IR QCD properties from ST, DS, and LQCD

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I. IR: existing tools
II. Our approach
III. Ward-Slavnov-Taylor identities
IV. Two solutions for Dyson Schwinger
V. Numerical solution of Dyson Schwinger
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Existing tools ?

- There are two sets of very usefus analytic relations to learn about QCD in the IR: Ward-Slavnov-Taylor (WST) identities and the infinite tower of Dyson-Schwinger (DS) integral equations. Lattice QCD give also essential numerical indications.

The best would be to have an analytic solution, however this is not possible:

- WST relates Green-Functions, not enough constraints.
- DS are too complicated, highly non linear, it is not known how many solutions exist, but there is presumably a large number.

Common way out ?

- Use truncated DS with some hypotheses about vertex functions and compare output to LQCD

Our approach

1- Combine informations from LQCD and analytic methods: not only using LQCd as an a posteriori check, but use it as an input for DSE. We believe that this allows a better control on systematic uncertainties of all methods.

2- Use WST identities (usually overlooked)

3- 1 and 2 are complemented with **mild** regularity assumptions about vertex functions

4- Take due care of the UV behaviour (known since QCD is asymptotically free) and use a well defined renormalisation procedure (no renormalisation at $\mu=0$ because of possible IR singularities).

Notations

- $G(p^2)$ is the gluon dressing function, $= p^2G^{(2)}(p^2)$, $G^{(2)}(p^2)$ being the gluon propagator, G LIKE GLUON

(frequent notation (fn): $D(p^2)$instead of $G^{(2)}(p^2)$)

- $F(p^2)$ is the ghost dressing function, $= p^2F^{(2)}(p^2)$, $F^{(2)}(p^2)$ being the ghost propagator, F LIKE FANTÔME

(fn: $G(p^2)$instead of $F^{(2)}(p^2)$)

- In the deep IR it is assumed $G(p^2) \propto (p^2)^{\alpha_G}$

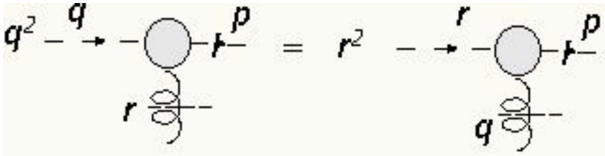
(fn: $p^2\,D(p^2) \propto (p^2)^{\alpha_D}$ or $(p^2)^{\delta gl}$; $\alpha_G=2\kappa$)

- In the deep IR it is assumed $F(p^2) \propto (p^2)^{\alpha_F}$

(fn: $p^2\,G(p^2) \propto 1/(p^2)^{\alpha_G}$ or $(p^2)^{\delta gh}$; $\alpha_F=-\kappa$)

IR Ghost propagator from WST identities

hep-ph/0007088, hep-ph/0702092



$$F(p^2)X(q,p;r) = F(r^2)X(q,r;p)$$

$$X(q,p;r) \;=\; a(q,p;r) - (r\cdot p)\; b(q,p;r) + (r\cdot q)\; d(q,p;r)$$

Cut lines: the external propagator has been cut; p,q,r : momenta

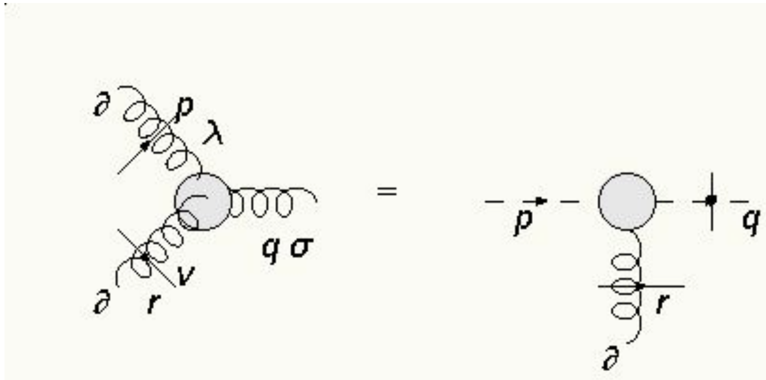
Identities valid for all covariant gauges

