p^2) = p^2 + iC_{\mathrm{A}}g^2 \int [dk]\,\Gamma^{\mu}\Delta_{\mu\nu}(k)\mathbb{\Gamma}^{\nu}(p, p+k, k)D(p+k)\,.$$

The SDE for the gluon-ghost vertex



Vertex decomposition

- Choose p_μ and k_μ as the two linearly independent four-vectors. The most generic fully dressed gluon-ghost vertex $\mathbb{\Gamma}^{bcd}_\nu(p,q,k)$ is expressed as



$$\mathbb{\Gamma}^{bcd}_\mu(p,q,k) \quad = \quad -gf^{bcd}\,\mathbb{\Gamma}_\mu(p,q,k)\,,$$

$$\mathbb{\Gamma}_\mu(p,q,k) \quad = \quad A(p^2,q^2,k^2)p_\mu + B(p^2,q^2,k^2)k_\mu\,,$$

The ghost equation

In the covariant gauges the full gluon propagator has the general form

$$\Delta_{\mu\nu}(k) = -i\left[\, g_{\mu\nu} - \frac{k_\mu k_\nu}{k^2}\right]\Delta(k^2) + \xi\,\frac{k_\mu k_\nu}{k^4}\,,$$

$$D^{-1}(p^2) = p^2 - C_{\rm A}\, g^2 \int [dk]\, \left[p^2 - \frac{(p\cdot k)^2}{k^2}\right] A(p^2,q^2,k^2)\, \Delta(k)\, D(p+k)$$
$$- C_{\rm A}\, g^2 \xi \int [dk]\, \frac{p\cdot k}{k^2} \left[A(p^2,q^2,k^2) + B(p^2,q^2,k^2) + \frac{p\cdot k}{k^2} A(p^2,q^2,k^2)\right]\, D(p+k)$$
$$- C_{\rm A}\, g^2 \xi \int [dk]\, B(p^2,q^2,k^2)\, D(p+k)$$

- Manifestly gauge-dependent quantity.

The ghost equation in Landau gauge

$$D^{-1}(p^2) = p^2 - C_A g^2 \int [dk] \left[p^2 - \frac{(p \cdot k)^2}{k^2} \right] A(p^2, q^2, k^2) \, \Delta(k) \, D(p+k)$$

- In the limit of $p \to 0$, $D^{-1}(0) = 0$ unless $A(p^2, q^2, k^2)$ has poles of the type $1/p^2$
- Lattice studies indicate that $A(p^2, q^2, k^2)$ contains no poles

  A. Cucchieri, T. Mendes and A. Mihara, JHEP **0412**, 012 (2004)
- Self-consistent with recents lattice results, showing a divergent ghost propagator in the Landau gauge

P. O. Bowman et al.,arXiv:hep-lat/0703022
A. Cucchieri and T. Mendes, arXiv:0710.0412 [hep-lat].
I. L. Bogolubsky, E. M. Ilgenfritz, M. Muller-Preussker and A. Sternbeck, arXiv:0710.1968 [hep-lat].

The ghost equation in the Feynman gauge

- In the Feynman gauge, $\xi = 1$, take the limit as $p \to 0$, assuming that $A(p^2, q^2, k^2)$ and $B(p^2, q^2, k^2)$ do not contain $(1/p^2)$ poles. The SDE for the ghost propagator reduces to

$$D^{-1}(0) = -C_{\rm A}\, g^2 \int [dk]\, B(0, k^2, k^2)\, D(k)$$

- $D^{-1}(0) \neq 0$ provide that non-perturbatively $B(0, k^2, k^2)$ does not vanish identically.

Electroweak sector: A reminder

- At tree-level, the ghost propagator associated with W-Bosons in the Electroweak sector are:

$$D_W(k) = \frac{1}{k^2}$$

Landau gauge

$(\xi = 0)$

$$D_W(k) = \frac{1}{k^2 - \xi M_W^2}$$

$$D_W(k) = \frac{1}{k^2 - M_W^2}$$

Feynman gauge

$(\xi = 1)$

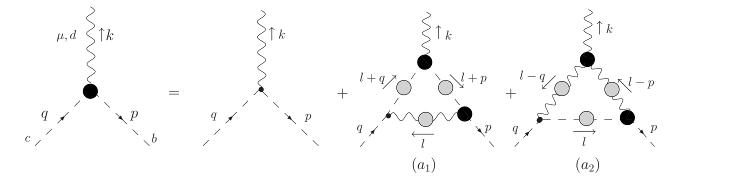
- Propagators ξ-dependent already at tree-level
- IR finite in Feynman gauge and IR divergent in Landau gauge..

