The PNJL model at imaginary chemical potential

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Motivation

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Lattice QCD

- Lattice QCD has sign problem for $Re(\mu) \neq 0$, but not for $Im(\mu) \neq 0$
- imaginary chemical potential $\mu = i\theta T$
- analytic continuation from $\mu^2 < 0$ to $\mu^2 > 0$

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Polyakov loop extended Nambu–Jona-Lasinio model

- calculations possible for real and imaginary chemical potential

Model setup

- PNJL model
- mean field
- two light quark flavors

- vacuum cutoff $\Lambda = 631.5$ MeV, bare quark mass $m_0 = 5.5$ MeV, scalar coupling $g_S = 4.385\Lambda^{-2}$
- following a series of publications by

Y. Sakai, K. Kashiwa, H. Kouno, M. Matsuzaki and M. Yahiro,
Phys. Rev. D **79**, 096001 (2009), *J. Phys. G: Nucl. Part. Phys.* **36**, 115010 (2009)
`arXiv:0902.0487` and `arXiv:0904.0925`.

Polyakov loop

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- $L(\vec{x}) = \mathcal{P} \exp \left[i \int_0^{1/T} d\tau \, A_4(\vec{x}, \tau) \right]$
- traced expectation values $\Phi = \frac{1}{N_c} \langle \mathrm{tr}\, L \rangle \qquad \bar{\Phi} = \frac{1}{N_c} \left\langle \mathrm{tr}\, L^\dagger \right\rangle$

Polyakov loop potentials

> polynomial [Ratti, Thaler, Weise (2006)]:

$$\frac{\mathcal{U}_{\mathrm{poly}}}{T^4} = -\frac{b_2(T)}{2}\Phi\bar{\Phi} - \frac{b_3}{6}(\Phi^3 + \bar{\Phi}^3) + \frac{b_4}{4}(\Phi\bar{\Phi})^2$$

> logarithmic [Rößner, Ratti, Weise (2007)]:

$$\frac{\mathcal{U}_{\log}}{T^4} = -\frac{a(T)}{2}\Phi\bar{\Phi} + b(T)\log\left[1 - 6\Phi\bar{\Phi} + 4(\Phi^3 + \bar{\Phi}^3) - 3(\Phi\bar{\Phi})^2\right]$$

> [Fukushima (2008)]:

$$\frac{\mathcal{U}_{\mathrm{Fuku}}}{T^4} = -bT\left(54e^{-a/T}\Phi\bar{\Phi} + \log\left[1 - 6\Phi\bar{\Phi} + 4(\Phi^3 + \bar{\Phi}^3) - 3(\Phi\bar{\Phi})^2\right]\right)$$

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extended $\mathbb{Z}_3$ symmetry

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- Roberge and Weiss (1986): QCD has a periodicity

$$\Omega_{QCD}(\theta) = \Omega_{QCD}(\theta + 2\pi k/3)$$

- Certain shifts in the imaginary chemical potential can be undone by a $\mathbb{Z}_3$ transformation. This “extended $\mathbb{Z}_3$ transformation” is given by

$$\begin{aligned} \theta &\rightarrow \theta + 2\pi k/3 \\ \Phi &\rightarrow \Phi \exp\left[-i2\pi k/3\right] \quad \text{with} \quad k \in \mathbb{Z}. \end{aligned}$$

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- $\Omega_{QCD}(\theta)$ and related quantities $(\sigma_f(\theta), \ldots)$ invariant under the extended $\mathbb{Z}_3$ transformation, have RW periodicity with period $\frac{2\pi}{3}$