- Assuming X regular when one momentum vanishes, the lhs is regular when $r \to 0$, then the ghost dressing has to be finite non zero:

F(0) finite non zero, α_F=0 (fn: $p^2G(p^2) \to$finite$\neq 0$,α_G=0 or δ_{gh} = 0, κ =0)

- *There is almost no way out, unless the ghost-gluon vertex is singular* when only one momentum vanishes (difficult without violating Taylor's theorem)
- Does this contradict DS equations ? Lattice ? We do not believe, see later

IR Gluon propagator from WST identities, hep-ph/0507104, hep-ph/0702092



∂: longitudinal propagator, the third gluon is transverse.

Cut lines: the external propagator has been cut,

p,q,r : momenta

λνσ: gluons polarisation

When q →0 it is easy to prove that the lhs vertex function vanishes under mild regularity assumptions. The rhs goes to a finite limit (see Taylor theorem).Then the transverse gluon propagator must diverge:

$G^{(2)}(0)=\infty$,

α_G<1 (frequent notation: $D(p^2) \to \infty$ α_D <1 or δ_{gl} <1; κ < 0.5)

or **α_G=1** with very mild divergences, to fit with lattice indications: $G^{(2)}(0)$ = finite$\neq$0

unless

The three gluon vertex diverges for one vanishing momentum. It might diverge only in the limit of Landau gauge.

In arXiv:0801.2762, Alkofer et al, it is proven from DS that α_G **>=** 0, **OK**

$F(p^2)^2G(p^2)$ is proportional to a MOM renormalised coupling constant, g^f in a definite scheme (Von Smekal).

Lattice indicates $\alpha_G \sim 1$, $\alpha_F \sim 0_-$, $F(\mu^2)^2G(\mu^2) \to 0$, $g^f(\mu) \to 0$

A frequent analysis of the ghost propagator DS equation

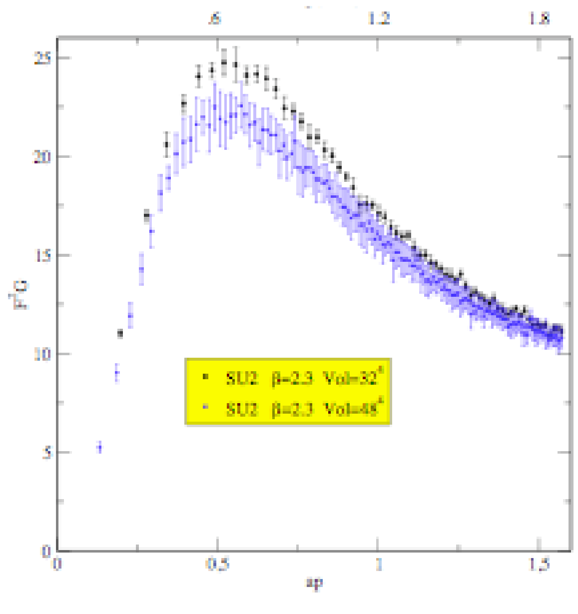
Leads to $2\alpha_F + \alpha_G = 0$ (fn: $\alpha_D = 2\,\alpha_G$ or $\delta_{gl} = -2\,\delta_{gl} = 2\kappa$) i.e. $F(p^2)^2G(p^2) \to ct$

