The PNJL model at imaginary chemical potential

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1 Motivation

At imaginary chemical potential QCD has an interesting symmetry, known as the Roberge-Weiss (RW) symmetry, resulting in the RW phase transition and periodicity [1]. Recent lattice QCD simulations have investigated this phase structure and found a mass dependence of the RW endpoint [2, 3].

As first-order phase transitions and second-order endpoints might influence the phase structure at real chemical potential, this is worth further effort.

In the Polyakov-loop extended Nambu–Jona-Lasinio (PNJL) model, we thus investigate the phase structure in the μ^2-T-plane. At imaginary quark chemical potential the PNJL model also features the RW symmetry and we find the RW periodicity as well as the RW phase transition.

The PNJL model at imaginary chemical potential has already been investigated by Sakai et al., e.g. [4, 5, 6]. In a two-flavor PNJL model we extend their work and study the order of the RW phase transition endpoint for different Polyakov-loop potentials and analyze its dependence on the relative strength of the potentials.

2 Model

We employ the PNJL model for two light quark flavors at real and imaginary chemical potential in mean-field approximation. Parameters are taken from [5].

$$\mathscr{L}_{\rm PNJL} = \bar\psi\left(i\gamma_\mu D^\mu - \hat m_f\right)\psi + \frac{g_S}{2}\left[(\bar\psi\psi)^2 + (\bar\psi i\gamma_5\tau_a\psi)^2\right] + \mathscr{U}_{\rm Polyakov}(\Phi[A],\bar\Phi[A],T)$$

We start with the logarithmic Polyakov-loop potential [7]:

$$\frac{\mathscr{U}_{\log}}{T^4} = -\frac{a(T)}{2}\Phi\bar\Phi + b(T)\log\left[1-6\Phi\bar\Phi+4(\Phi^3+\bar\Phi^3)-3(\Phi\bar\Phi)^2\right]$$

3 Roberge-Weiss periodicity and phase transition

The PNJL model has the same symmetry as QCD at imaginary chemical potential $\mu = i\theta T$: Certain shifts in the imaginary chemical potential can be undone by a $\mathbb{Z}_3$ transformation. This "extended $\mathbb{Z}_3$ transformation" is given by

$$\theta \;\rightarrow\; \theta + 2\pi k/3$$
$$\Phi \;\rightarrow\; \Phi\exp\left[-i2\pi k/3\right] \quad \text{with} \quad k\in\mathbb{Z}.$$

A convenient definition is the "modified Polyakov loop",

$$\Psi = \Phi\exp^{i\theta} \qquad \bar\Psi = \bar\Phi\exp^{-i\theta}$$

which is then invariant under the extended $\mathbb{Z}_3$ transformation.

It can easily be shown that the PNJL model is invariant under the extended $\mathbb{Z}_3$ transformation and thus possesses the RW periodicity at imaginary chemical potential.

At large temperatures the system undergoes a first-order "RW transition" between different $\mathbb{Z}_3$ sectors at $\theta = (2k+1)\pi/3$. θ-even quantities show a cusp, θ-odd quantities have a jump. For low temperatures this transition is a crossover. The behavior of the order parameters, especially at the RW transition, is shown in figs. 1 and 2.