Approximations to the vertex equation

- $B(0,k^2,k^2)$ is determined by

$$B(0,k^2,k^2) = \lim_{p\to 0}\left[\frac{1}{k^2}k^\mu\,\mathbb{\Gamma}_\mu(p,q,k)\right]$$

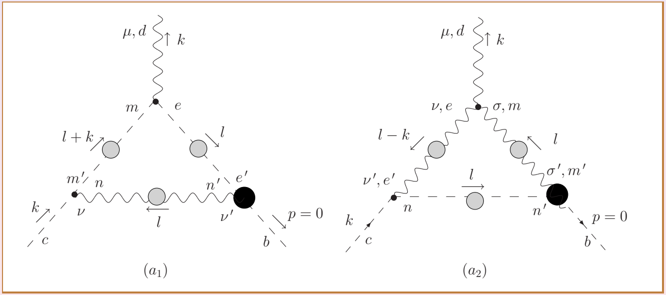
- Expanding ghost-gluon scattering kernel, in terms of the 1PI fully dressed three-particle vertices of the theory, neglecting diagrams that contain four-point functions.



Approximations to the vertex equation

- Linearize the equation: replace the vertices by their tree-level expressions $\mathbb{\Gamma}_\mu \to \Gamma_\mu$ and $\mathbb{\Gamma}_{\mu\nu\sigma} \to \Gamma_{\mu\nu\sigma}$

Then the diagrammatic representation will be



The vertex equation

- We arrive at the following linear and homogeneous equation

$$B(0,x,x) = \frac{C_{\rm A}\, g^2}{128\pi^2}\left\{\frac{1}{x}\big[D(x)-\Delta(x)\big]\int_0^x dy\, y^2 B(0,y,y)D(y)\right.$$
$$+\int_x^\infty dy\,(x-2y)B(0,y,y)D(y)\big[D(y)-\Delta(y)\big]+2\int_x^\infty dy\, y\, B(0,y,y)D(y)\Delta(y)$$
$$\left.-\frac{2}{x}D(x)\int_0^x dy\, y^2 B(0,y,y)D(y)+4\,\Delta(x)\int_0^x dy\, y\, B(0,y,y)D(y)\right\}.$$

- Setting $D(x)=\Delta(x)=1/x$ (perturbative expression), we obtain the equation that describes the asymptotic behavior of $B(0,x,x)$ given by

Asymptotic behavior of $B(0,x,x)$

$$B(0,x,x) = \lambda \int_x^{\infty} dy \, \frac{B(0,y,y)}{y}$$

where $\lambda = C_{\rm A}\, g^2/64\pi^2$.

- Converting it into a first-order differential equation,leads to the following asymptotic behavior

$$B(0,x,x) = \sigma x^{-\lambda}$$

where σ is an arbitrary dimensionful parameter (to be saturated by a non-perturbative mass like Λ_{QCD} whose the perturbative expansion vanishes to all orders.).

Our approach

- Ideally, one should solve the system of SDE for the gluon, ghost and the vertex. This is technically too complicated. Instead we study the system of SDE making various IR finite Ansätze for $\Delta(k)$ and $D(k)$ and check if they can be self-consistently implemented.

$$\Delta(k^2) = \frac{1}{k^2 + m^2(k^2)}\,,$$

$$D(k^2) = \frac{1}{k^2 + M^2(k^2)}\,,$$

where $m^2(k^2)$ and $M^2(k^2)$ act as dynamically generated masses.

J. M. Cornwall, Phys. Rev. D **26**, 1453 (1982).
A. C. Aguilar and J. Papavassiliou, JHEP **0612**, 012 (2006)
A. C. Aguilar and J. Papavassiliou, EPJA **35**, 189-205 (2008)

Gluon Propagator

- Crucial characteristic is that $m^2(k^2)$ is not "hard", but depends non-trivially on the momentum k^2. $m^2(k^2)$ is a monotonically decreasing function, starting at a non-zero value in the IR and dropping "sufficiently fast" in the deep ultraviolet (UV).
- The mass displays a logarithmic running

$$m^2(k^2) = m_0^2 \left[\ln\left(\frac{k^2 + \rho\, m_0^2}{\Lambda^2}\right) \Big/ \ln\left(\frac{\rho\, m_0^2}{\Lambda^2}\right) \right]^{-1-\gamma_1},$$