and $F(p^2) \to \infty$

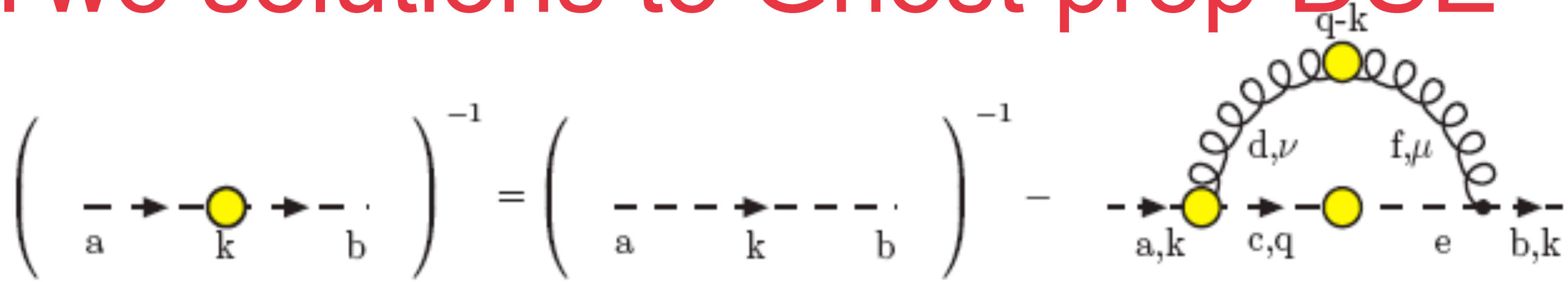
In contradiction with lattice

This is a strong, non truncated DS equation

So what ?



Two solutions to Ghost prop DSE



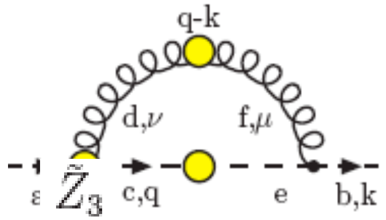
- The non-truncated ghost propagator DS equation
- It is also a WST equation !!!
- We will first prove that there are two types of solutions,

I. $2\alpha_F + \alpha_G = 0$ (fn: $\alpha_D = 2\alpha_G$ or or $\delta_{gl} = -2\ \delta_{gl} = 2\kappa$; « conformal solution »)

$F(p^2)^2 G(p^2) \rightarrow ct \neq 0$; In disagreement with lattice

II. $\alpha_F = 0$ (fn: $\alpha_G = 0$, « disconnected solution ») $F(p^2) \rightarrow ct \neq 0$ In fair agreement with lattice, see recent large lattices: I.L. Bogolubsky, et al. arXiv:0710.1968 [hep-lat], A. Cucchieri and T Mendes arXiv:0710.0412 [hep-lat], and in agreement with WST

- We will next show via a numerical study that solution I (II) are obtained when the coupling constant is non-equal (equal) to a critical value.



hep-ph/0507104, hep-ph/06040

- From anomalous dimensions it is easy to see that the loop is UV divergent. It needs a careful renormalisation (the subscript R stands for renormalised)

or to use a subtracted DSE with two different external momenta, thus cancelling the UV divergence.

$$\frac{1}{F_R(k^2)} - \frac{1}{F_R(k')^2} = -N_c g_R^2 \tilde{z}_1 \int \frac{d^4q}{(2\pi)^4} \left(1 - \frac{(k.q)^2}{k^2\, q^2}\right) \left[\frac{G_R((q-k)^2) H_{1R}(q,k)}{((q-k)^2)^2} - \frac{G_R((q-k')^2) H_{1R}(q,k')}{((q-k')^2)^2}\right] F_R(q^2)$$

The conclusion is the same. Let us make the argument with the more familiar unsubtracted form (although we have used mainly the subtracted form)

$$\frac{1}{F_R(k^2)} = \tilde{Z}_3 - N_c g_R^2 \tilde{z}_1 \int \frac{d^4q}{(2\pi)^4} \left(1 - \frac{(k.q)^2}{k^2\, q^2}\right) \left[\frac{G_R((q-k)^2) H_{1R}(q,k)}{((q-k)^2)^2}\right] F_R(q^2)$$

- $\tilde{Z}_3$ is the ghost prop renormalisation. It cancels the UV divergence. When $k \longrightarrow 0$ the lhs $\propto (k^2)^{-\alpha_F}$, $\tilde{Z}_3$ is independent of k.

I. If $\alpha_F < 0$, taking $k \rightarrow 0$, $\tilde{Z}_3$ has to be matched by the integra $\tilde{Z}_3 = g^2$ Integral(k=0), $g^2 = N_c g_R^2 z_1$

J. C. R. Bloch, Few Body Syst. **33** (2003) 111 [arXiv:hep-ph/0303125]

This leads to a well defined value for the coupling constant and the relation

$2\alpha_F + \alpha_G = 0$ (fn: $\alpha_D = 2\,\alpha_G$), $F(p^2)^2 G(p^2) \rightarrow ct \neq 0$, follows from a simple dimensional argument.