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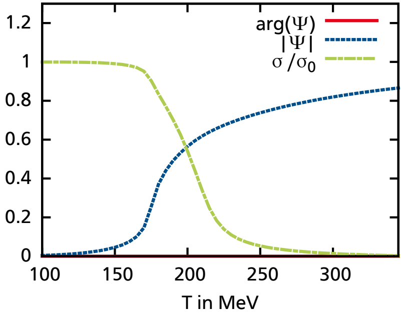
- $\Omega_{QCD}(\theta)$ and related quantities ($\sigma_f(\theta), \ldots$) invariant under the extended $\mathbb{Z}_3$ transformation, have RW periodicity with period $\frac{2\pi}{3}$
- RW phase transition at high temperatures and $\theta = \frac{\pi}{3}, \frac{3\pi}{3}, \ldots$

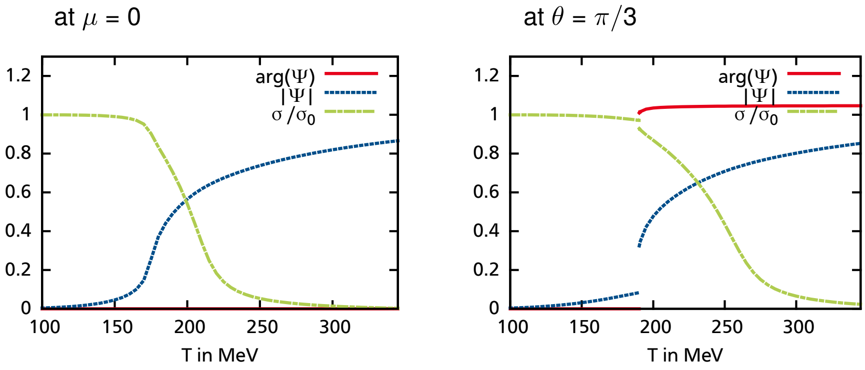
Order parameters

at $\mu = 0$



chiral and deconfinement crossover transition

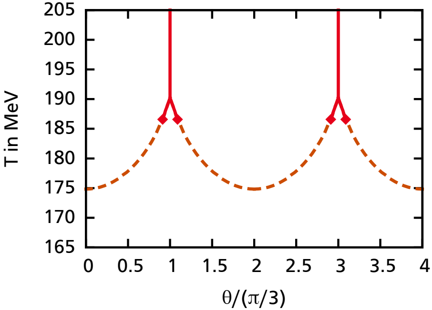
Order parameters



chiral and deconfinement crossover transition

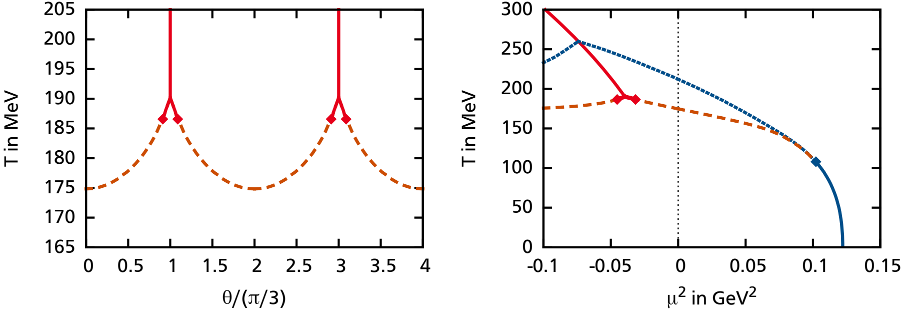
for convenience: modified Polyakov loop: $\Psi = \Phi e^{i\theta}$ (complex!)

Phase diagram



RW transition/1st order deconfinement transition
crossover deconfinement transition

