More on Gribov copies in Landau-gauge

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St. Goar, Germany

UNI GRAZ

Overview

- Landau gauges in the non-perturbative domain

Supported by the FWF and the DFG

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- Gauge-fixing on the lattice

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- Propagators
  - Impact on the propagators in 2d
  - Impact on the propagators in 3d

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- Landau gauges in the non-perturbative domain
- Gauge-fixing on the lattice
- Propagators
  - Impact on the propagators in 2d
  - Impact on the propagators in 3d
  - Calculations in 4d (yet) too expensive

Supported by the FWF and the DFG

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- Correlation functions are in general gauge-dependent
  - Gauge-fixing is required
- Gauge-dependent quantities are interesting
  - Manifestation of confinement mechanism
    - Gribov-Zwanziger scenario
    - Topologial mechanisms

Unique gauge-fixing [For an introduction: Sobreiro & Sorella, 2005]

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  - Landau gauge: $\partial_\mu A_\mu^a = 0$

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  - There are gauge-equivalent configurations which obey the same local gauge-condition: Gribov copies
- There are no known local gauge conditions, which lead to a unique gauge configuration
  - Non-local conditions possible, but impractical outside lattice gauge theory

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- Choice: Leave the global color symmetry unfixed

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  - Average over the elements in the first Gribov region: Minimal Landau gauge

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  - Zwanzigers conjecture: In an infinite volume minimal and absolute Landau gauge should coincide
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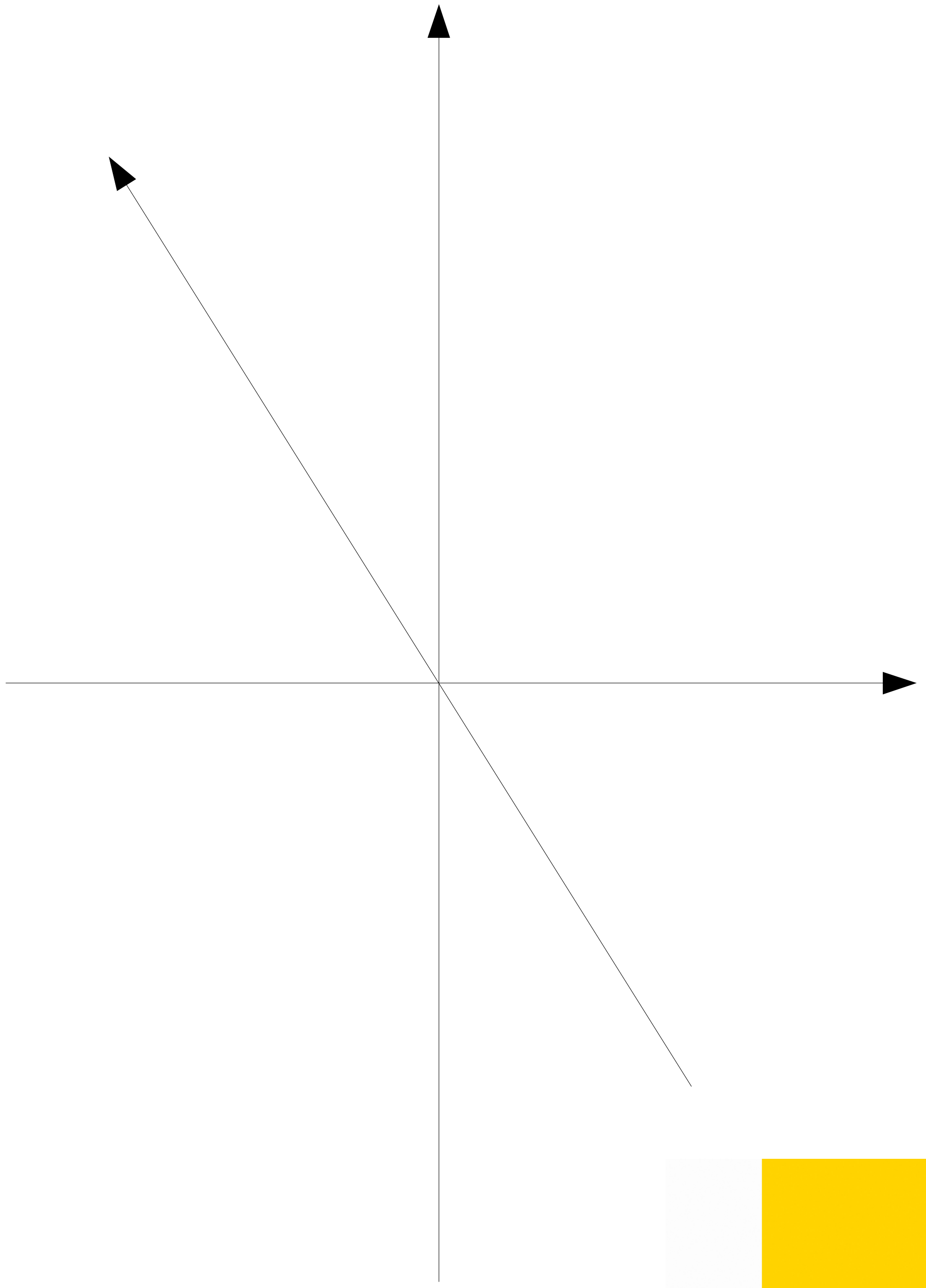
  - Does not apply to lattice calculations in a finite volume
- Here:
  - Compare absolute and minimal Landau gauge

Configuration space (artist's view)



Configuration space (artist's view)

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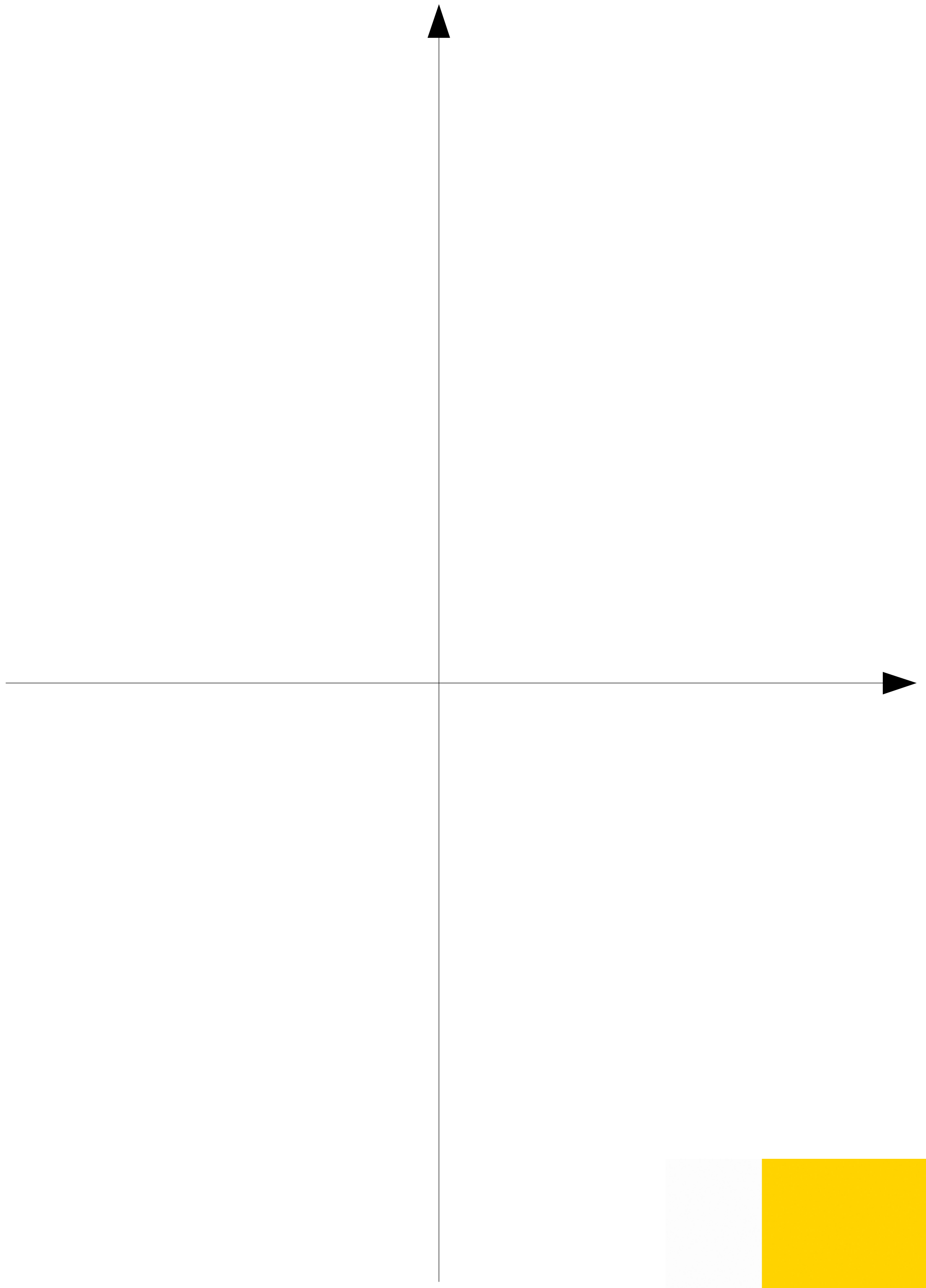
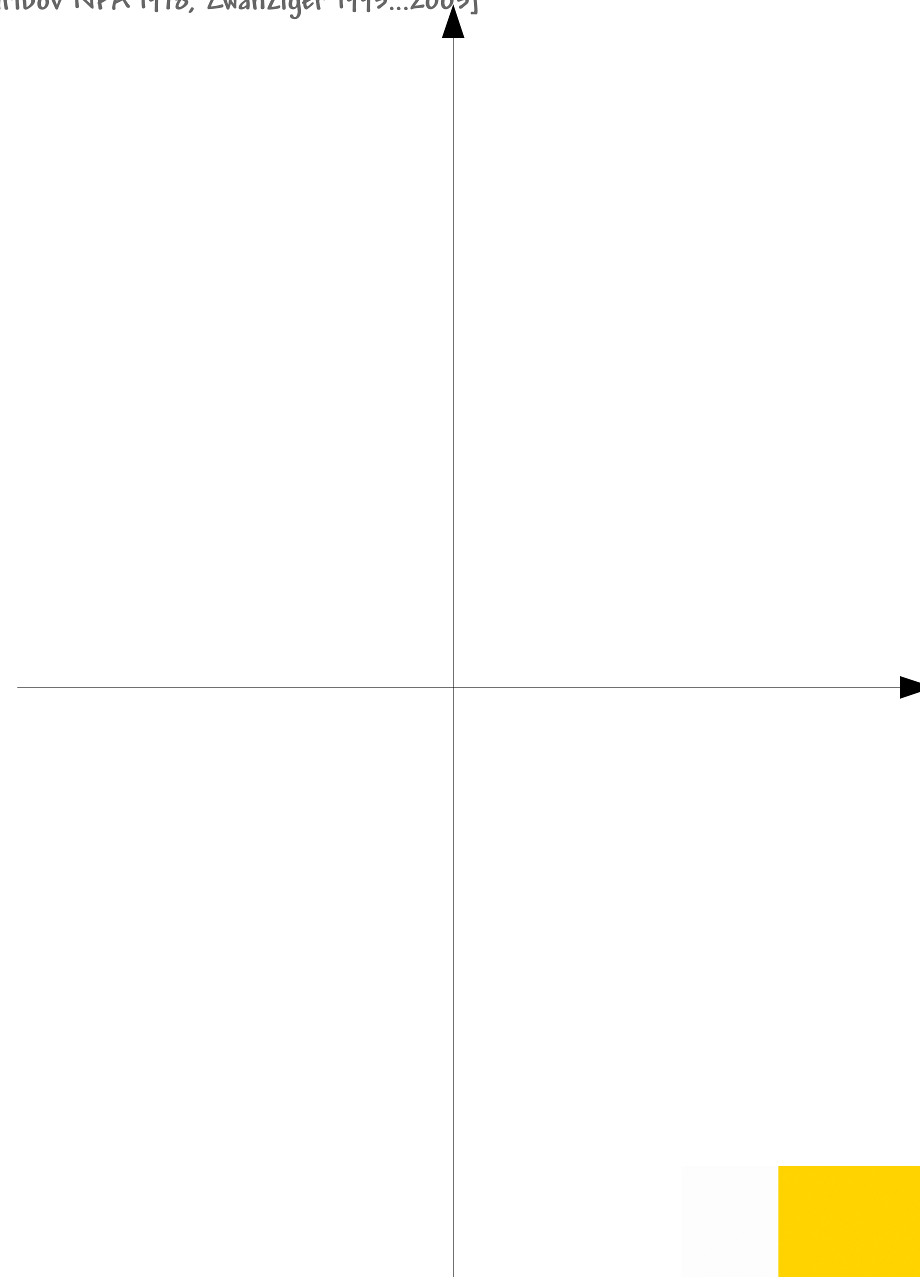
  - Only residual gauge orbits left



