Aspects of the PNJL Model at Imaginary Chemical Potential

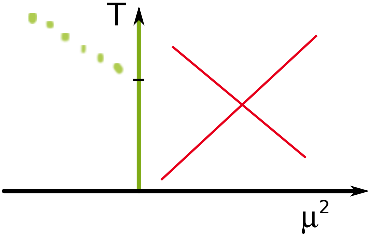
Outline

1. Motivation
2. The PNJL model at imaginary chemical potential
3. Extrapolation of crossover transition lines
4. [Roberge–Weiss transition endpoint]

Motivation

Lattice QCD

- Lattice QCD has sign problem for *Re*$(\mu) \neq 0$, but not for *Im*$(\mu) \neq 0$
- imaginary chemical potential $\mu = i\theta T$
- use extrapolation from $\mu^2 < 0$ to $\mu^2 > 0$



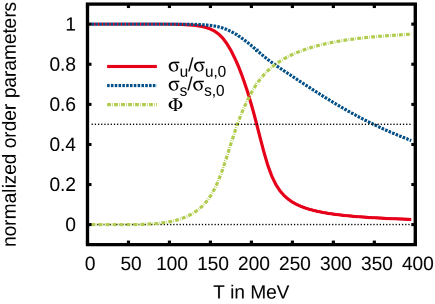
Motivation

PNJL model

- calculations possible for real and imaginary chemical potential
- perform extrapolation from $\mu^2 < 0$ to $\mu^2 > 0$ and compare with direct calculations
- 2 quark flavors: [Sakai et. al. (2008 & 2009)]
- here: 2+1 quark flavors

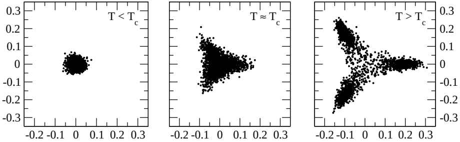
PNJL Model

- chiral condensate σ: (approximate) order parameter for chiral transition
- Polyakov loop Φ: (approximate) order parameter for deconfinement transition



Interplay: $\mathbb{Z}_3$ symmetry in pure gauge limit

- $\mathbb{Z}_3$ is center of SU(3) gauge group
- low T: only one ground state ($\Phi = 0$)
- high T: three degenerate ground states ($\Phi \propto \exp\left[i\frac{2\pi k}{3}\right]$), $\mathbb{Z}_3$ symmetry is spontaneously broken
- first order transition at $T_c = 270$ MeV



from: Hagen, Bruckmann, Bilgici, Gattringer; PoS LAT2007

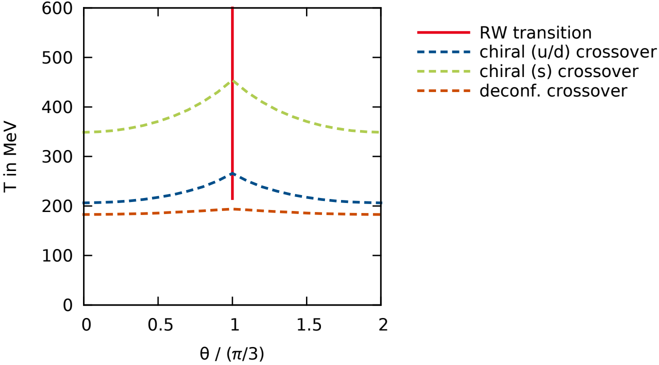
Special properties at imaginary chemical potential

- imaginary chemical potential $\mu = i\theta T$
- Roberge & Weiss (1986): QCD has periodicity $\Omega_{QCD}(\theta) = \Omega_{QCD}(\theta + 2\pi k/3)$, connected by $\mathbb{Z}_3$ transformation
- "extended $\mathbb{Z}_3$ transformation":

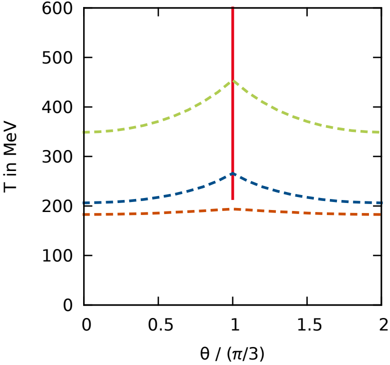
$$\Phi \to \Phi e^{-i2\pi k/3} \text{ and } \theta \to \theta + 2\pi k/3$$

