IV The Renormalization group

basic idea

Statistical physics is based on the idea of reducing the number of degrees of freedom to describe a system:

microstate	macrostate
$N \sim 10^{23}$ variables	$\sim 5-6$ variables (pressure, temperature, ...)

the transition from a microstate to a macrostate description involves coarse graining operations in which exact distributions are replaced by the most probable distribution

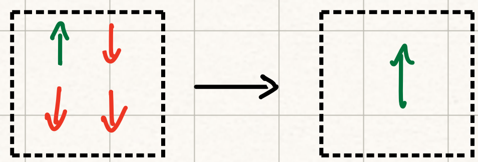
the Renormalization Group (RG) provides a systematic tool to progressively coarse grain a microscopic description of a system by means of a series of transformations, while each RG transformation (in real space) typically involves two steps:

1.) coarse grain the degrees of freedom of a microscopic system (e.g. by introducing "block spins")

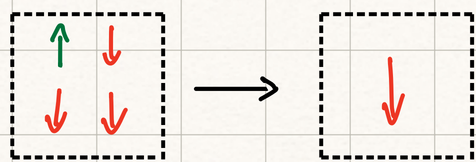
2.) Rescale basic variables such as lengths

note: the coarse graining process is NOT unique!
consider for example a spin system in $D=2$ dimensions,
we can form block spins by grouping together blocks of
4 spins each and assign a spin value to the block spin by:

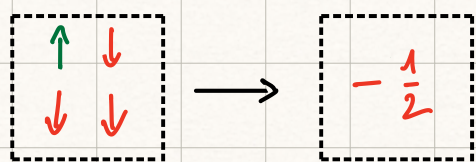
1.) choosing one spin in each block (e.g. top left corner) and assign that value to the corresponding block spin ("decimation")



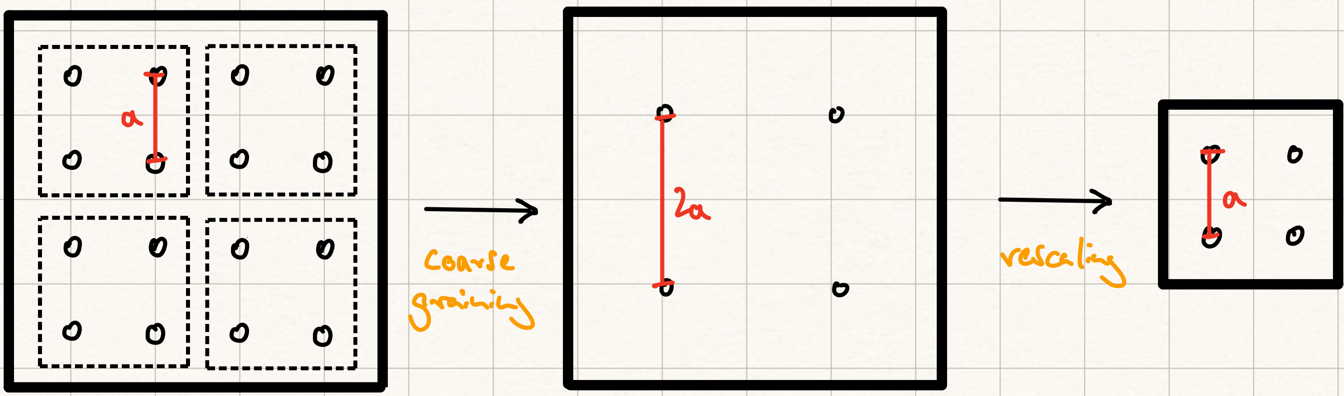
2.) use the most common value of spin in the block and assign it to the corresponding block spin ("majority rule"). if equal, toss a coin!