Configuration space (artist's view) [Gribov NPA 1978, Zwanziger 1993...2003]

- Minimal Landau gauge



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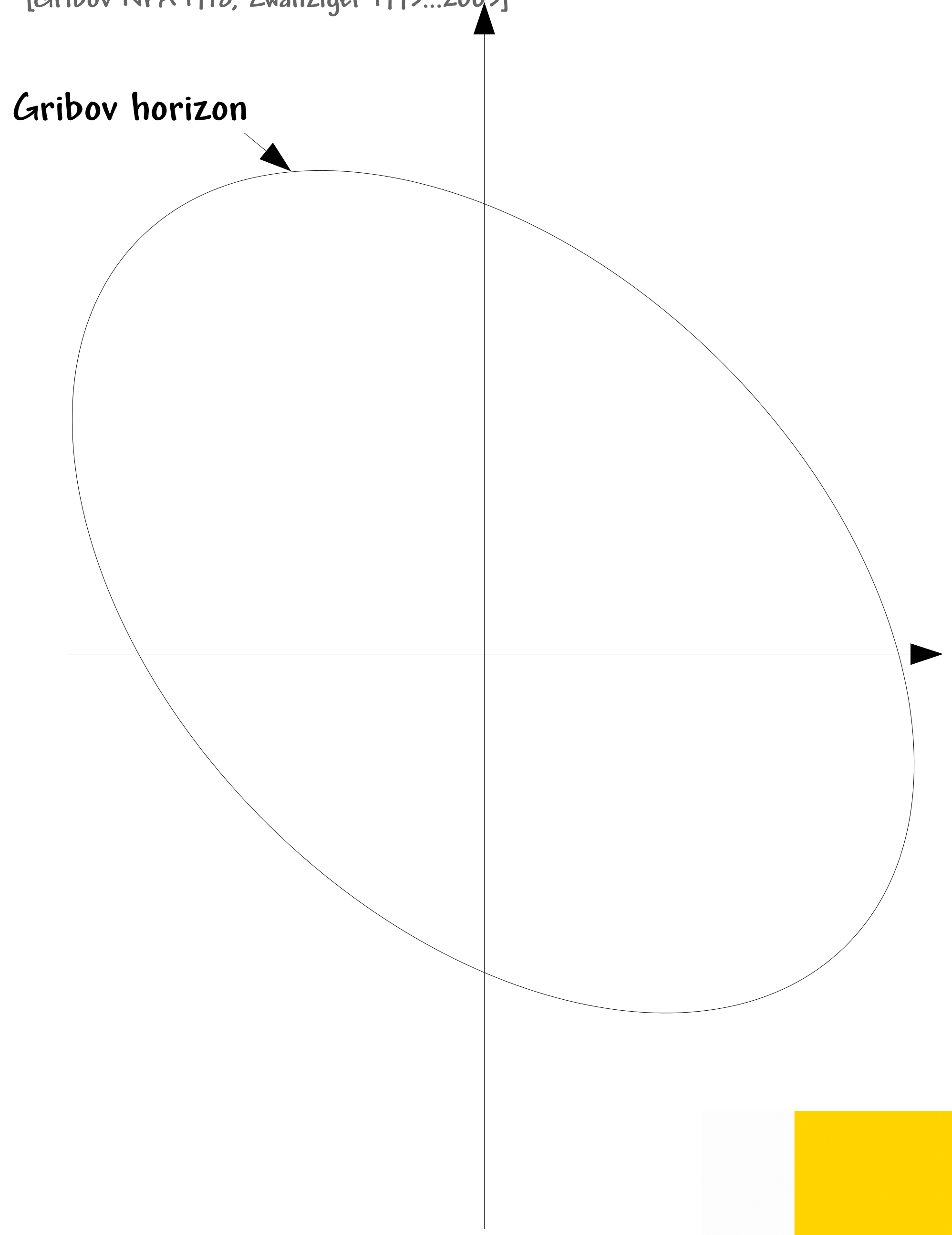

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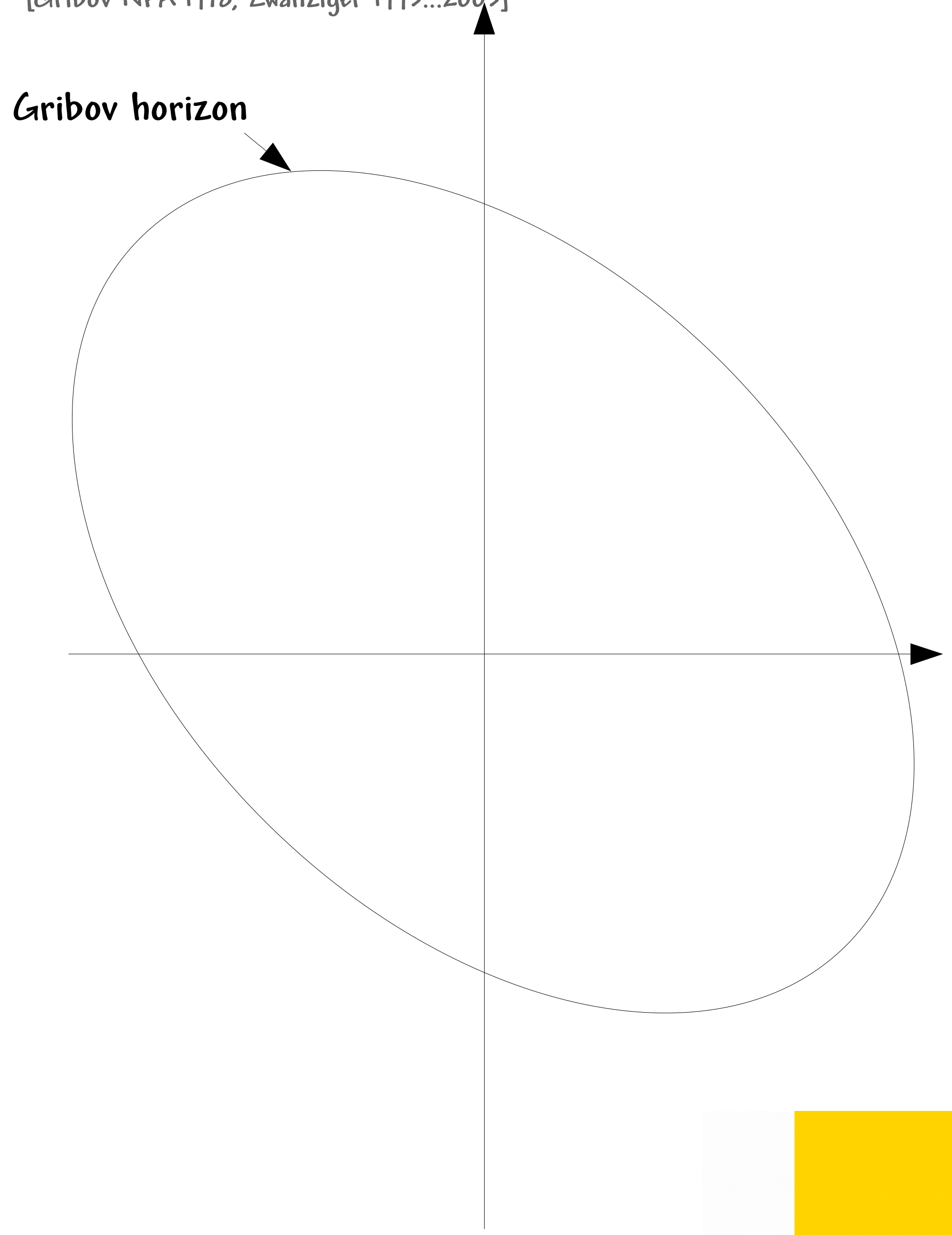