Phase diagram



RW transition/1st order deconfinement transition

crossover deconfinement transition

1st order/crossover chiral transition

Dependence on Polyakov loop potentials

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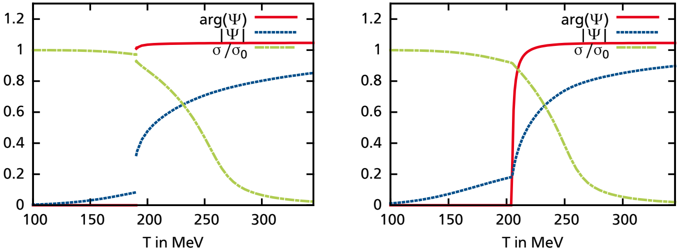
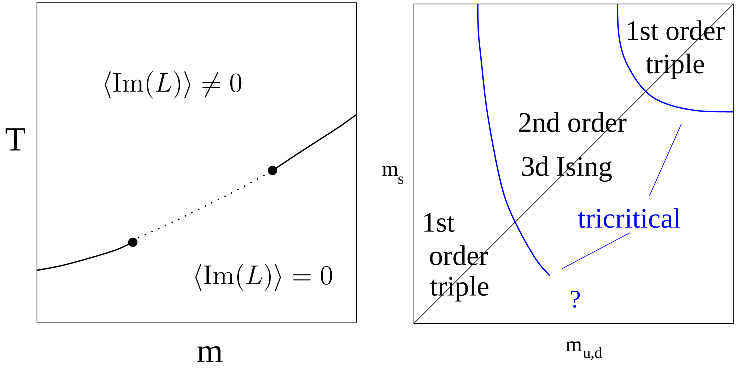
- for logarithmic Polyakov loop potential: always 1st order
- for polynomial Polyakov loop potential: always second order
- for Fukushima-type Polyakov loop potential: 2nd order for small quark masses, 1st order for very large quark masses

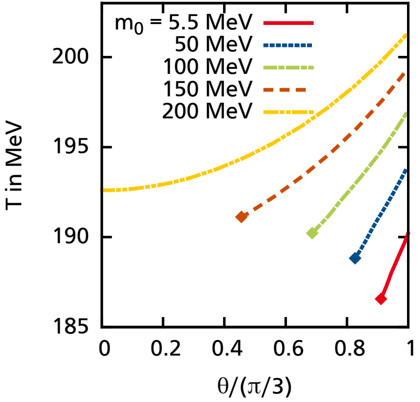


Order of the RW transition endpoint
Results from the Lattice



P. de Forcrand and O. Philipsen,
Phys. Rev. Lett. **105**, 152001 (2010),
arXiv:1004.3144.

Mass dependence (logarithmic Polyakov loop potential)



Mass dependence (logarithmic Polyakov loop potential)

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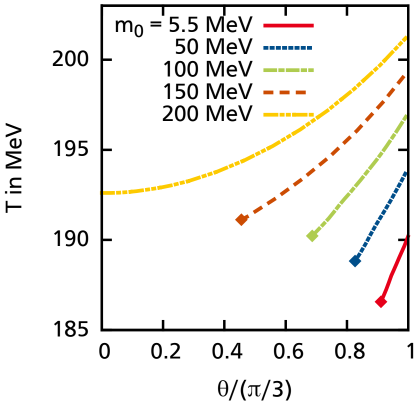
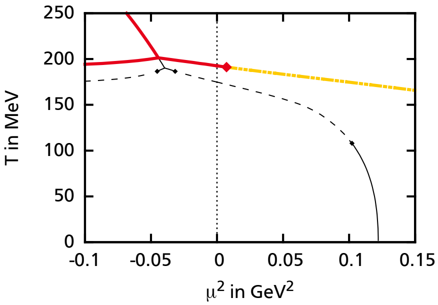
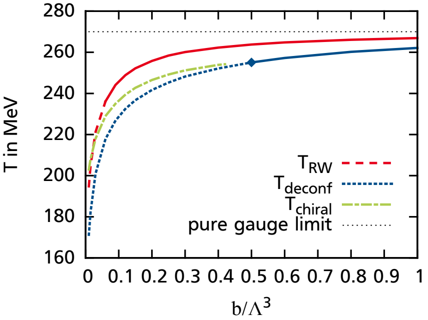


- m_0 = 200 MeV
- leg reaches into real chemical potential region



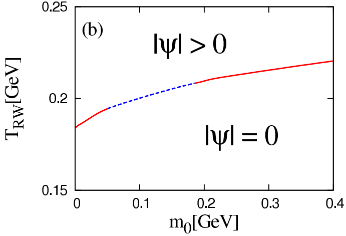
Change relative strength (Fukushima Polyakov loop potential)