II. If $\alpha_F = 0$, the same integral is equal to: $\tilde{Z}_3 - 1/F_R(0) = g^2$ Integral(k=0),

the coupling constant now also depends on $F_R(0)$ which is finite non zero.

In the small k region, $F_R(k^2) = F_R(0) + c\,(k^2)^{\alpha'_F}$ and now the dimensional argument gives $\alpha'_F = \alpha_G$.

If $\alpha_G = 1$ then $F_R(k^2) = F_R(0) + c\,k^2 \log(k^2)$

To summarise

I. If $\alpha_F < 0$, $2\alpha_F + \alpha_G = 0$, $F(p^2)^2 G(p^2) \rightarrow ct \neq 0$ and fixed coupling constant at a finite scale; $\alpha_G = -2\,\alpha_F = 2\kappa$

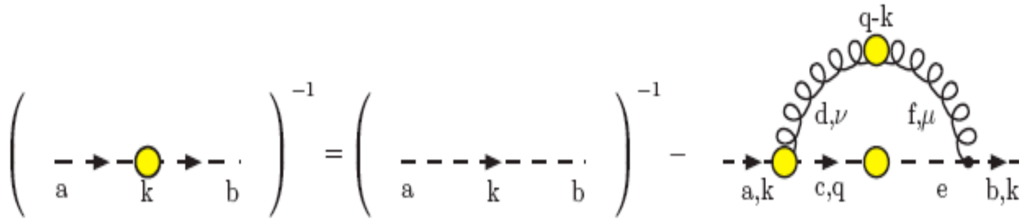
From arXiv:0801.2762, Alkofer et al, $-0.75 \leq \alpha_F \leq -0.5$, $1 \leq \alpha_G \leq 1.5$

II. if $\alpha_F = 0$, $F(p^2) \rightarrow ct \neq 0$ $\alpha'_F = \alpha_G$ and no fixed coupling constant

Notice: solution II agrees with WST

And, better and better with lattice !!

Numerical solutions to Ghost prop DSE



$$\frac{1}{F_R(k^2)} - \frac{1}{F_R(k')^2} = -N_c g_R^2 \widetilde{z}_1 \int \frac{d^4q}{(2\pi)^4} \left(1 - \frac{(k.q)^2}{k^2\, q^2}\right) \left[\frac{G_R((q-k)^2) H_{1R}(q,k)}{((q-k)^2)^2} - \frac{G_R((q-k')^2) H_{1R}(q,k')}{((q-k')^2)^2} \right] F_R(q^2)$$

- To solve this equation one needs an input for the gluon propagator G_R (we take it from LQCD, extended to the UV via perturbative QCD) and for the ghost-ghost-gluon vertex H_{1R}: regularity is usually assumed from Taylor identity and confirmed by LQCD.
- To be more specific, we take H_{1R} to be constant, and G_R from lattice data interpolated with the α_G=1 IR power. For simplicity we subtract at k'=0. We take μ=1.5 GeV.The equation becomes

$$\frac{1}{\widetilde{F}(k^2)} = \frac{1}{\widetilde{F}(0)} - \int \frac{d^4q}{(2\pi)^4} \left(1 - \frac{(k.q)^2}{k^2\, q^2}\right) \left[\frac{G_R((q-k)^2)}{((q-k)^2)^2} - \frac{G_R((q)^2)}{((q)^2)^2} \right] \widetilde{F}(q^2)$$

where $\widetilde{F}(k)=g(\mu)\, F_R(k,\mu)$. Notice that $\widetilde{F}(\mu)=g(\mu)$, with g defined as

$g^2=N_c g_R^2 \widetilde{z}_1 H_{1R}= N_c g_B^2 Z_3 \widetilde{Z}_3^2 H_{1B}$

- ~We find one and only one solution for any positive value of F(0). F(0)=∞ corresponds to a critical value:

$g_c^2 = 10\pi^2/(F_R^2(0)\, G_R^{(2)}(0))$ (fn: $10\pi^2/(D_R(0) \lim p^2 G_R(p^2))$

J. C. R. Bloch, Few Body Syst. **33** (2003) 111 [arXiv:hep-ph/0303125]

- This critical solution corresponds to $F_R(0)=\infty$, It is the solution I, with $2\alpha_F + \alpha_G=0$, $F(p^2)^2G(p^2) \to ct \neq 0$, a diverging ghost dressing function and a fixed coupling constant.