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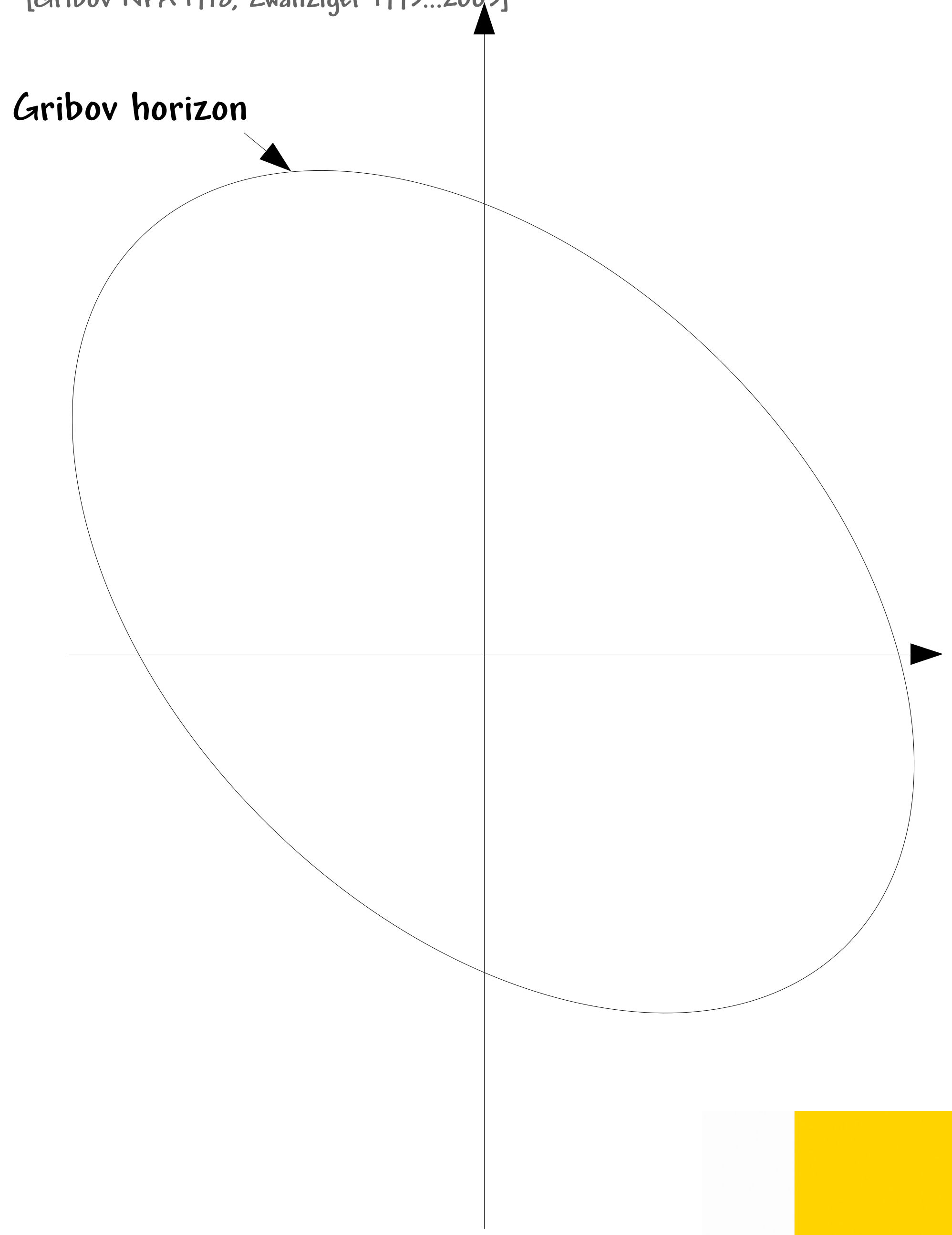
- All propagators are positive semi-definite inside this region



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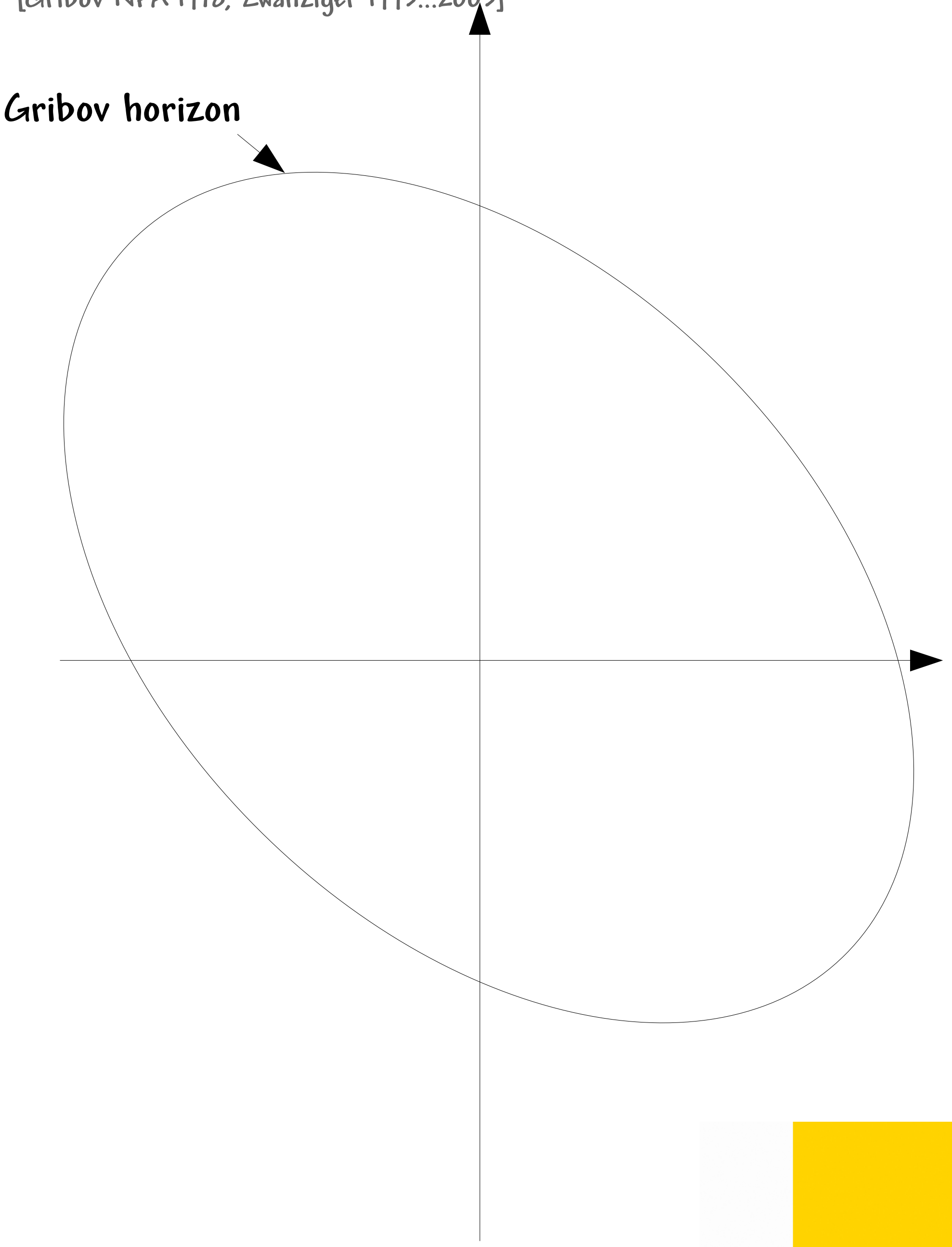

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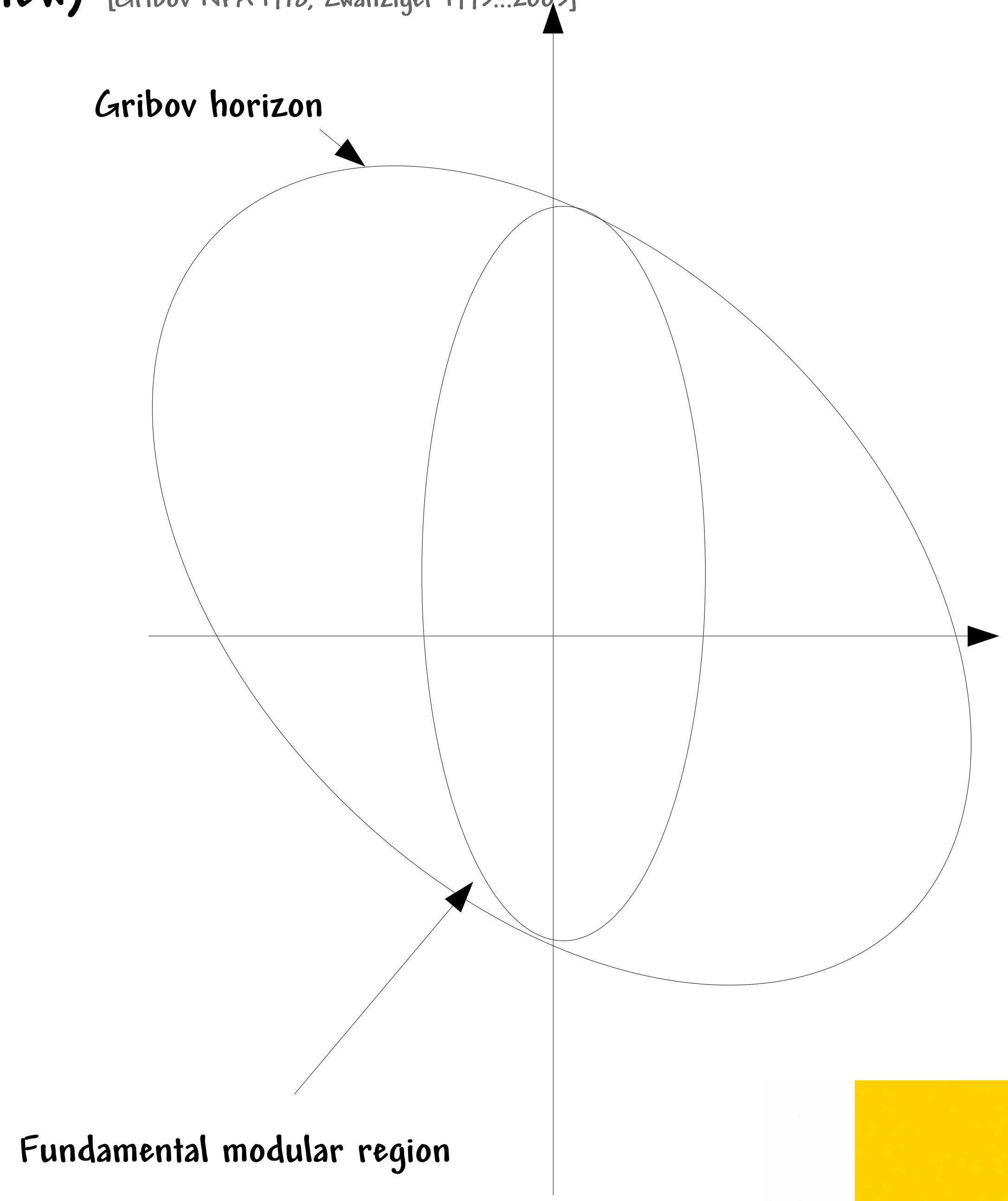