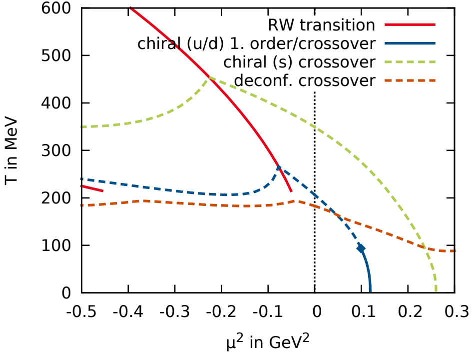
- invariant in QCD and PNJL: thermodynamic potential $\Omega(\theta)$, derived quantities $(\sigma_f(\theta), \dots)$
- RW periodicity with period $\frac{2\pi}{3}$
- RW phase transition at high temperatures and $\theta = \frac{\pi}{3}, \frac{3\pi}{3}, \dots$

Phase diagram

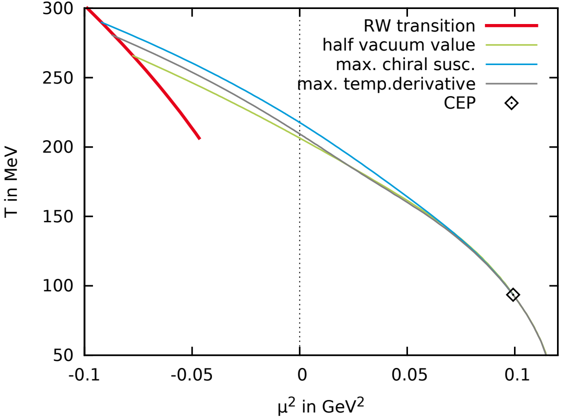


Phase diagram





Chiral crossover transition



Extrapolation of the chiral crossover line

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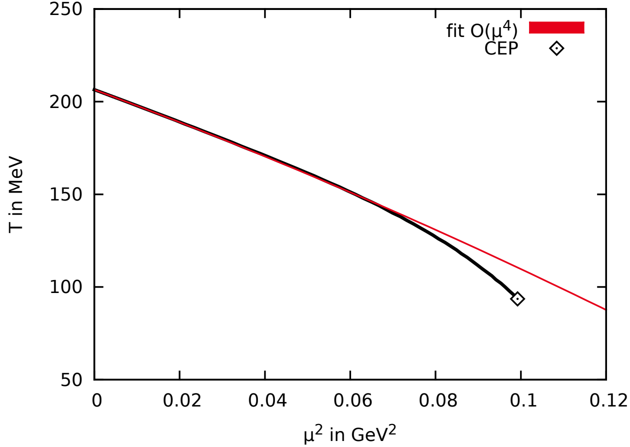
Crossover criterion: half vacuum value



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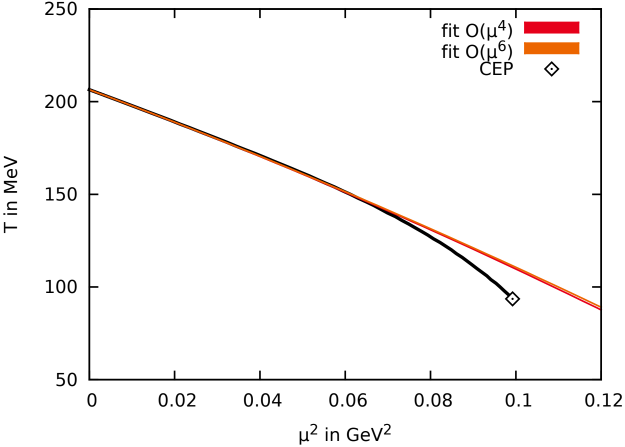

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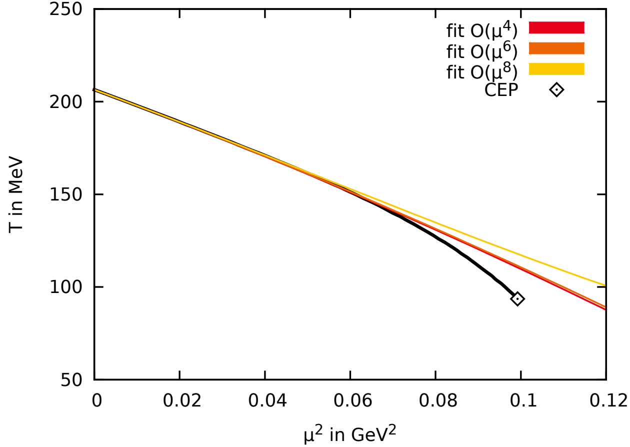