- The non-critical solutions, have $F_R(0)$ finite, i.e. $\alpha_F = 0$, the behaviour $F_R(k^2)=F_R(0) + c\, k^2 \log(k^2)$ has been checked.

- Not much is changed if we assume a logarithmic divergence of the gluon propagator for $k \to 0$: $F_R(k^2)=F_R(0) - c'\, k^2 \log^2(k^2)$

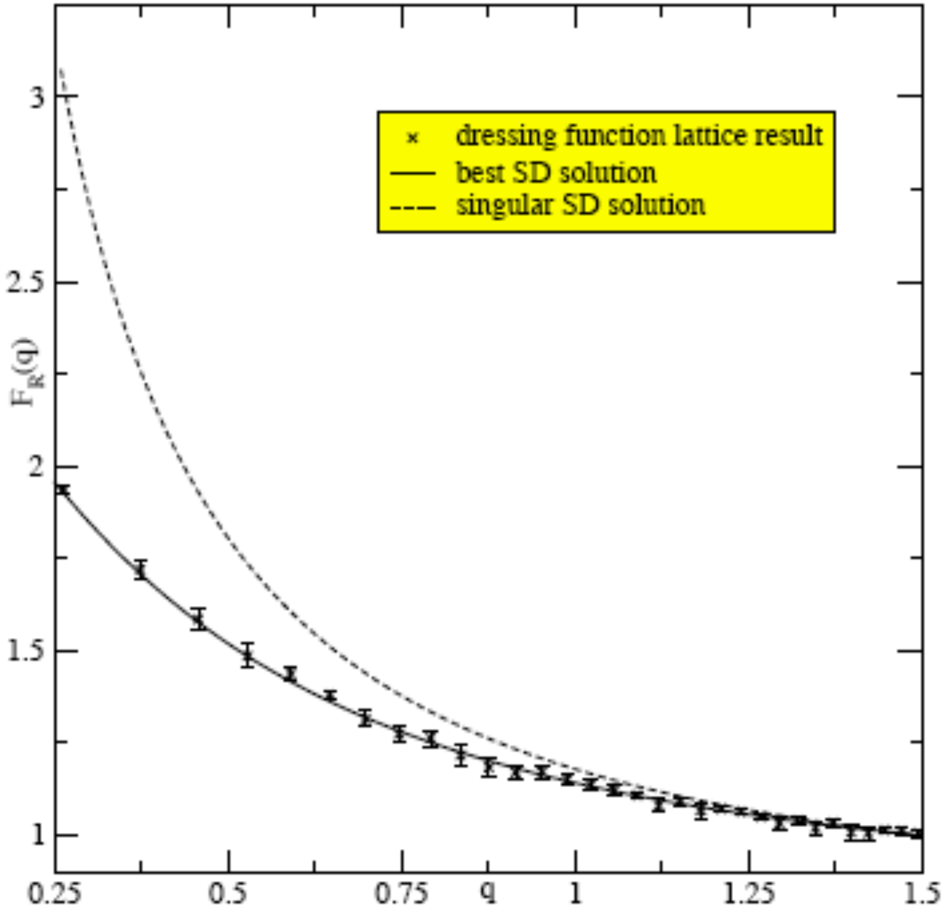


Figure 2: Comparison between the lattice SU(3) data at $\beta = 5.8$ and with a volume 32^4 for the ghost form factor and our continuum SD prediction renormalised at $\mu = 1.5\ GeV$ for $\tilde{g}^2 = 29$. (solid line) ; the agreement is striking ; also shown is the singular solution at $\tilde{g}^2 = 33.198....$ (broken line), which is obviously excluded.