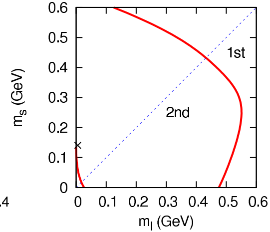
- $\frac{\mathcal{U}_{\mathrm{Fuku}}}{T^4} = -bT\left(54e^{-a/T}\Phi\Phi^* + \log\left[1 - 6\Phi\Phi^* + 4(\Phi^3 + \Phi^{*3}) - 3(\Phi\Phi^*)^2\right]\right)$
- pure gauge limit $\lim_{b\to\infty} T_{\mathrm{RW/deconf/chiral}} = T_c = 270\ \mathrm{MeV}$



Recent development: EPNJL model

- entanglement PNJL model
- Polyakov loop dependent coupling $g_S(\Phi) = g_S\left[1 - \alpha_1\Phi\bar{\Phi} - \alpha_2(\Phi^3 + \bar{\Phi}^3)\right]$





Y. Sakai, T. Sasaki, H. Kouno and M. Yahiro,
Phys. Rev. D **82**, 076003 (2010),
`arXiv:1006.3648` and `arXiv:1105.3959`.

Summary

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- PNJL calculations at imaginary μ possible
- mass dependence of RW endpoint not correctly reproduced, need improved model
- need to pin down Polyakov loop potential

Thank you!

References

- Y. Sakai, K. Kashiwa, H. Kouno, M. Matsuzaki and M. Yahiro,
  Phys. Rev. D **79**, 096001 (2009),
  arXiv:0902.0487.
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PNJL model

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- Model Lagrangian
$$\mathcal{L}_{PNJL} = \bar{\psi}\left(i\gamma_\mu D^\mu - m_f\right)\psi + \frac{g_S}{2}\left[(\bar{\psi}\tau_a\psi)^2 + (\bar{\psi}i\gamma_5\tau_a\psi)^2\right] + g_D\left[\det\left(\bar{\psi}(1-\gamma_5)\psi\right) + \det\left(\bar{\psi}(1+\gamma_5)\psi\right)\right] + \mathcal{U}_{Polyakov}(\Phi[A], \bar{\Phi}[A], T)$$
- Mean field approximation
$$\mathcal{L}_{MF} = \bar{\psi}\left(i\gamma_\mu D^\mu - M_f\right)\psi + g_S\left(\sigma_u^2 + \sigma_d^2 + \sigma_s^2\right) + 4g_D\sigma_u\sigma_d\sigma_s + \mathcal{U}_{Polyakov}(\Phi, \bar{\Phi}, T)$$
- chiral condensates σ_f: order parameters for chiral transition
- thermodynamic potential $\Omega(T, \mu; \sigma_f, \Phi, \bar{\Phi})$
- stationary conditions (“gap equations”): $\frac{\partial\Omega}{\partial X} = 0 \qquad X = \left\{\sigma_{u,d,s}, \Phi, \bar{\Phi}\right\}$
- regularized by three-momentum cut-off

Interplay: $\mathbb{Z}_3$ symmetry in pure gauge limit

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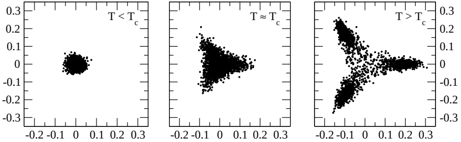
- $\mathbb{Z}_3$ is center of SU(3) gauge group
- low T: only one ground state ($\Phi = 0$)
- high T: three degenerate ground states ($\Phi \propto \exp\left[i\frac{2\pi k}{3}\right]$), $\mathbb{Z}_3$ symmetry is spontaneously broken
- first order transition at $T_c = 270$ MeV



from: Hagen, Bruckmann, Bilgici, Gattringer; PoS LAT2007

Close to transition endpoint