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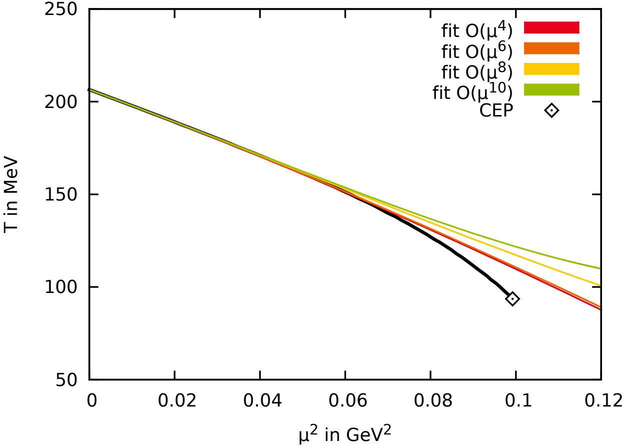

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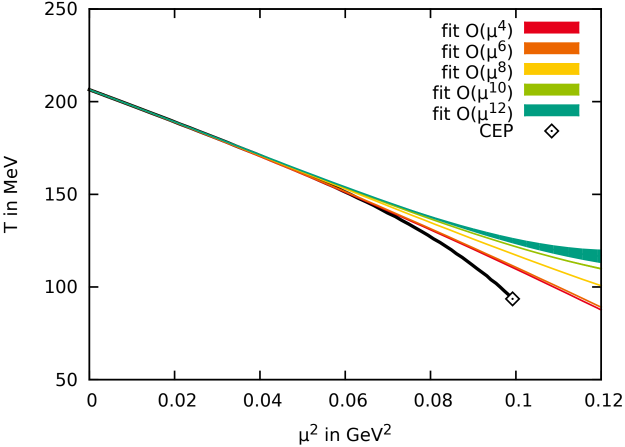
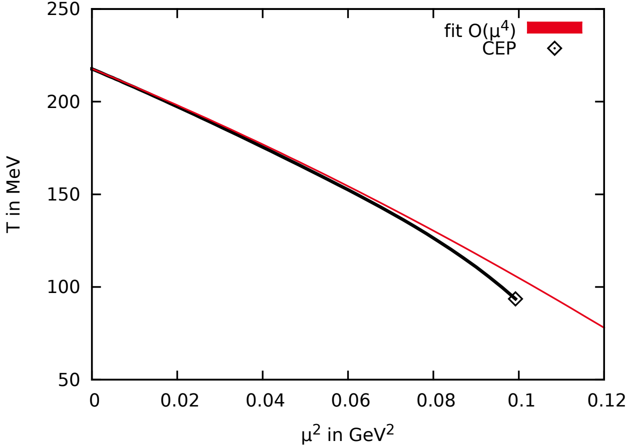


Extrapolation of the chiral crossover line

Crossover criterion: maximal chiral susceptibility



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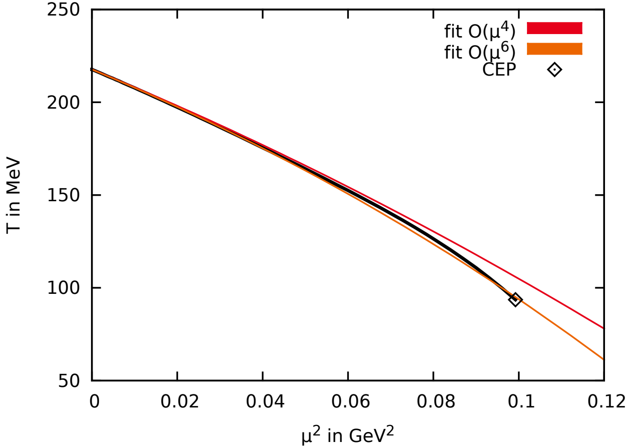