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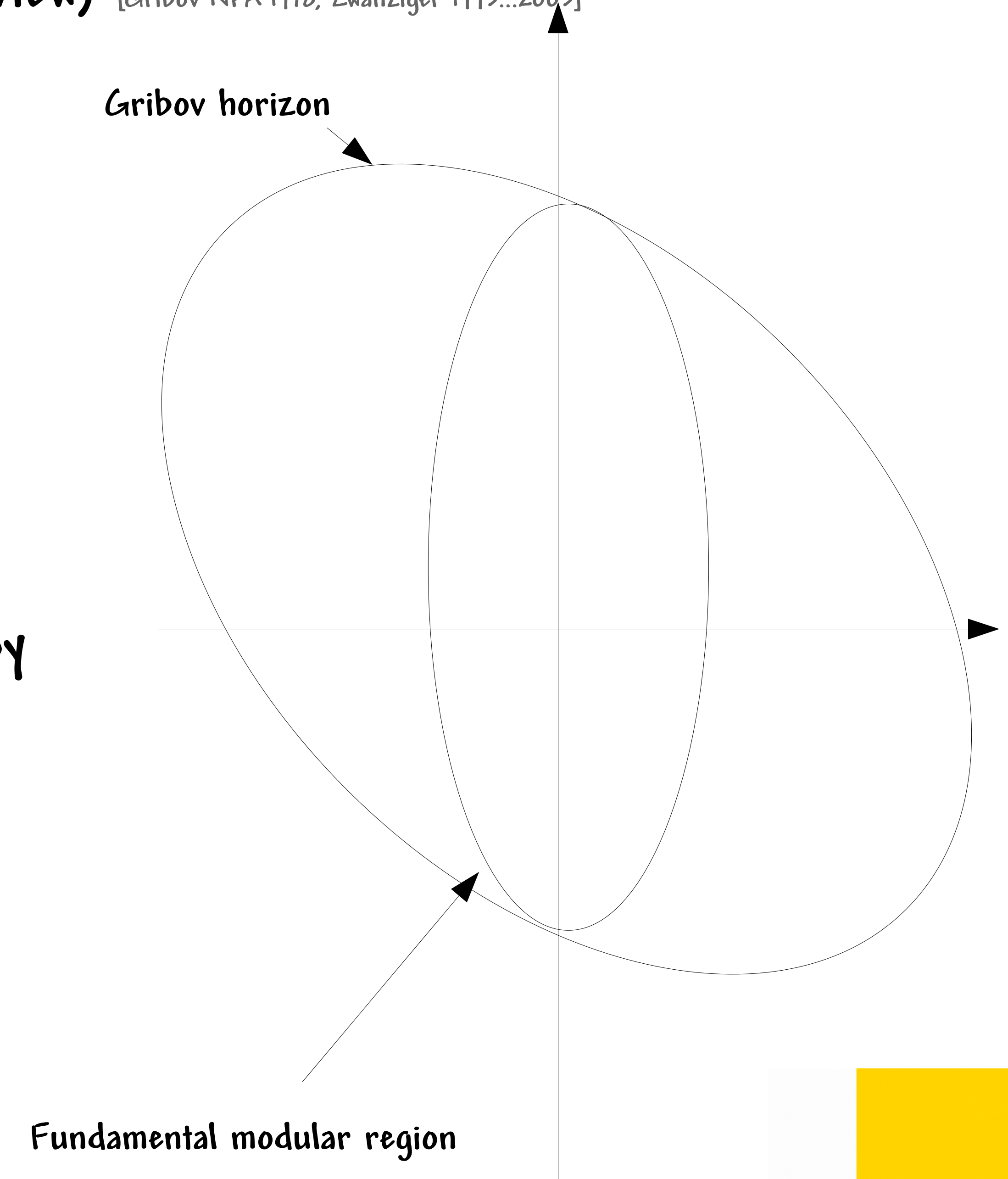

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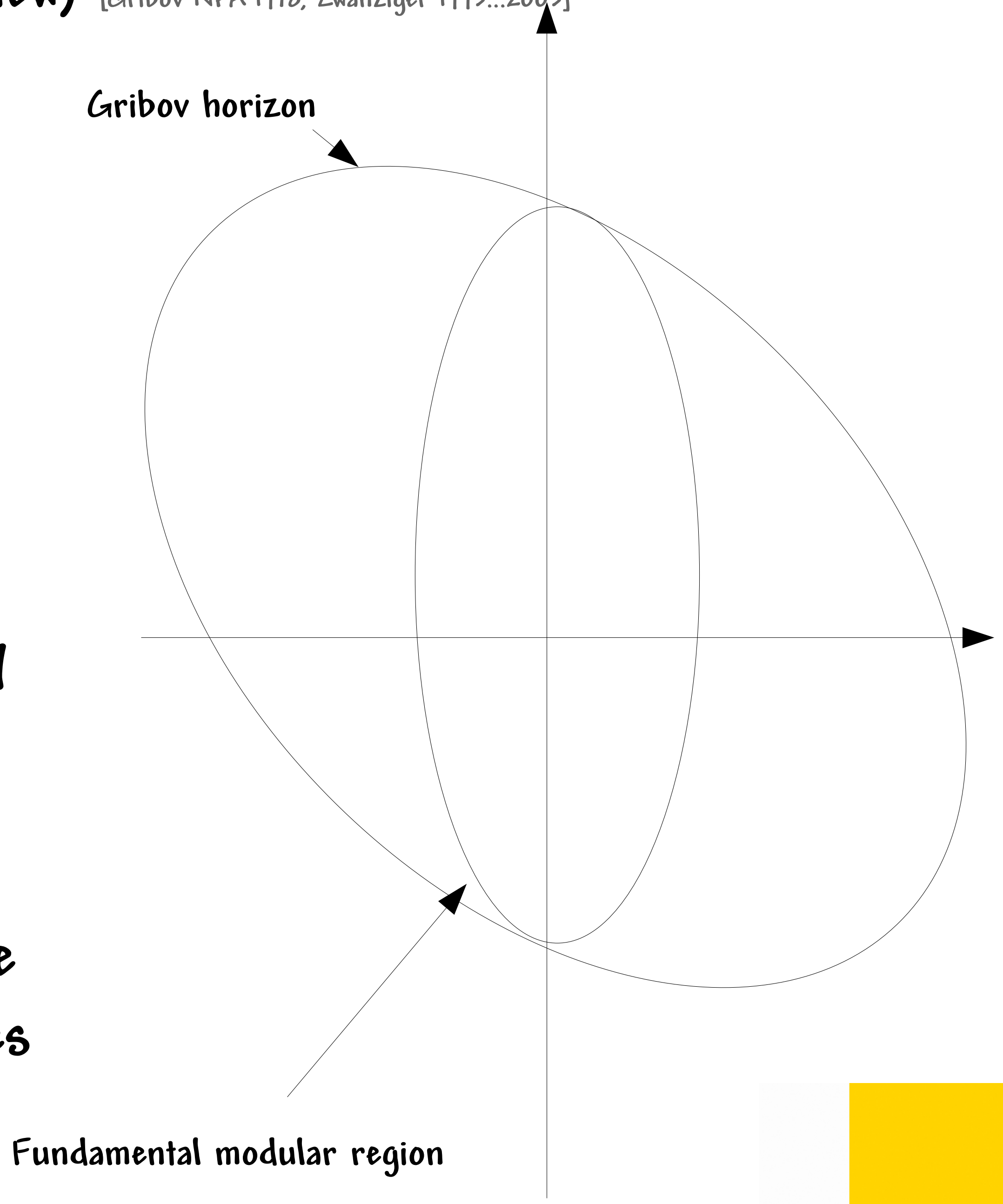
- Equivalent: Take the representative of the gauge orbit, which minimizes the trace of the gluon propagator

  $$\int dp D_{\mu\mu}^{aa}(p)$$



Gauge-fixing on the lattice

- Take the requirement of finding an absolute minimum literally by recasting (Landau-) gauge-fixing into a minimization problem
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  - Various approximate approaches have been developed
    - Mostly used: Restart algorithms
    - Highly sophisticated methods available
    - Here: Evolutionary algorithm
      - Very versatile and adapts to a configuration
      - Takes lattice Gribov copies also into account
      - Particular advanatge: Lower dimensionality

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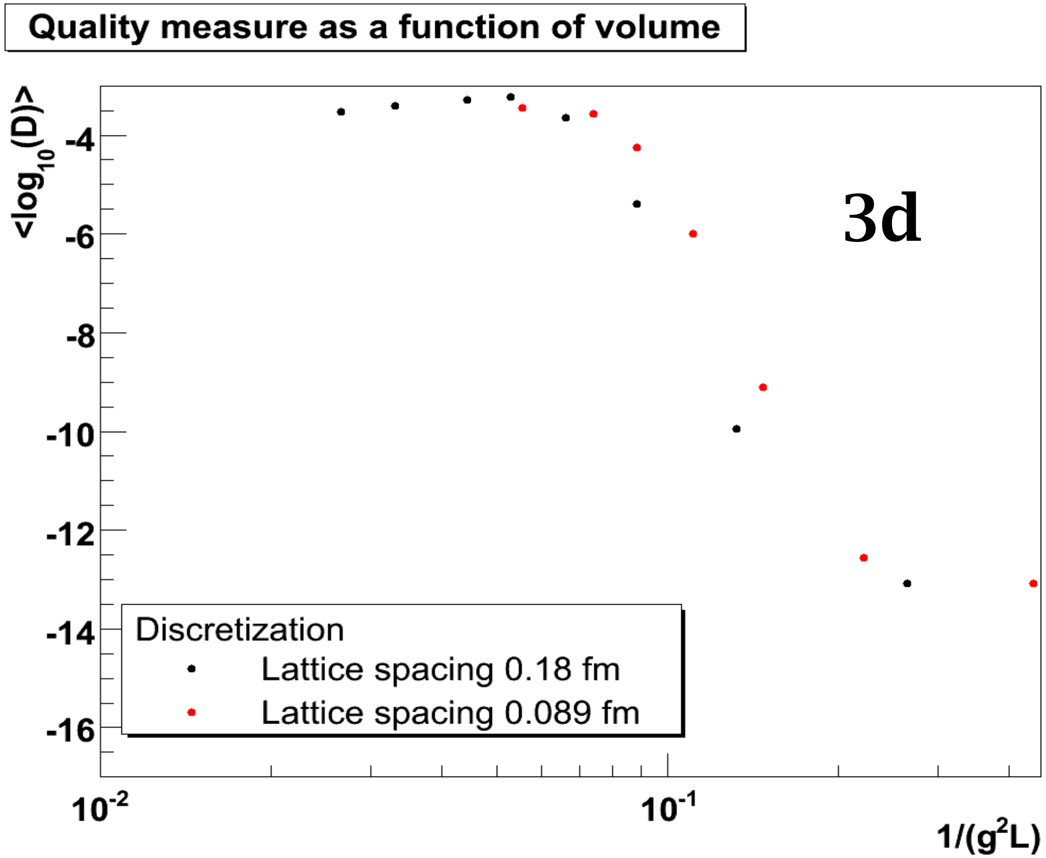
  - Number of Gribov copies grows exponentially with volume [Zwanziger 1993...2003]
  - NP-complete problem - All gauge-fixing algorithms will require an amount of time to find the absolute minimum which grows exponentially with volume
  - No termination criterion known

Artifact-dependence of Gribov copies [Maas, unpublished]