- On dimensional grounds $\rightarrow$ $m^2(k^2)$ could be connected with OPE operator $\langle A^2_{\rm min} \rangle$.
(The *gauge-invariant non-local* condensate of dimension two obtained through the minimization of $\int d^4x (A_\mu)^2$ over all gauge transformations)

Gluon Propagator

- Other possibility: Power-law running of the form

$$m^2(k^2) = \frac{m_0^4}{k^2 + m_0^2}\left[\ln\left(\frac{k^2 + \rho\, m_0^2}{\Lambda^2}\right) \Big/ \ln\left(\frac{\rho\, m_0^2}{\Lambda^2}\right)\right]^{\gamma_2 - 1},$$

- Connection with the local gauge-invariant condensate $\langle G^2 \rangle$
- OPE formalism:

$$m^2(k^2) \sim \frac{\langle G^2 \rangle}{k^2}$$

J. M. Cornwall, Phys. Rev. D **26**, 1453 (1982).
M. Lavelle, Phys. Rev. D **44**, 26 (1991).
A. C. Aguilar and J. Papavassiliou, EPJA **35**, 189-205 (2008).

Ghost Propagator

- For the ghost propagator we study these three cases:
- Hard mass - Simplest (naive) case

$$M^2(k^2) = M_0^2$$

- Logarithmic running

$$M^2(k^2) = M_0^2 \left[\ln\left(\frac{k^2 + \rho\, M_0^2}{\Lambda^2}\right) \Big/ \ln\left(\frac{\rho\, M_0^2}{\Lambda^2}\right) \right]^{-1-\kappa_1}$$

- Power-law running

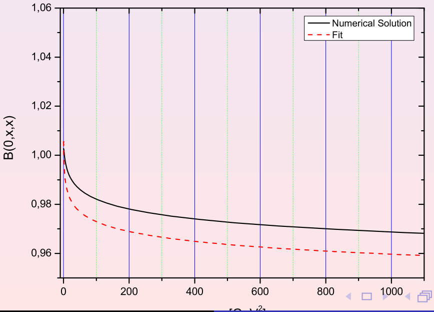
$$M^2(k^2) = \frac{M_0^4}{k^2 + M_0^2} \left[\ln\left(\frac{k^2 + \rho\, M_0^2}{\Lambda^2}\right) \Big/ \ln\left(\frac{\rho\, M_0^2}{\Lambda^2}\right) \right]^{\kappa_2 - 1}$$

Solution

- Plugging these propagators in the equation for $B(0,x,x)$, we solve it numerically. The solutions can be fitted by the simple expression

$$\boxed{B(0,x,x) = \frac{\sigma}{[x+M^2(x)]^{\lambda}}\,,}$$

For large x the above expression goes over the asymptotic solution



Regularization

- Substituting the solution for $B(0, x, x)$ back to the ghost-equation

$$\begin{aligned} D^{-1}(0) &= -C_{\rm A} g^2 \int [dk]\, B(0, k^2, k^2)\, D(k) \\ &= -C_{\rm A} g^2 \sigma \int [dk] \frac{1}{[k^2 + M^2(k^2)]^{1+\lambda}} \end{aligned}$$

- Divergent integral: at large k^2 it goes as $(\Lambda_{\rm UV})^{1-\lambda}$. Can be made UV finite by subtracting from it its perturbative value, i.e. the vanishing integral

$$\int [dk] (k^2)^{-\alpha} = 0\,,$$

for $\alpha = 1 + \lambda$. (the standard dimensional regularization result valid for any value of α)

Regularization

- For self-consistency we also have:

$$D^{-1}(0) = M_0^2\,,$$

Then, after the regularization procedure, it follows that

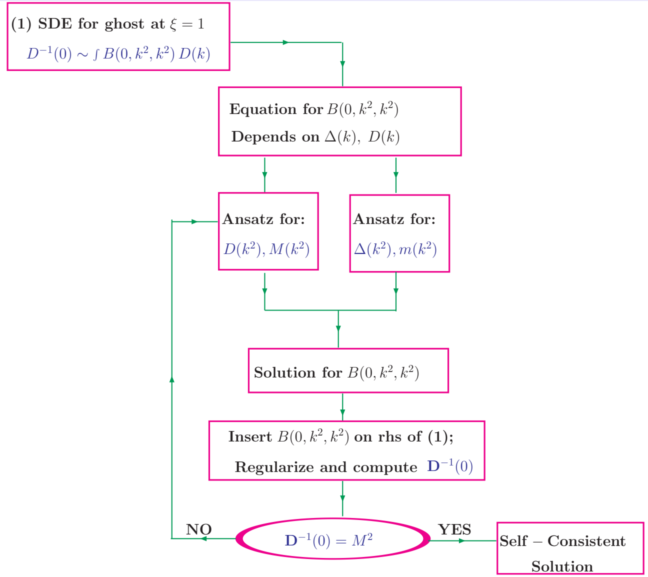
$$\begin{aligned} M_0^2 &= -C_{\rm A}\, g^2\, \sigma \int [dk] \left(\frac{1}{[k^2 + M^2(k^2)]^{1+\lambda}} - \frac{1}{(k^2)^{1+\lambda}} \right) \\ &= -C_{\rm A}\, g^2 \sigma \int \frac{[dk]}{[k^2 + M^2(k^2)]^{1+\lambda}} \left(1 - \left[1 + \frac{M^2(k^2)}{k^2} \right]^{1+\lambda} \right) . \end{aligned}$$

- The resulting integral

$$\int dy \, \frac{M^2(y)}{y^{1+\lambda}}$$

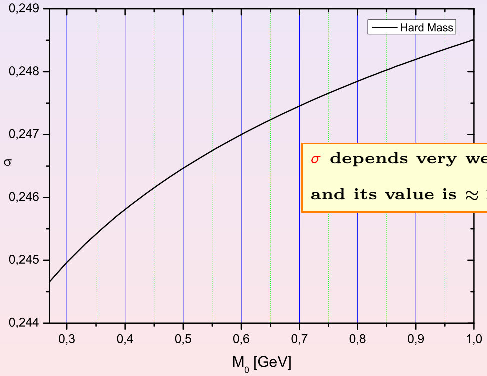
converges even for the less favorable case of a constant $M^2(y)$ (since $\lambda > 0$).

Schematic view



- Self-consistency implemented by relating appropriately σ with M_0 (you give me a σ, I give you a M_0)

- For a hard mass: $$\sigma \approx \frac{(1-\lambda)}{4}\Lambda^{2\lambda}\left[1+\lambda\ln\left(\frac{M_0^2}{\Lambda^2}\right)\right],$$



σ depends very weakly on M_0, and its value is $\approx 1/4$.

- Logarithmic running

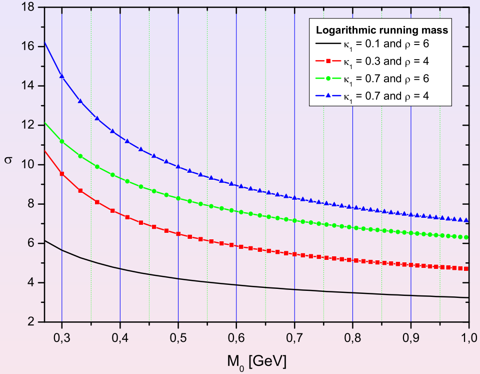


Figure: σ as function of M_0, when $M^2(k^2)$ runs logarithmically

- Power-law running

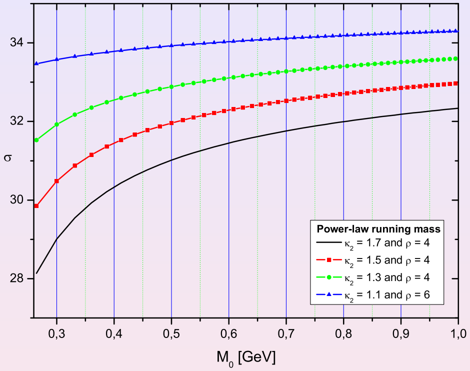


Figure: σ as function of M_0, when the power-law running is assumed for $M^2(k^2)$.

Summary

- It is possible to obtain from the SDEs of QCD an IR finite ghost propagator in the Feynman gauge
- The longitudinal component of the gluon-ghost vertex, which is inert in the Landau gauge, assumes a central role, allowing for $D(0)$ to be finite
- We do not need to impose the presence of massless poles of the type $1/p^2$.

- Solve the gluon-ghost system in the Feynman gauge.