3.) average over the values of the spins in the block spin and assign mean value to the block spin



as a result of either of these operations the number of lattice sites in each dimension is reduced by a factor of 2



there are basically two kinds of coarse graining implementations:

1.) <u>real space RG</u>: RG transformations applied to a discrete system on a lattice in coordinate space

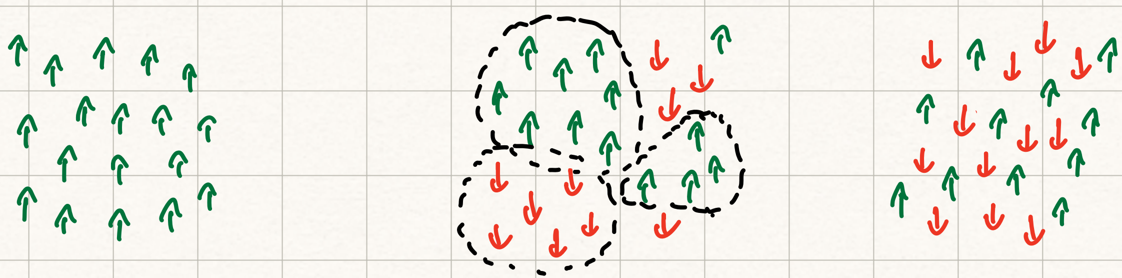
2.) <u>momentum-space RG</u>: RG transformations applied to a continuous system in momentum (Fourier) space

RG transformations applied repeatedly to a system

What happens after $N \to \infty$ of such transformations?

consider the 2d Ising model at 3 different temperatures:

$T = 0$	$T \lesssim T_c$	$T \to \infty$



perfect long-range order	correlation length of size $\xi(T)$	complete disorder
1	2	3

(1) all spins are aligned, system will not change, no matter how much we coarse grain, system invariant under RG transformations

$\Rightarrow$ "fix point" at $T=0$

(3) all spins randomly oriented, again system will look identical at every coarse graining scale

$\Rightarrow$ fix point at $T \to \infty$

(2) spins are a random sea with islands of correlated spins, each only exists for a brief amount of time, correlation length: $\xi(T)$

Question: are there RG fix points around T_c? *

for this we need to discuss the physical interpretation of ξ in more detail.

reminder: $$G(i,j) = \langle (\sigma_i - \langle \sigma_i \rangle)(\sigma_j - \langle \sigma_j \rangle) \rangle$$
$$= \langle \sigma_i \sigma_j \rangle - \langle \sigma_i \rangle \langle \sigma_j \rangle$$
$$\equiv \exp\left(-\frac{|i-j|}{\xi}\right)$$

Consider the 3 different temperatures above:

(1) all spin perfectly aligned: $\sigma_i = \langle \sigma_i \rangle$

$\Rightarrow G(i,j) = 0 \; , \; \xi(T=0) = 0$

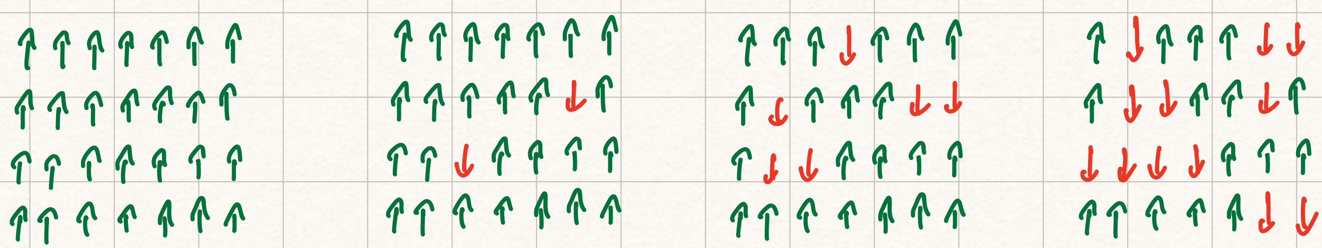
(3) all spins randomly oriented : $\langle \sigma_i \rangle = 0$

no correlation between two spins : $\langle \sigma_i \sigma_j \rangle = 0$

$\Rightarrow G(i,j) = 0 \; , \; \xi(T \to \infty) = 0$

ξ measures length scales over which fluctuations are correlated, not spins themselves!

(2) as T is increased from $T=0$, individual spins will start to flip in a sea of spins pointing in the opposite direction, i.e. $\xi(T)$ very small. as we approach T_c the size of spin clusters in the sea of spins can increase and will have a certain distribution of sizes characterized by ξ



$T=0$	$T_1 > 0$	$T_2 > T_1$	$T_3 < T_c$
$\xi = 0$	$\xi_1 > 0$	$\xi_2 > \xi_1$	$\xi_3 > \xi_2$