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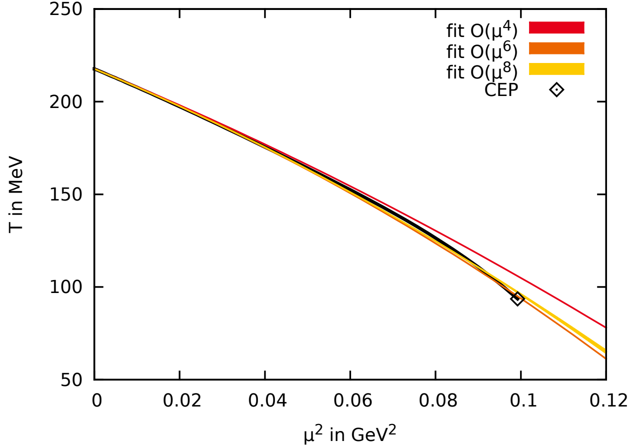
Crossover criterion: maximal chiral susceptibility



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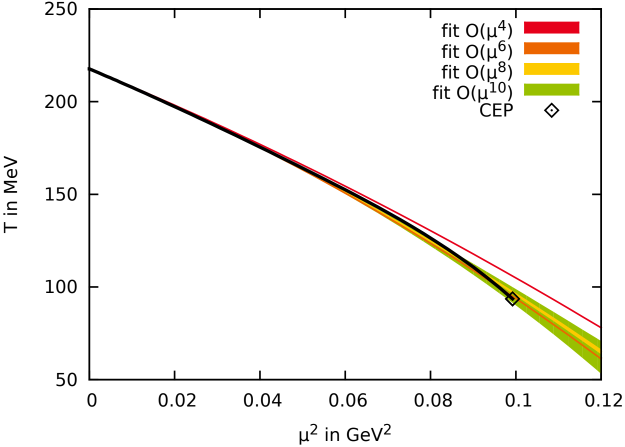
Crossover criterion: maximal chiral susceptibility



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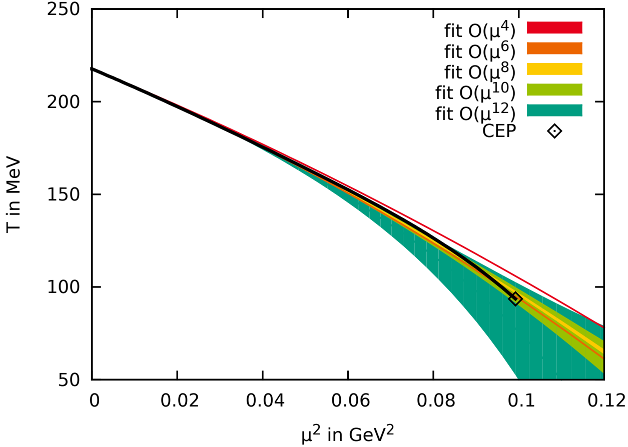
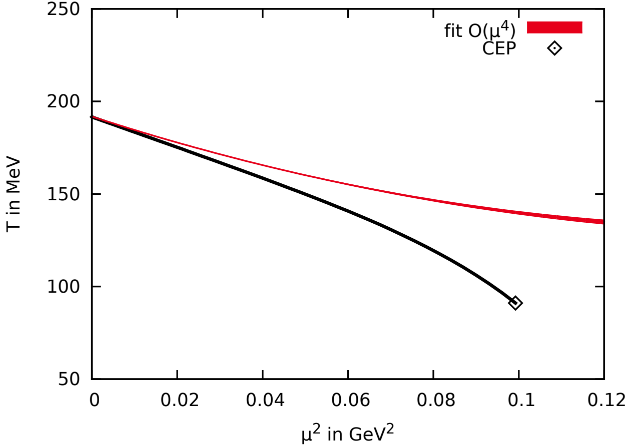


Extrapolation of the chiral crossover line

Crossover criterion: max. chiral susc. (different regularization)