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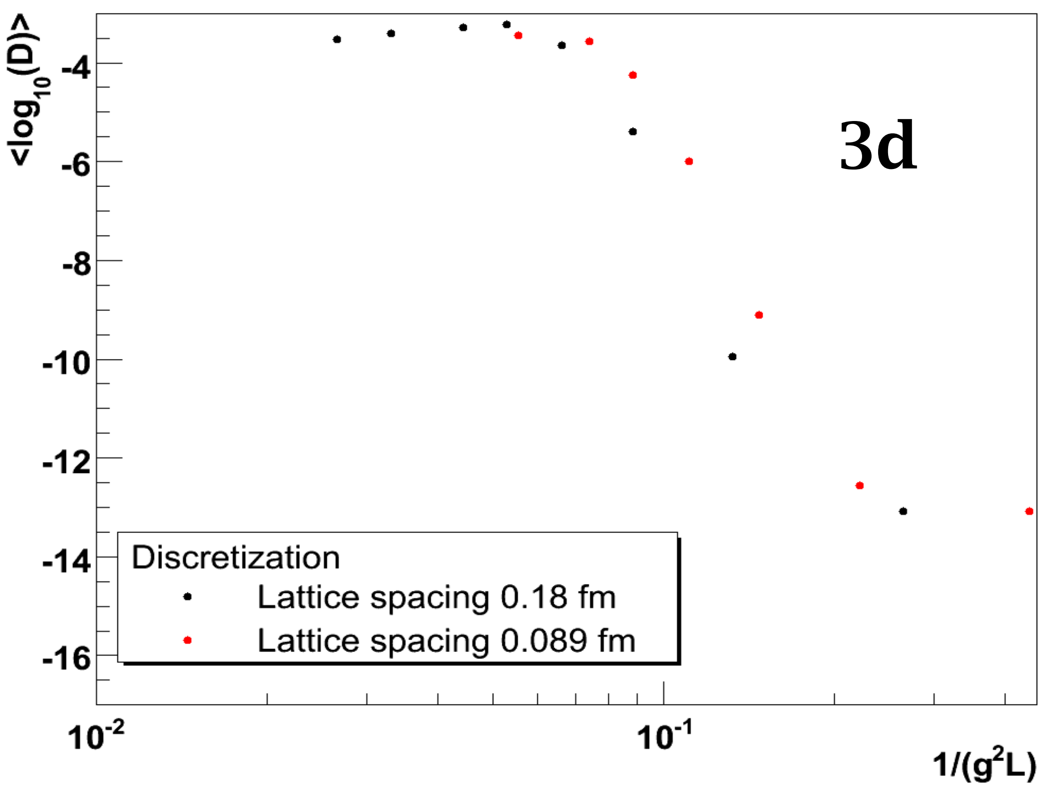
Quality measure as a function of volume



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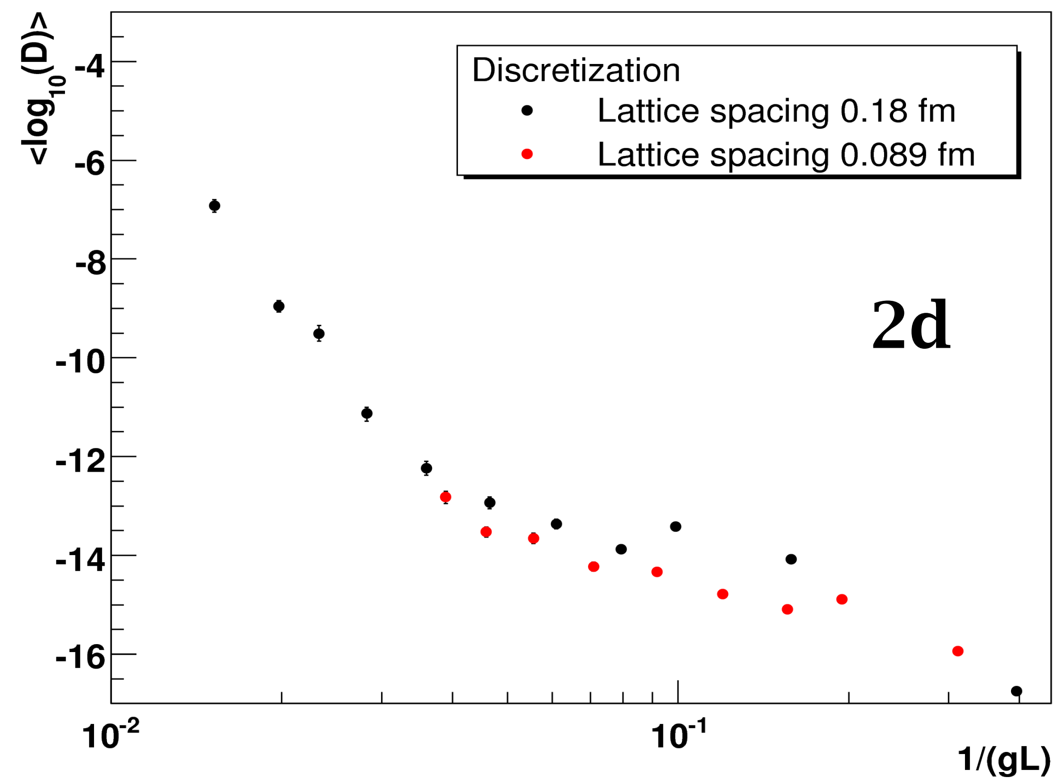

- Strong (exponental) increase with volume before saturation in 3d
- Small impact of discretization
- Gribov effect significantly weaker in two dimensions

Impact on the propagators

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  - Zwanzigers conjecture applies to the gluon propagator
- Important quantities
  - Describe gluons
  - Non-perturbative information are encoded

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- Ghost linked to the Faddeev-Popov operator

$$D_G^{ab}(x-y) \sim \langle (\partial_\mu D_\mu^{ab})^{-1} \rangle = \langle (\partial_\mu (\delta^{ab} \partial_\mu - g f^{abc} A_\mu^c))^{-1} \rangle$$

Minimal Landau gauge in two dimensions

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  [Maas 2007]
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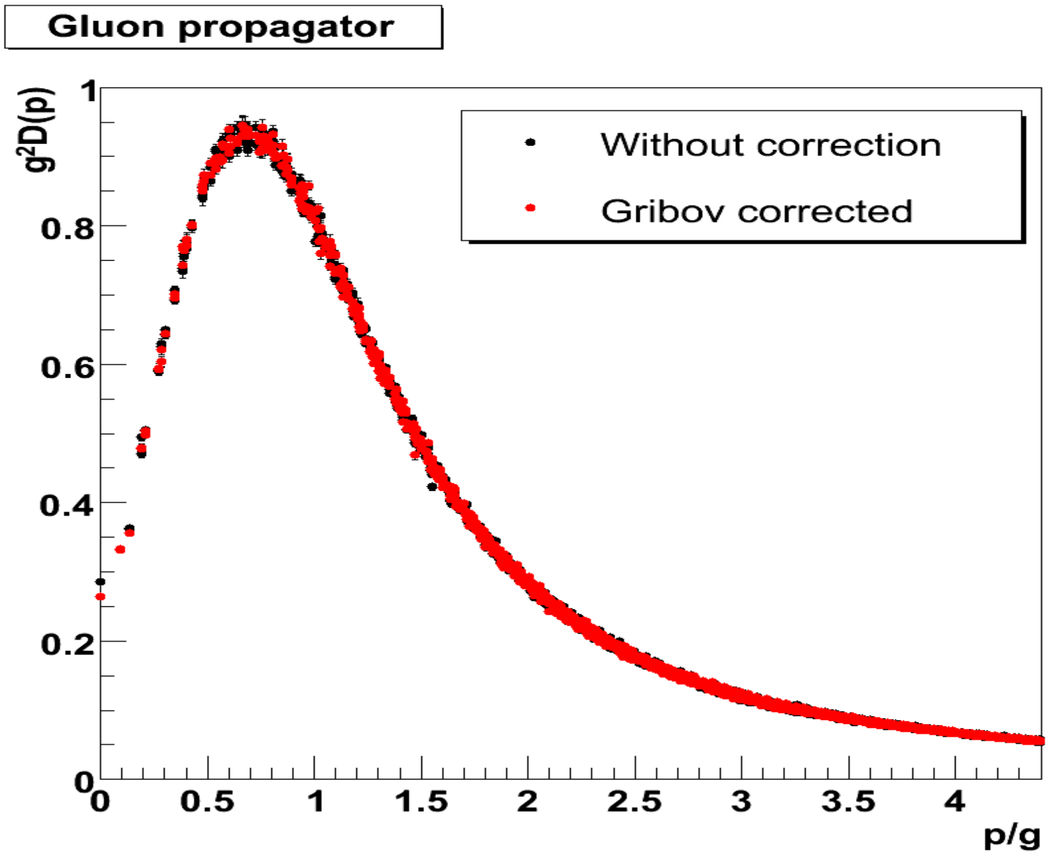
  - Both behave as power-laws in the far infrared
  - Quantitative agreement with predictions from functional calculations [Lerche et al. 2002, Zwanziger 2002, Huber et al. 2007, Pawlowski et al. 2004]

Impact on the propagators [104^2, beta=38.7, Maas, unpublished]