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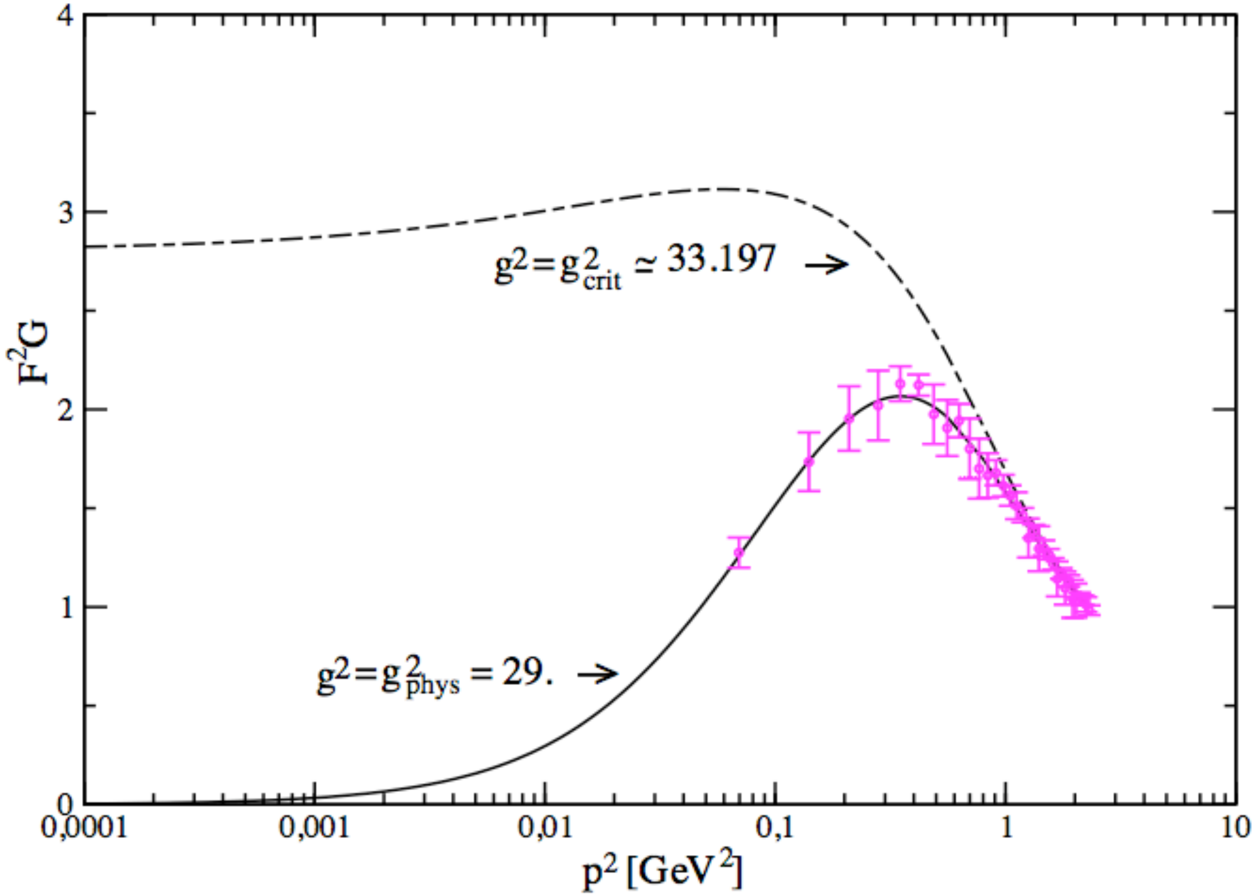
e-Print: arXiv:0801.2721 [hep-ph]

- The input gluon propagator is fitted from LQCD. The DSE is solved numerically for several coupling constants. The resulting F_R is compared to lattice results. For $g^2=29$, i.e. solution II ($F_R(0)$ finite, $\alpha_F=0$) the agreement is striking. The solution I ($F_R(0)$ infinite, $2\alpha_F + \alpha_G=0$) , dotted line, does not fit at all.

$F^2(p)G(p)$: the dotted line corresponds to the critical coupling constant. It is solution I, goes to a finite non zero value at $p \to 0$; the full line corresponds to the g2 which fits best lattice data. It corresponds to Solution II, $F^2(p)G(p) \to 0$ when $p \to 0$.