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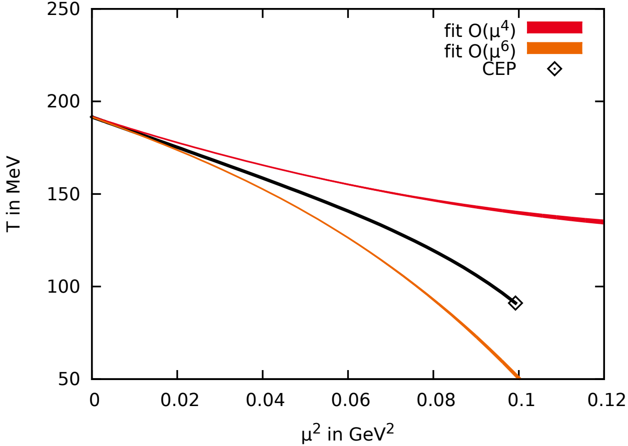

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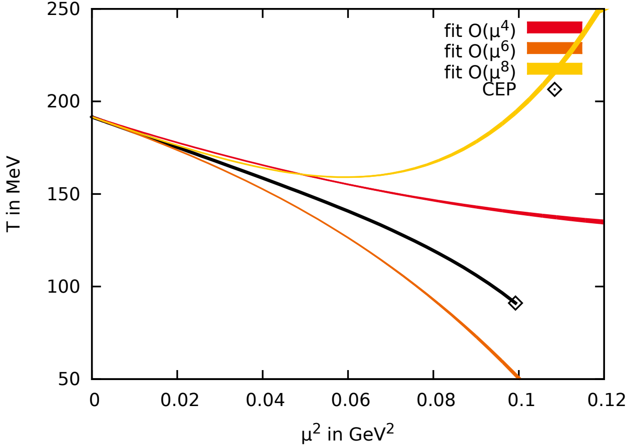

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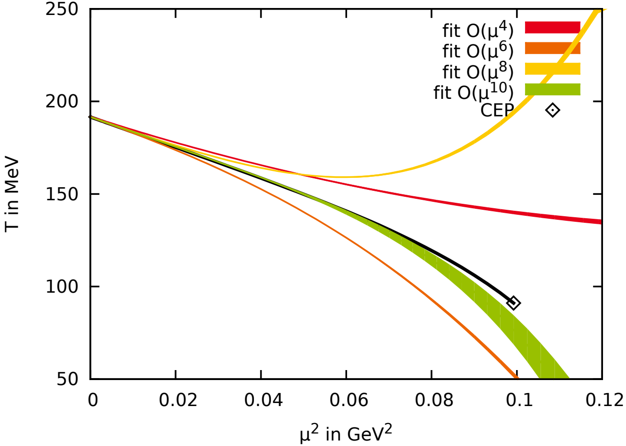

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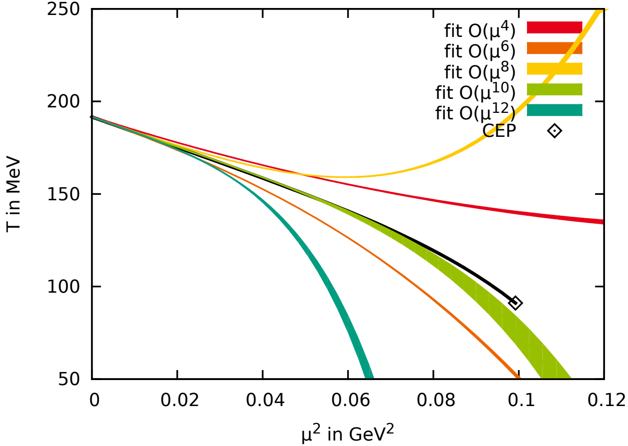


Summary and outlook

Summary

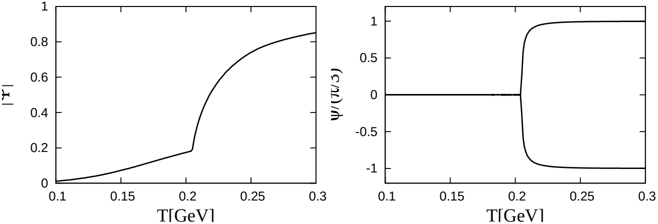
- PNJL model has RW symmetry & periodicity
- extrapolation of crossover lines from imaginary to real chemical potential possible
- not always reliable

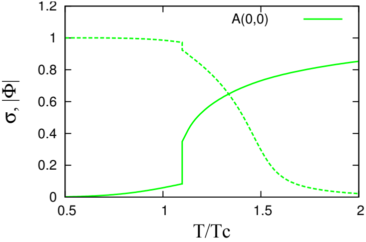
Outlook

- more sophisticated fit ansätze, . . .
- comparison with results from lattice QCD

