Phasenübergänge und die Renormierungsgruppe

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questions:
- physics students?
- completed theoretical physics I - IV ?
- language English / German ?
- field theory knowledge? not necessary !!
- further questions?

- please provide feedback if things can be improved or are unclear (too fast? too slow? requests for exercise problems? ...)
- please ask questions! use opportunity of having a course with only ~ 20 people! discussions very welcome

format: lectures always 2 hours, exercise problems every 2-3 weeks (home problems), discussion of problems on indicated date

recommended literature

* Lectures on phase transitions and the Renormalization Group
  Nigel Goldenfeld
  very readable book, discusses most of topics covered in this course + further details, focus on phase transitions, less RG
  highly recommended!

* Introduction to Renormalization Group methods in physics
  Creswick, Farach, Poole
  discussion of RG methods in various contexts, also some non-standard topics

* Statistical Mechanics

Pathria, Beale

discussion of basic concepts of statistical physics + modern applications like early universe, compact (but ~~detailed~~ with details) discussion of RG methods

* Phase transitions and the RG

Zinn-Justin

covers advanced topics, pre-knowledge of field theory recommended, nice reference for technical details

* Introduction to the Functional Renormalization Group

Kopietz, Bartosch, Schütz

phenomena in statistical physics can be divided into two categories

1.) consituents of system can be regarded as noninteracting; thermodynamic quantities can be determined based on single particle states of constituents

- specific heat of gases and solids
- blackbody radiation
- electron theory of metals
- chemical reactions
- Bose-Einstein condensation

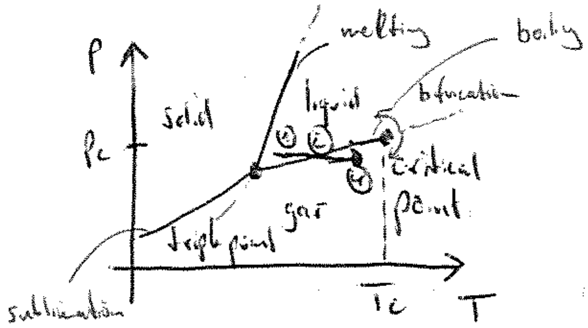
↑ usually studied in Theo IV

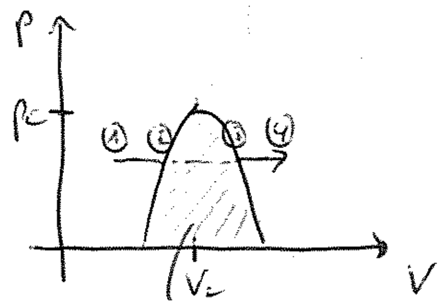
2) interactions between constituents play a key role, fluctuations need to be treated with care, thermodynamic quantities exhibit singularities → phase transitions

- condensation of gases
- coexistence of phases
- ferromagnetism
- superconductivity
- critical phenomena

⇑ subject of this course

consider liquid-gas phase transition, work at <u>constant pressure</u>





- start in liquid phase, apply heat ⇒ temperature increases and system expands ①
- when liquid begins to boil ② temperature remains constant while adding heat; volume still increases (latent heat)
- from ② to ③ fraction of gas and volume increases until all liquid has turned into gas
- further heating leads to temperature increase ④

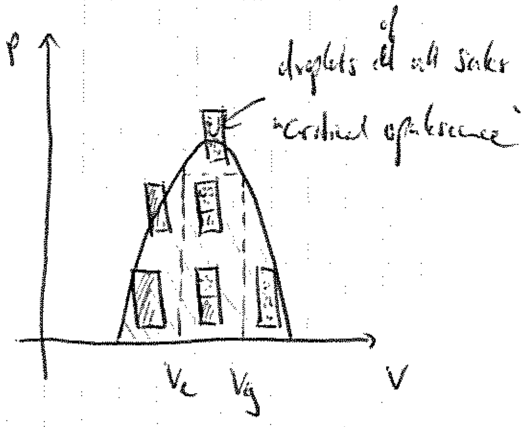
* critical point bifurcation point in phase diagram: 1 phase → 2 phase

around T_c the volume difference between liquid and gas becomes smaller, i.e. the width of coexistence region decreases in p-V

one finds

$$V_g - V_\ell \sim |T - T_c|^\beta$$

↑ critical exponent



similarly $C_V \sim |T - T_c|^{-\alpha}$

$$|p_g - p_\ell| \sim |V_g - V_\ell|^\delta$$

the correlation length ξ describes the spatial extend of fluctuations in a physical quantity
if there is a distribution of droplet sizes the correlation length describes the characteristic size; for $T \to T_c$ $\xi \to \infty$
⇒ critical opacity

⇒ theoretical description of physics around critical points very challenging

Why bother about such a small region in phase diagram?

* very interesting physics happening around critical points, microscopic details washed out due to diverging correlation length, and very different systems show identical critical behavior ⇒ universality
  ↳ only depend on range of int., dimension, symmetries of system
* study of critical phenomena triggered development of very powerful new methods ⇒ Renormalization Group
  was later applied to many different areas: chaos, fractals, networks, quantum field theory, many-body theory (all systems that involve degrees of freedom at very different length scales)
  ↳ Kenneth Wilson, Nobel prize 1982 for development of RG and application to critical phenomena, later methods heavily used in elementary particle physics

Outline

1. Review of relevant parts of Stat Phys

* ensembles, partition function, thermodynamic potentials, phase space, density matrix, ergodicity

2. Phase transitions

* definition of phases and phase boundaries, Ehrenfest classification, phase transitions in one and two dimensions, spontaneous symmetry breaking and ergodicity breaking, exact solution for 1d-Ising model, correlation length

3. Mean field theory

* critical phenomena in Ising model, liquid-gas transitions, exponents
  Van-der Waals equation of state, Landau theory of phase transitions (effective theories) universality, coarse graining, inclusion of correlations, breakdown of Landau-theory at low dimension

4. Introduction to the Renormalization Group

* block spins / coarse graining, fixed points, relevant/marginal/irrelevant couplings, application of RG to 1d Ising model, generalization to 2d, anomalous dimension, role of microscopic length scales

5. Applications of RG to different systems

* topological phase-transitions (Kosterlitz-Thouless phase transitions), Percolation, sol-gel transition, chaos, networks, Nuclear physics …
  depending on time