Conclusions

- Ward-Slavnov-Taylor identities imply at p=0, a finite non zero ghost dressing function and an infinite gluon propagator, unless respectively one ghost-gluon vertex, one three gluon vertex becomes infinite when one momentum vanishes.
- The ghost propagator Dyson-Schwinger equation allows for two types of solutions,

I) with a divergent ghost dressing function and a finite non zero F^2G, i.e. the relation $2\alpha_F + \alpha_G = 0$ (fn: $\alpha_D = 2\,\alpha_G$), « conformal solution »;

II) with a finite ghost dressing function and the relation $\alpha_F = 0$ (fn: $\alpha_G = 0$), « decoupled solution » and a vanishing F^2G

Lattice QCD clearly favors II) which **also** agrees with Ward-Slavnov-Taylor identities. The gluon propagator seems rather finite and non-zero although arXiv:0710.1968 might provide a mild indication of an increase in the deep IR.

To be done: check further propagators and vertex functions in Yang-Mills and in unquenched QCD. Generalise the combined use of WST, DS and LQCD.

Understanding confinement is a major scientific challenge. The recent results call for renewed approach to this problem.

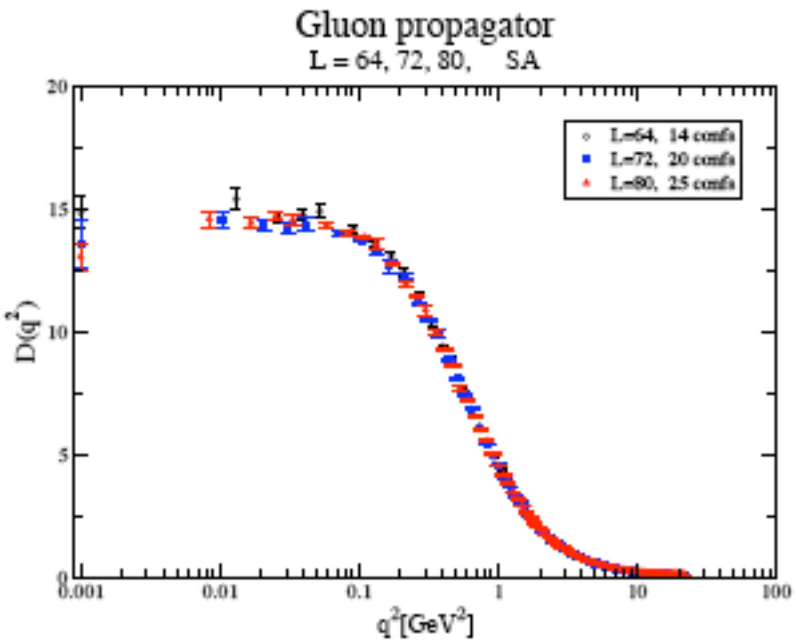
Back-up slide What do we learn from big lattices ?

N^3	a_G	N^4	a_G	N^3	a_G	N^4	a_G
140^3	0.073(4)	48^4	0.093(7)	140^3	0.13(2)	48^4	0.19(4)
200^3	0.051(3)	56^4	0.063(6)	200^3	0.06(2)	56^4	0.18(4)
240^3	0.003(3)	64^4	0.049(9)	240^3	0.10(2)	64^4	0.17(4)
320^3	-0.021(9)	80^4	0.052(5)	320^3	0.01(5)	80^4	0.15(2)
		112^4	0.038(6)			112^4	0.10(7)
		128^4	0.016(5)			128^4	0.06(3)

Table 1: Table for the ghost propagator IR exponent a_G, in the 3d and 4d cases, obtained using either the two smallest nonzero momenta (left) or the third and fourth smallest nonzero momenta (right).