Alternative topic:
RW transition endpoint

Motivation
RW endpoint in the PNJL model





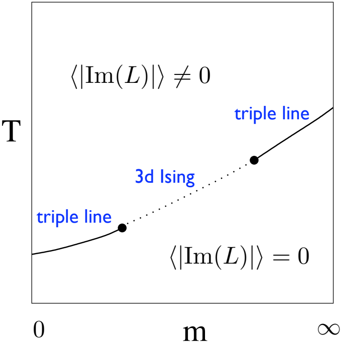
top: Fukushima Polyakov loop potential, “The RW transition is second order at the endpoint.”

Kouno et al., arXiv:0904.0925

bottom: Logarithmic Polyakov loop potential, 1st order.

Sakai et al., arXiv:0902.0487

Motivation
RW endpoint in Lattice QCD



- 1st order for low and high quark masses
- 2nd order for intermediate quark masses

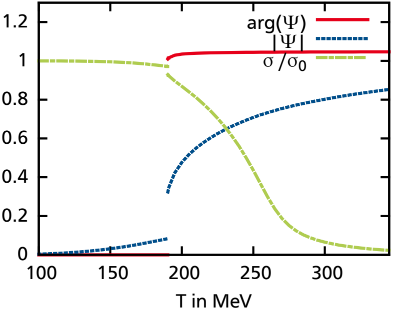
D'Elia/Sanfilippo, arXiv:0909.0254 (N_f = 2)

picture from: Philipsen, FAIR Lattice Days 2009 (N_f = 3)

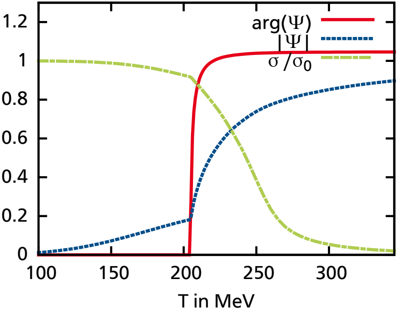
Our study of the RW transition endpoint

- PNJL model with $N_f = 2$
- redo and extend calculations by Kouno/Sakai et al.
- investigate dependence on quark masses and Polyakov loop potential

RW transition revisited

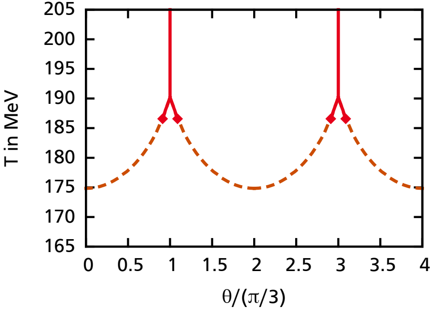


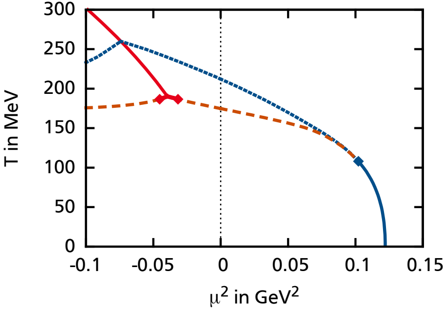
Logarithmic potential
First order ✓



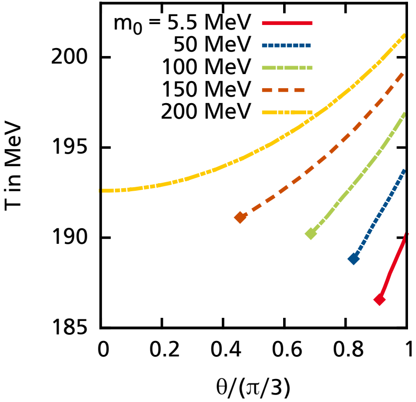
Fukushima potential
Second order ✓

RW legs
(logarithmic Polyakov loop potential)





RW legs at higher m_0 (logarithmic Polyakov loop potential)