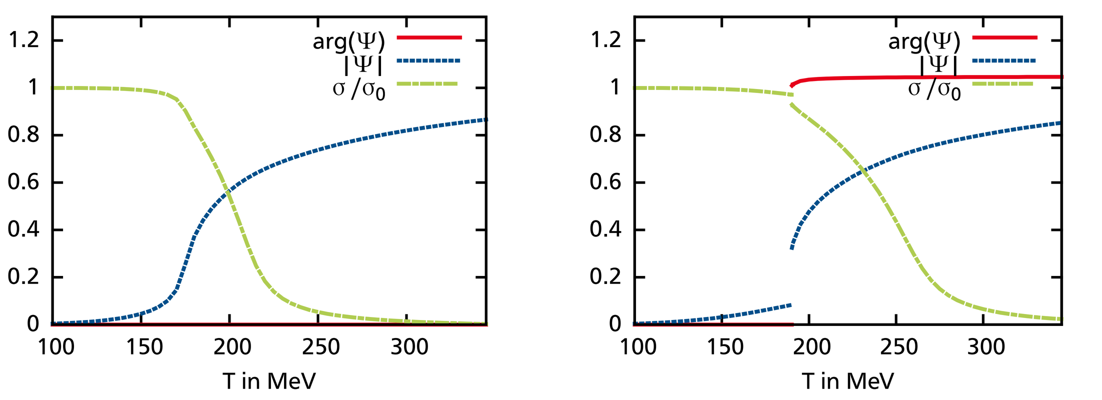

Fig. 1: Modified Polyakov-loop variables and the normalized chiral condensate at $\theta = 0$ (left) $\theta = \pi/3$ (right) as function of temperature.

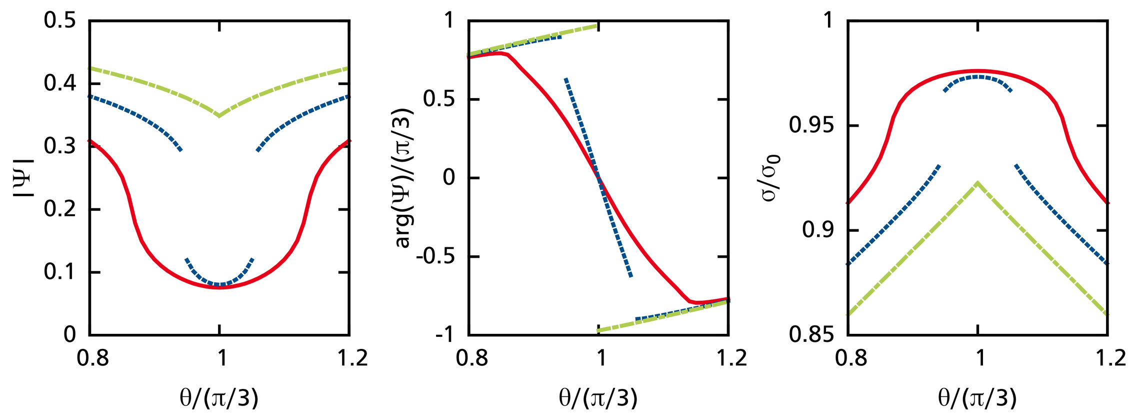

Fig. 2: Dependence of the modified Polyakov-loop variables and the normalized chiral condensate on θ for different temperatures around $T_{RW} = 190.3$ MeV (solid red: $T = 185$ MeV, dashed blue: 188 MeV, dot-dashed green: 191 MeV).

4 Roberge-Weiss endpoint

Lattice QCD simulations at imaginary chemical potential for two and three quark flavors have shown that the order of the RW endpoint depends on the quark masses [2, 3]. For low and high masses, the transition is of first order. In the intermediate mass range the transition changes to second order with tricritical points in between. A first-order transition at large quark masses can be expected from the limit of SU(3) gauge theory. If the transition at $\theta = \pi/3$ "ends" in a first-order transition, there must be first-order lines departing from it.

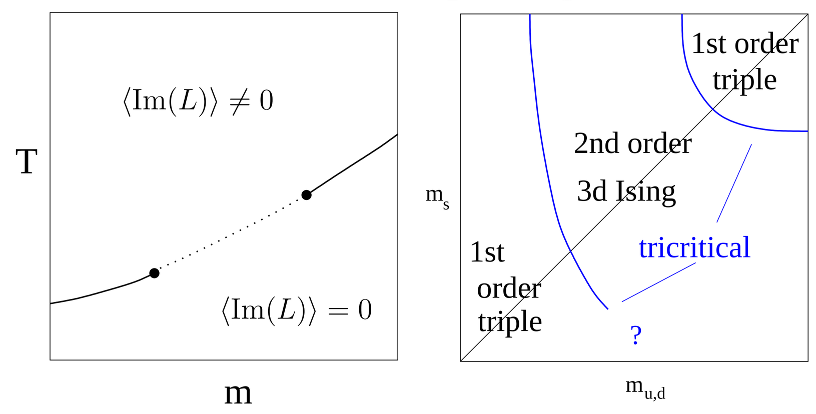

Fig. 3: $T-m$ phase diagram for $N_f = 3$ showing lines of triple points (solid) ending in tricritical points, connected with second-order line, and "Columbia plot" for RW endpoint at $\theta = \pi/3$, both from [3].