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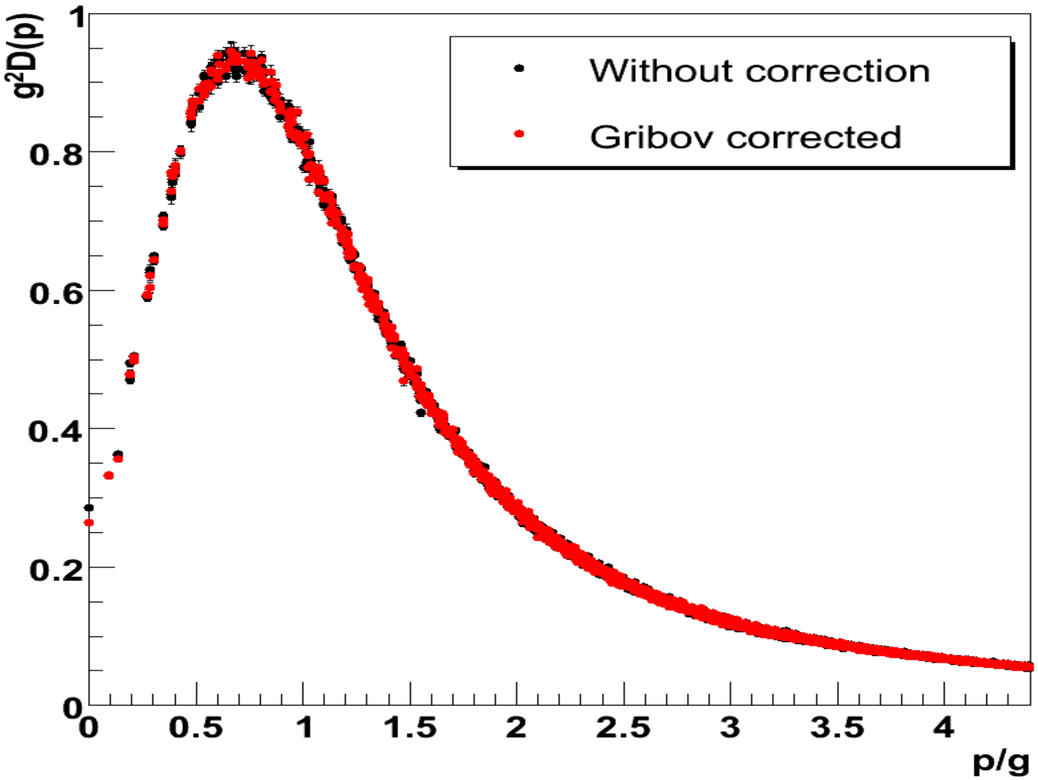
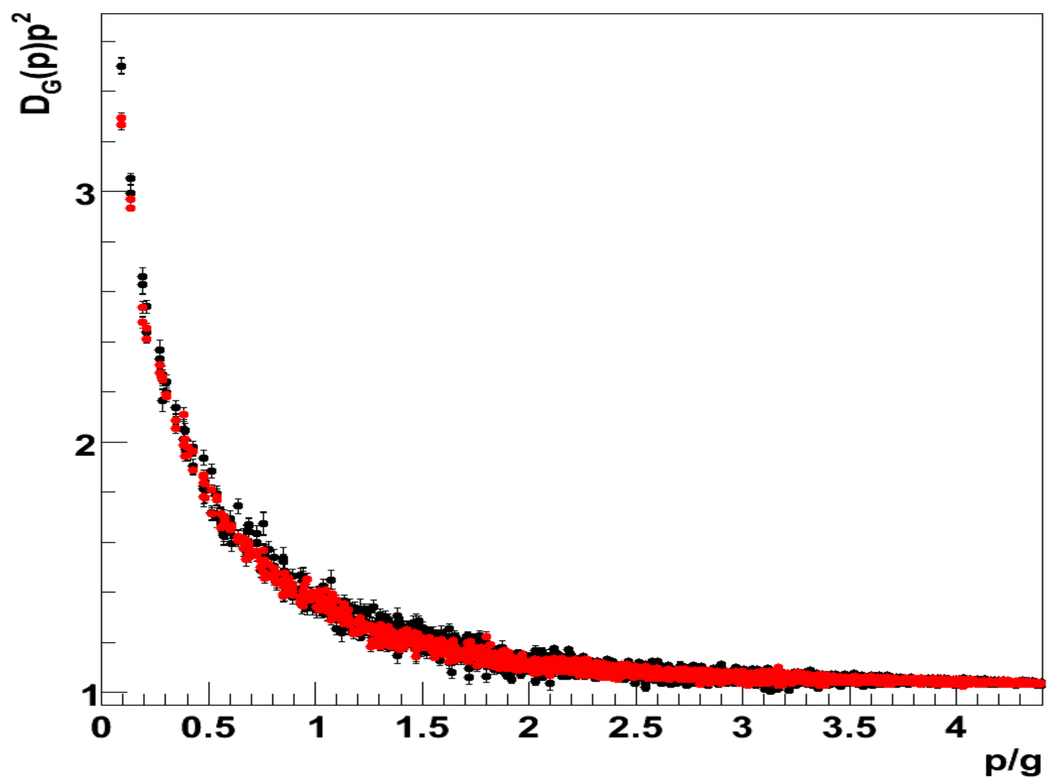
Gluon propagator



Ghost dressing function



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- Ghost propagator somewhat less enhanced

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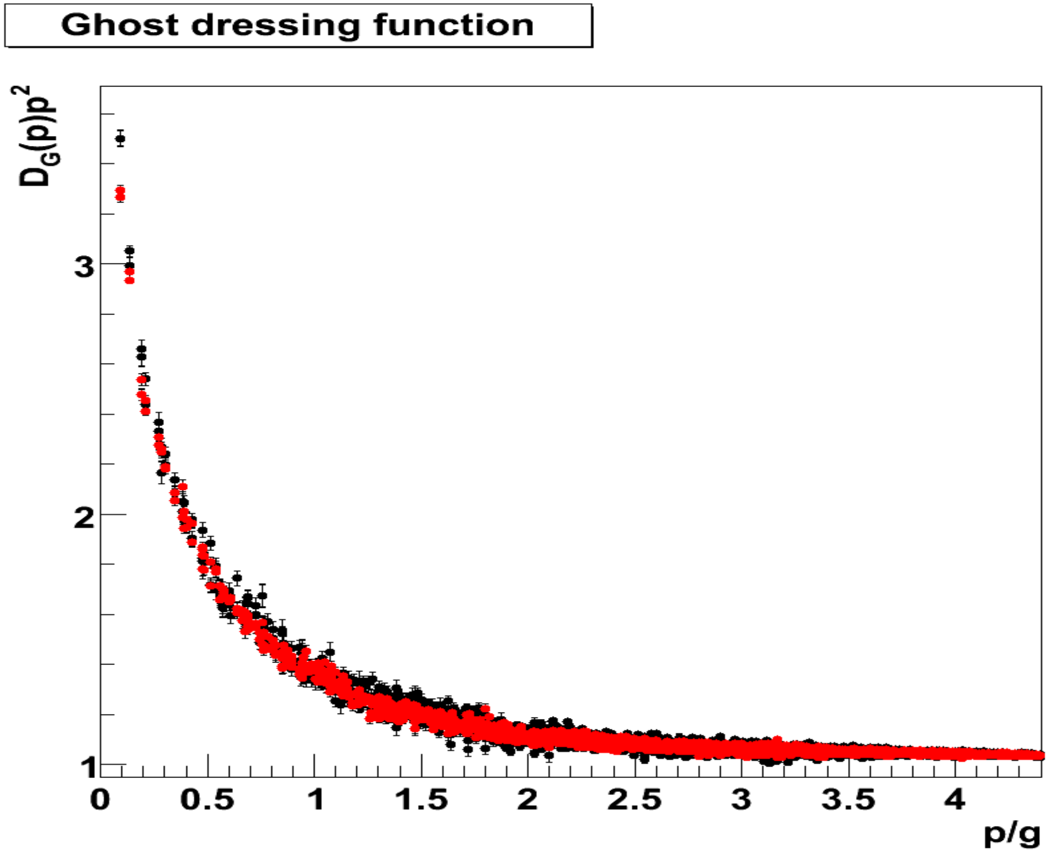

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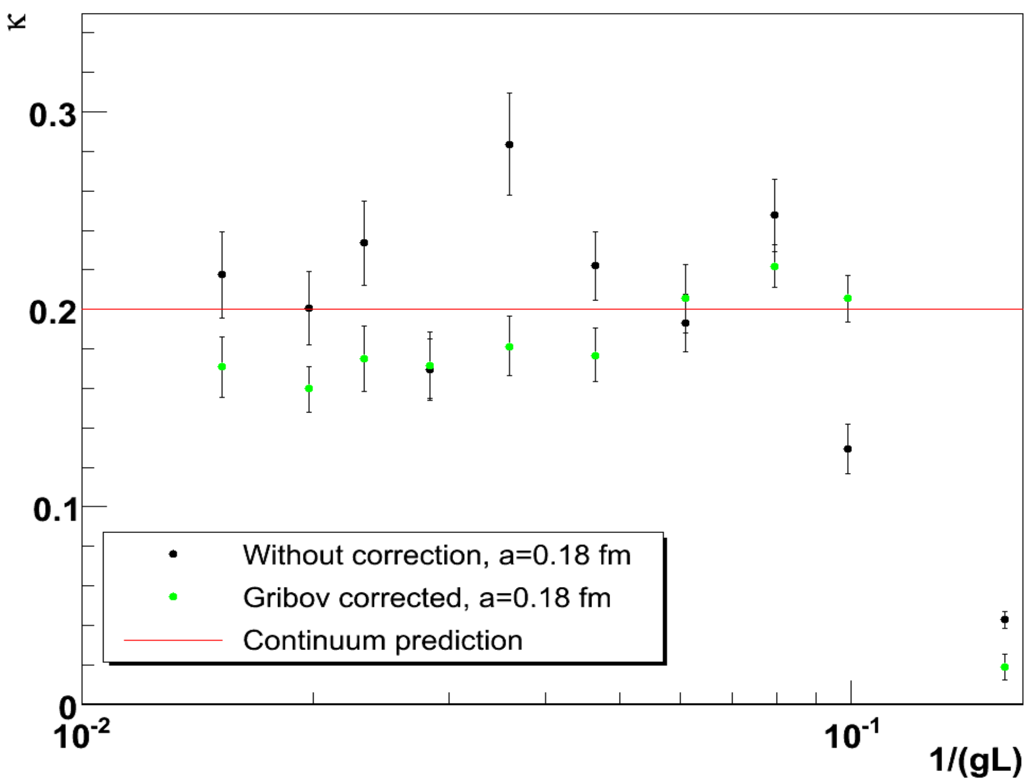
Ghost exponent



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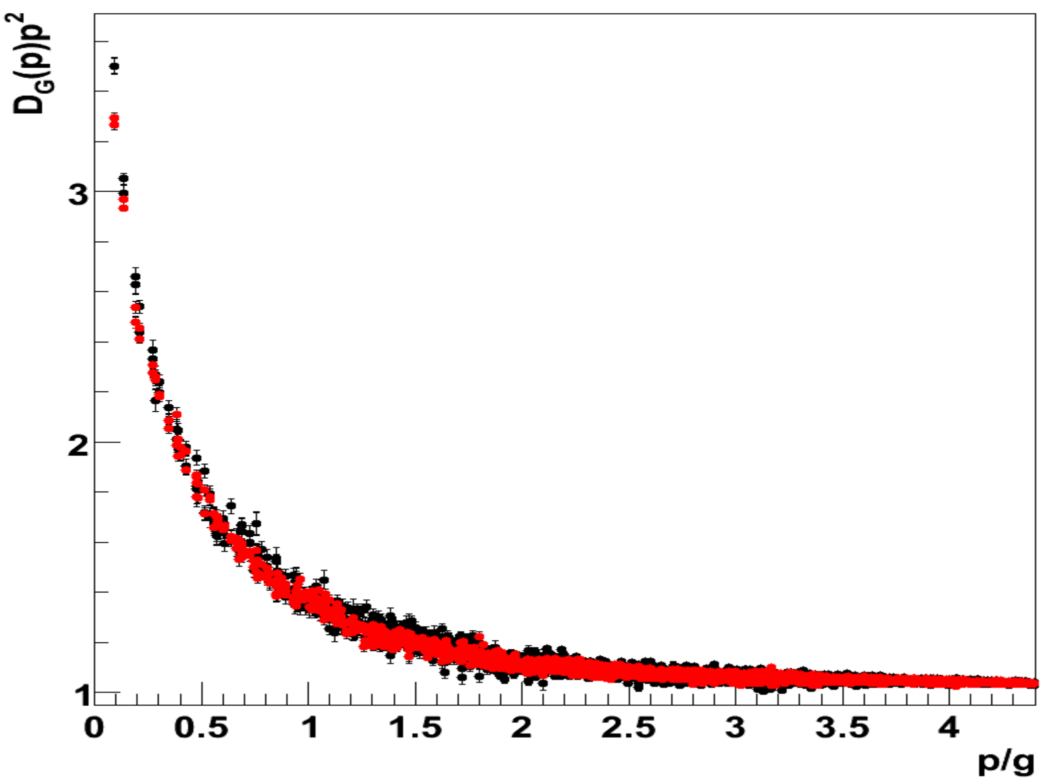