Attilio Cucchieri, Tereza Mendes.Published in PoS (LATTICE 2007) 297. arXiv:0710.0412 [hep-lat]

here $\alpha_G = - \alpha_F$ (in our notations) = $- \delta_{gh}$

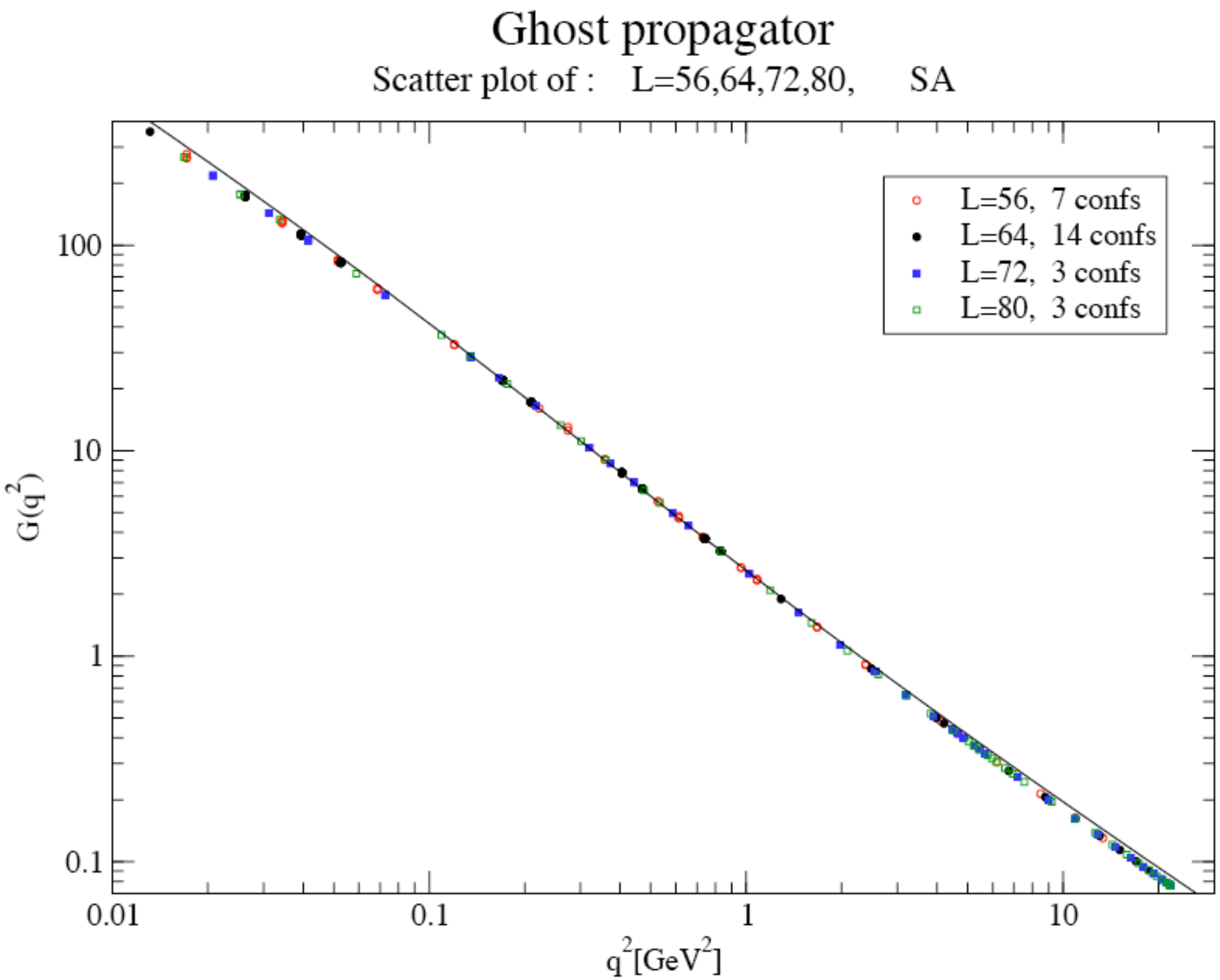


I.L. Bogolubsky, E.M. Ilgenfritz, M. Muller-Preussker, A. Sternbeck Published in PoS(LATTICE-2007)290. arXiv:0710.1968 [hep-lat]

They find α_F=-0.174 which seems at odds with both α_F=0 and $\alpha_F \leq$-0.5

But the fit is delicate, the power behaviour is dominant, if ever, only on a small domain of momenta.

Back-up slide



comparison of the lattice data of ref arXiv:0710.1968 with our solution arXiv:0801.2721.