The PNJL phase diagram in fig. 4 shows first-order lines departing from the triple point. Crossover lines are determined by the maximum of the chiral susceptibility and the inflection point of the Polyakov-loop absolute value as function of temperature.

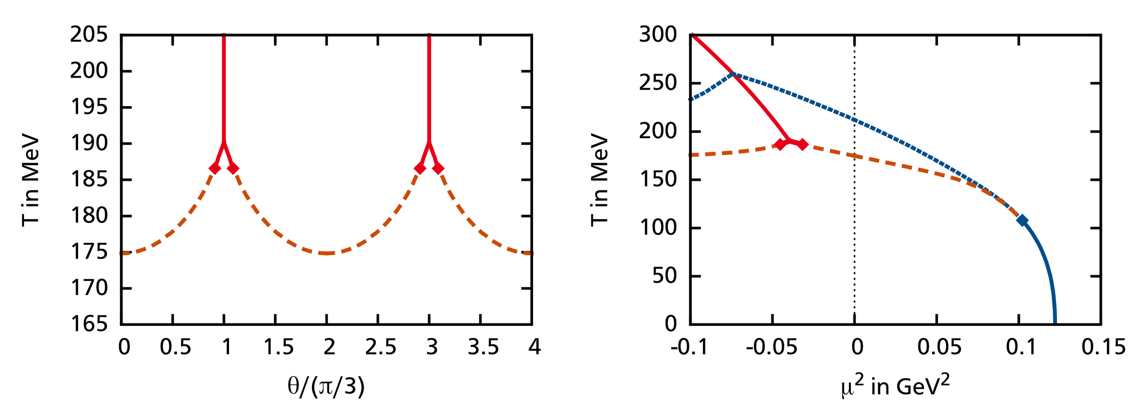

Fig. 4: Phase diagram in the $\theta-T$ (left) plane and the μ^2-T (right) plane. Solid (dashed) lines denote first-order (crossover) transitions. Dots represent second-order endpoints. Color coding: red: RW transition, orange: deconfinement transition, blue: chiral transition.

Mass dependence

Using the logarithmic potential we find the RW endpoint to be a triple point independent of the quark masses, contrary to lattice results. Increasing the quark masses makes the "RW legs" grow, see fig. 5. For bare quark masses larger than about $m_0 \approx 180$ MeV the first-order lines reach across the $\mu = 0$ axis. This scenario is shown in fig. 6 in comparison with the standard-parameter results.

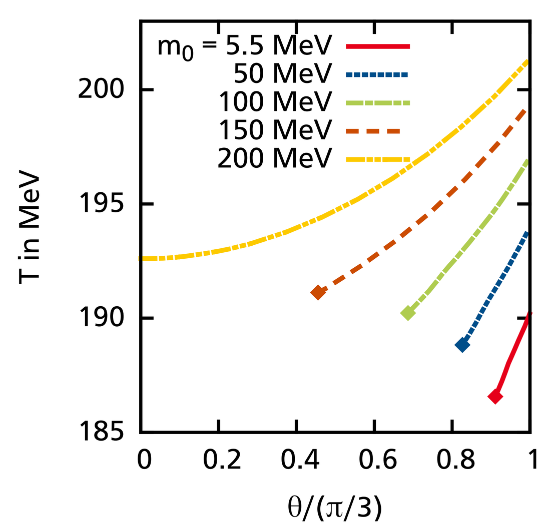

Fig. 5: RW "legs" in the $\theta-T$ phase diagram for different values of the bare quark mass m_0.