- Exponent in both cases compatible with predictions
- Finite volume correction much less severe in the absolute Landau gauge

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- Ghost propagator less suppressed
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  - Finite volume corrections less severe
- Predicted relation between gluon and ghost infrared behavior seems to be still valid
  - More statistics and volume needed

Minimal Landau gauge in three dimensions

- Same qualitative predictions as in two dimensions
  [Lerche et al. 2002, Zwanziger 2002, Huber et al. 2007, Pawlowski et al. 2004]
  - Vanishing gluon propagator and enhanced ghost propagator

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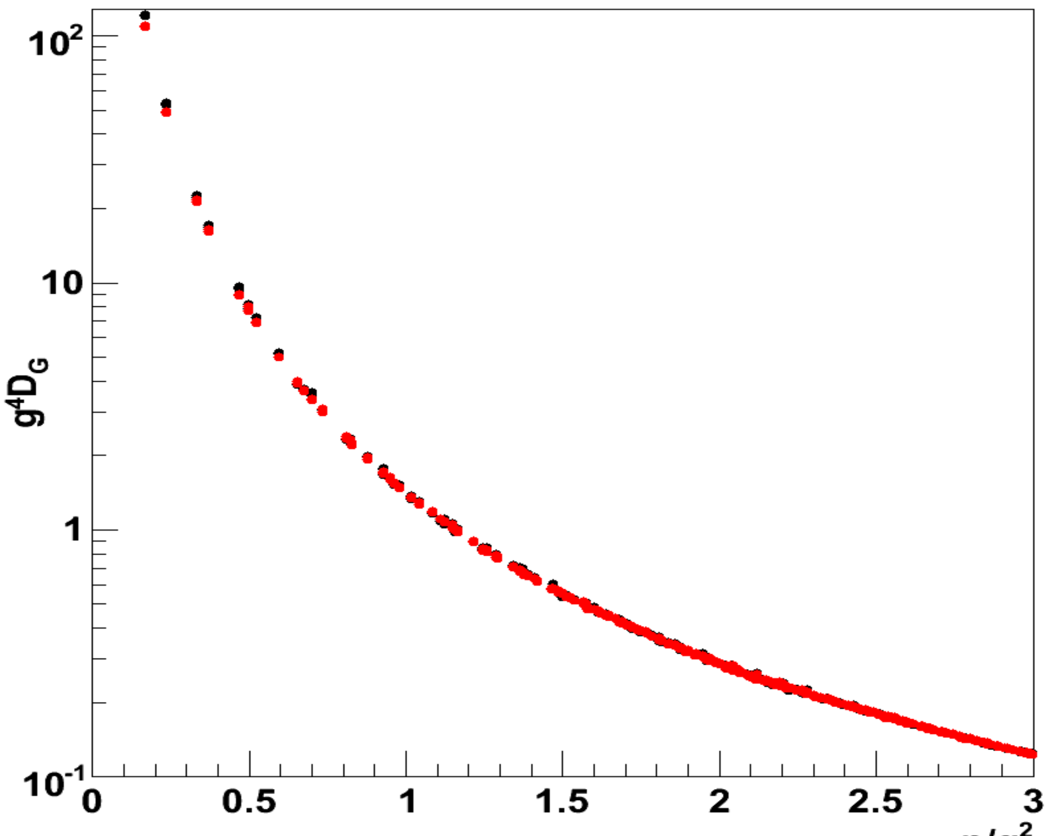
- Lattice results in the asymptotic regime [Cucchieri, et al. 2007/2008]
    - Infrared finite gluon propagator
    - Tree-level-like ghost
- Qualitative agreement to functional predictions in an intermediate momentum window

Impact on the ghost propagator [40³, beta=4.24, Maas, unpublished]

Ghost propagator



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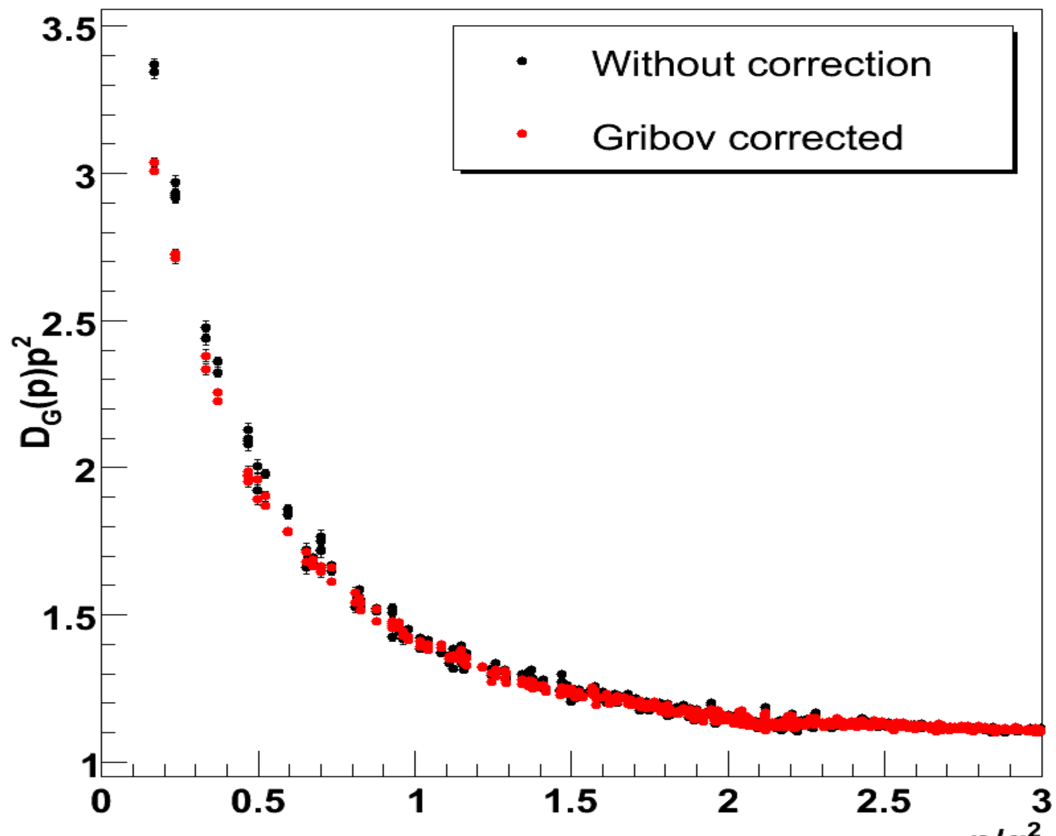

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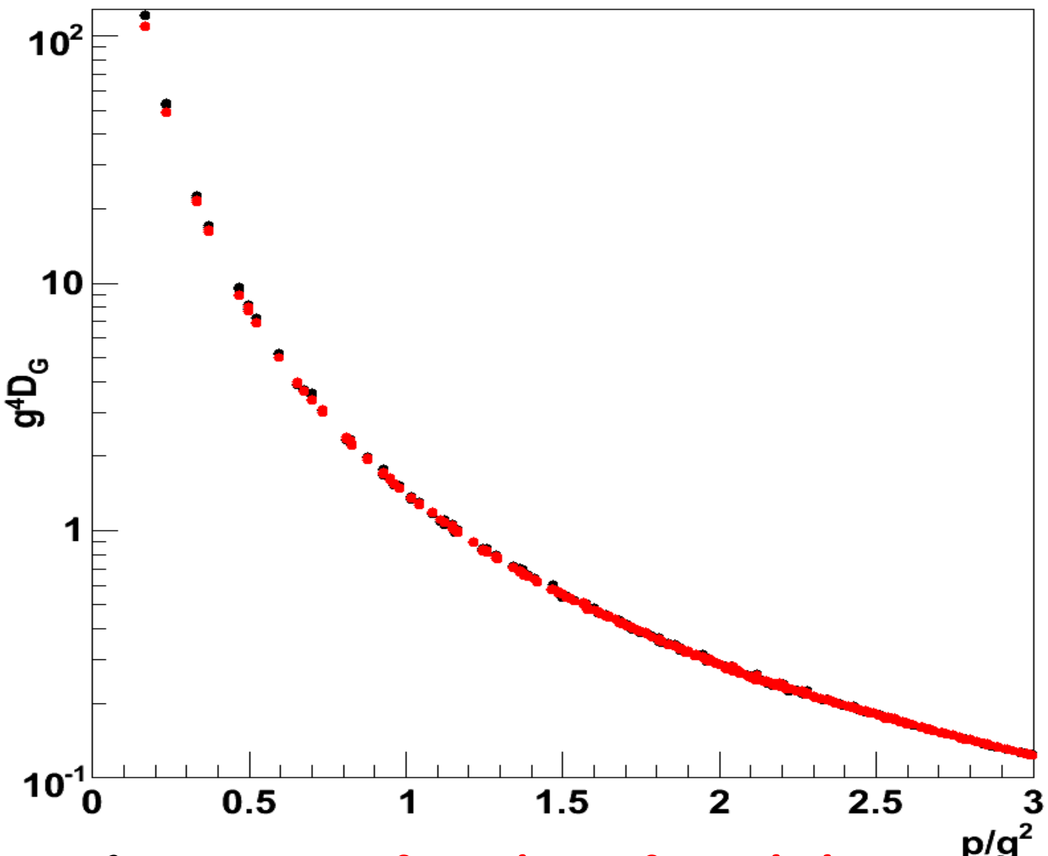

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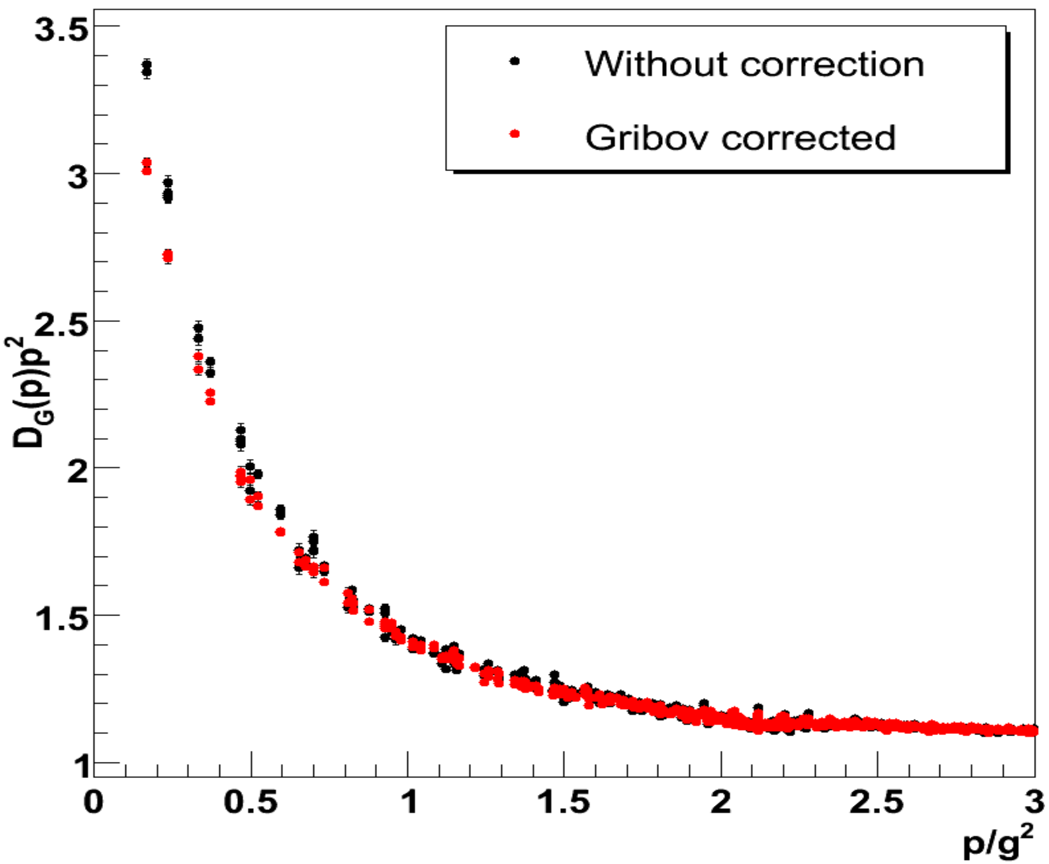

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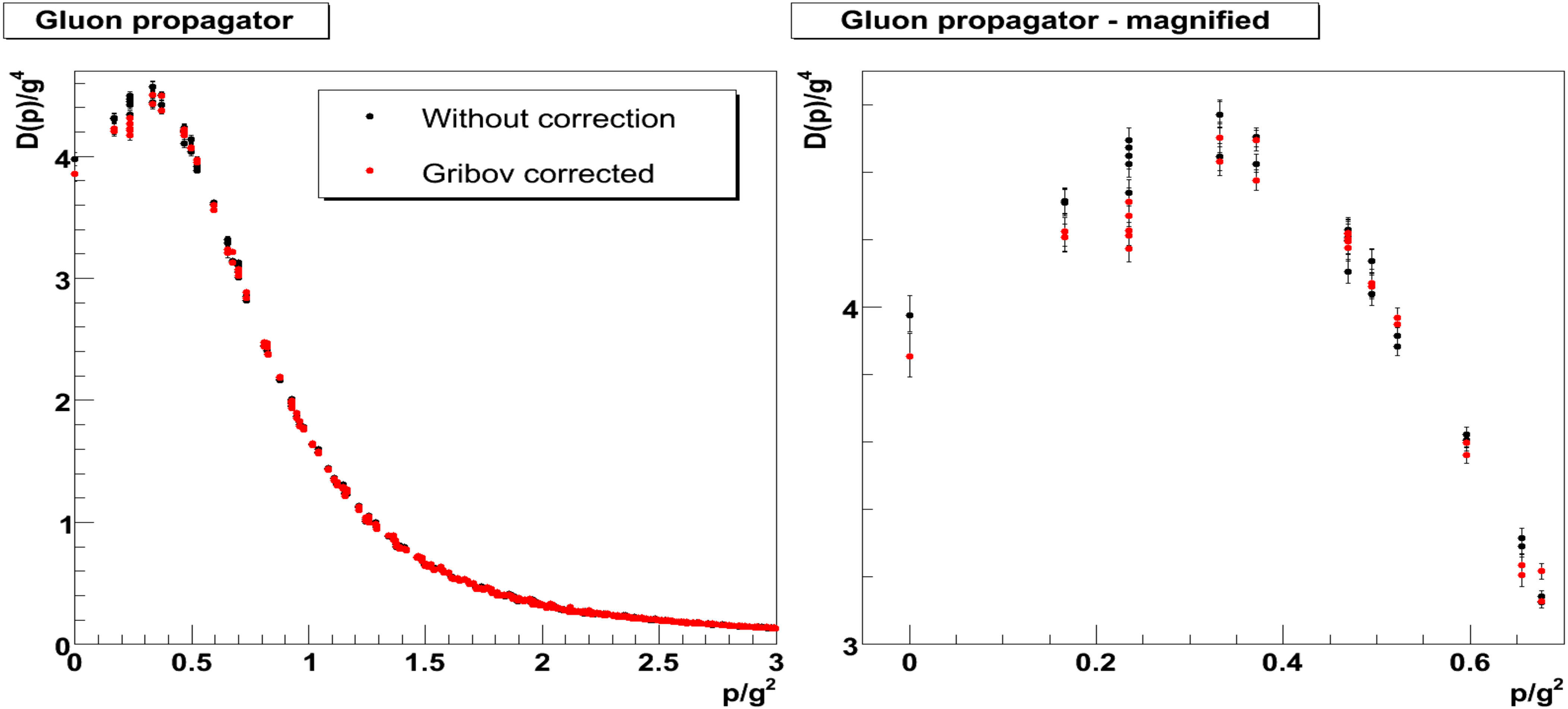
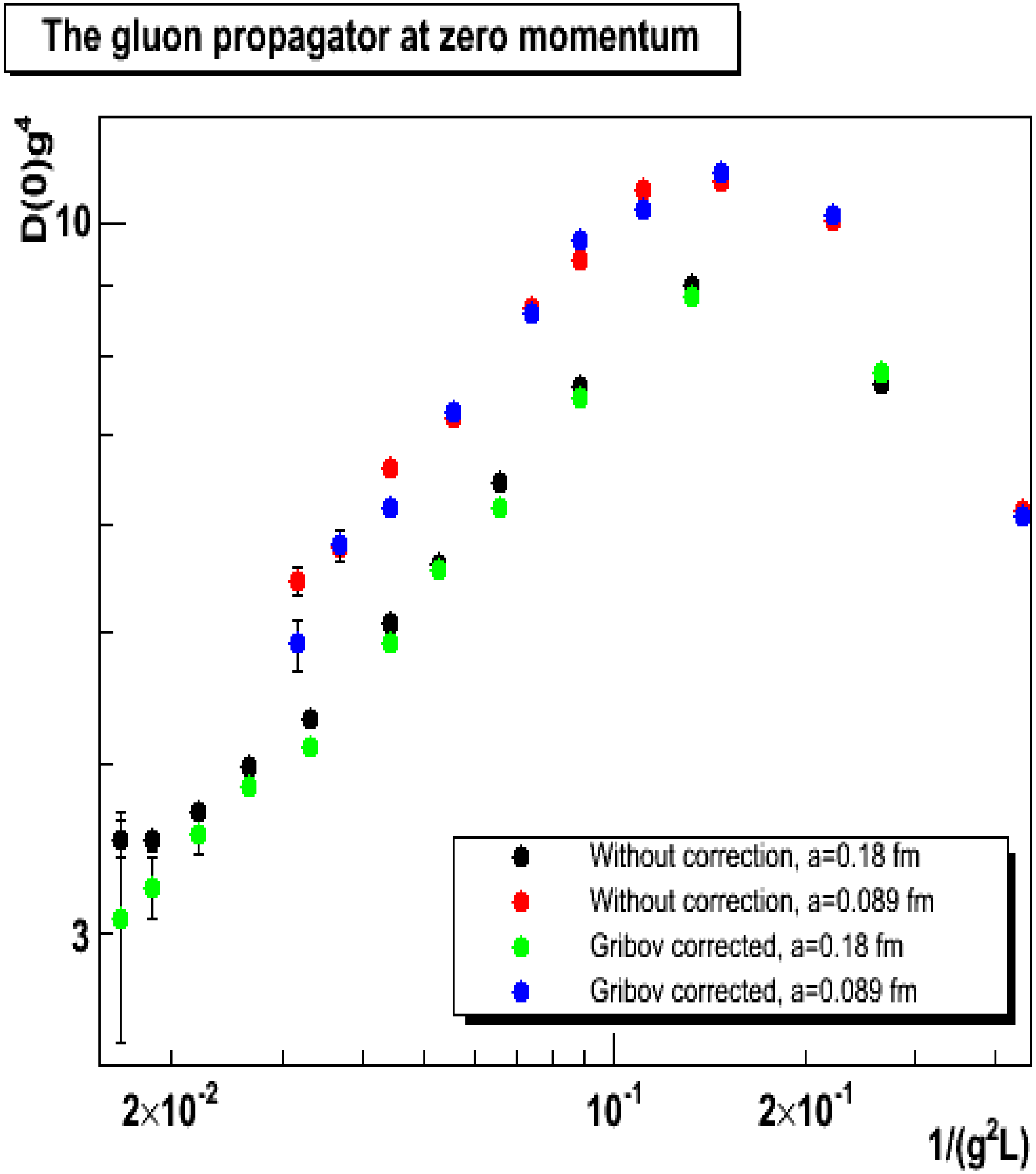
- Impact rate depends likely non-monotonically on the volume
- Asymptotic limit? - Finite volume artifacts more intricate than in two dimensions

Impact on the gluon propagator [40^3, beta=4.24, Maas, unpublished]



- Small effect
  - Most pronounced in the far infrared, and decays with increasing momentum

Impact on the gluon propagator [Maas, unpublished]



- Changes the asymptotic behavior of the gluon propagator at zero momentum

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  - Volumes are larger than (10 fm)^3
- Gluon propagator stronger infrared suppressed
  - Gauge requirement of minimal gluon propagator is even fulfilled locally
  - Possibly infrared vanishing
- Ghost propagator strongly affected – asymptotic limit unclear
- Quantitative and qualitative impact

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- In four dimensions even more impact to be expected

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- Which gauges lead in the continuum to the same correlation functions?

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- Add new gauge transformations
  - Randomly generated, but still to local Landau gauge inside the first Gribov horizon

Evolutionary algorithm [Maas, unpublished]

- Interpret a gauge transformation as the genetic code
- Begin with a population of gauge transformations to local Landau gauge into the first Gribov region
- Add new gauge transformations and mix existing ones
  - Take two, which belong to the more successful population
    - Lower minimum of the gauge-fixing functional
  - Create a new one, by taking half the elements from one and the other one from the other
    - Random, which element is from which

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- Begin with a population of gauge transformations to local Landau gauge into the first Gribov region
- Add new gauge transformations and mix/change existing ones
  - Create a new gauge transformation by changing a copy of a successful gauge transformation randomly at a random number of points in space-time – point mutations

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- Add new gauge transformations and mix/change existing ones and discard ineffective ones
  - The half of the ancestor generation with the hightes local minima
  - This half is replaced by the new/mixed/changed ones

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- Add new gauge transformations and mix/change existing ones and discard ineffective ones to get a new generation
- Repeat, until no improvement is found in a new generation
- Not guaranteed to find the absolute minimum
  - But a very successful approach in many applications
  - Requires for optimization more knowledge on the shape of local and absolute minima