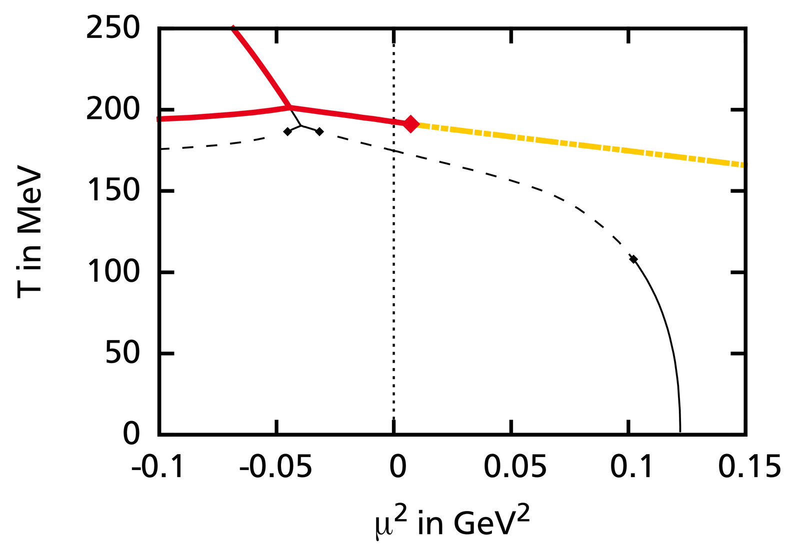

Fig. 6: Phase diagram in the μ^2-T plane. Black lines show the RW, deconfinement and chiral transitions for the standard value of $m_0 = 5.5$ MeV. Solid red (dashed yellow) lines show first-order RW/deconfinement (deconfinement crossover) transitions for a high bare quark mass $m_0 = 200$ MeV.

The authors of [8] report to reproduce the lattice QCD findings within the EPNJL model (using the logarithmic Polyakov loop potential), an extension of the PNJL model which uses a Polyakov-loop dependent coupling g_S.

other Polyakov loop potentials

Next we analyze the behavior of other Polyakov-loop potentials. The polynomial parametrization [9] leads to a second-order transition for all quark masses.

The Fukushima-type Polyakov-loop potential [10] gives a second-order transition for small quark masses and changes to first order for very high quark masses where the PNJL model is not applicable any more. Thus we shift the system towards the pure gauge limit by other means – by increasing the global factor b of the Fukushima-type Polyakov-loop potential.

$$\frac{\mathscr{U}_{\rm F}}{T^4} = -bT\left(54e^{-a/T}\Phi\bar\Phi + \log\left[1-6\Phi\bar\Phi+4(\Phi^3+\bar\Phi^3)-3(\Phi\bar\Phi)^2\right]\right)$$

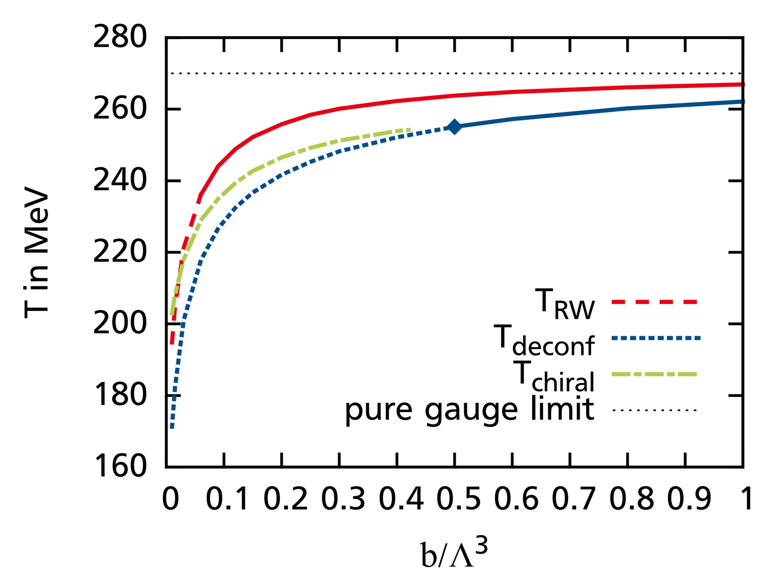

Fig. 7: The dependence of the transition temperatures and the order of the transition on parameter b. First-order (crossover) transitions are depicted as solid (dashed) lines.

Indeed, the RW endpoint changes from second to first order at about $b = 0.09\Lambda^3$ (default value for $N_f = 2$ is $b = 0.015\Lambda^3$). For $b > 0.5\Lambda^3$ the RW "legs" reach across the temperature axis. With increasing b all transition temperatures approach the pure-gauge limit of $T_c = 270$ MeV.

5 Outlook

- Calculate $N_f = 2+1$ "Columbia plot" for the order of the RW endpoint.

References

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