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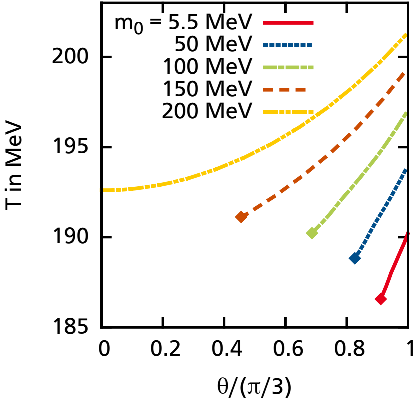
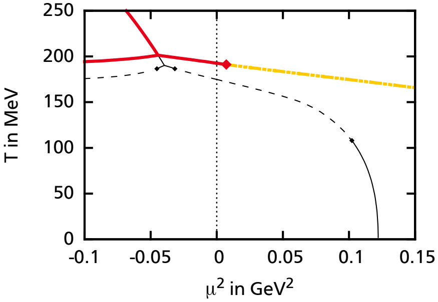
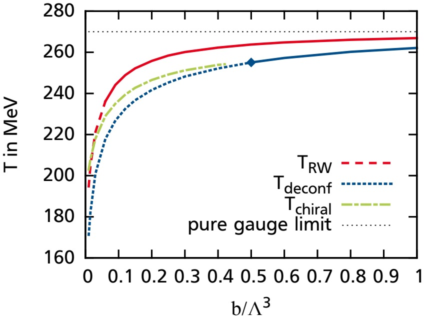


- m_0 = 200 MeV
- leg reaches into real chemical potential region



Change relative strength
(Fukushima Polyakov loop potential)

- $\frac{\mathcal{U}_{\text{Fuku}}}{T^4} = -bT\left(54e^{-a/T}\Phi\Phi^* + \log\left[1 - 6\Phi\Phi^* + 4(\Phi^3 + \Phi^{*3}) - 3(\Phi\Phi^*)^2\right]\right)$
- pure gauge limit $\lim_{b\to\infty} T_{\text{RW/deconf/chiral}} = T_c = 270\text{ MeV}$



Summary and outlook

Summary

- RW endpoint is of 1st or 2nd order depending on Polyakov loop potential
- relative strength of the Polyakov loop potential is important

Outlook

- "Columbia style"-plot: order of RW endpoint depending on quark masses ($N_f = 2 + 1$)
- ...

Thanks for your attention!

Polyakov loop extended Nambu–Jona-Lasinio model

- Lagrangian
$$\begin{aligned}\mathcal{L}_{PNJL} = {} & \bar{\psi}\left(i\gamma_\mu D^\mu - m_f\right)\psi \\ & + \frac{g_S}{2}\left[(\bar{\psi}\tau_a\psi)^2 + (\bar{\psi}i\gamma_5\tau_a\psi)^2\right] \\ & + g_D\left[\det\left(\bar{\psi}(1-\gamma_5)\psi\right) + \det\left(\bar{\psi}(1+\gamma_5)\psi\right)\right] \\ & + \mathcal{U}_{\text{Polyakov}}(\Phi[A], \bar{\Phi}[A], T)\end{aligned}$$

- in mean field approximation

$$\mathcal{L}_{MF} = \bar{\psi}\left(i\gamma_\mu D^\mu - M_f\right)\psi + g_S\left(\sigma_u^2 + \sigma_d^2 + \sigma_s^2\right) + 4g_D\sigma_u\sigma_d\sigma_s + \mathcal{U}_{\text{Polyakov}}(\Phi, \bar{\Phi}, T)$$

Polyakov loop

- $L(\vec{x}) = \mathcal{P} \exp \left[i \int_0^{1/T} d\tau \; A_4(\vec{x}, \tau) \right]$
- traced expectation values $\Phi = \frac{1}{N_c} \left\langle \mathrm{tr}\, L \right\rangle \qquad \bar{\Phi} = \frac{1}{N_c} \left\langle \mathrm{tr}\, L^\dagger \right\rangle$

Polyakov loop potentials

- polynomial [Ratti, Thaler, Weise (2006)]:

$$\frac{\mathcal{U}_{\text{poly}}}{T^4} = -\frac{b_2(T)}{2} \Phi \Phi^* - \frac{b_3}{6} (\Phi^3 + \Phi^{*3}) + \frac{b_4}{4} (\Phi \Phi^*)^2$$

- logarithmic [Rößner, Ratti, Weise (2007)]:

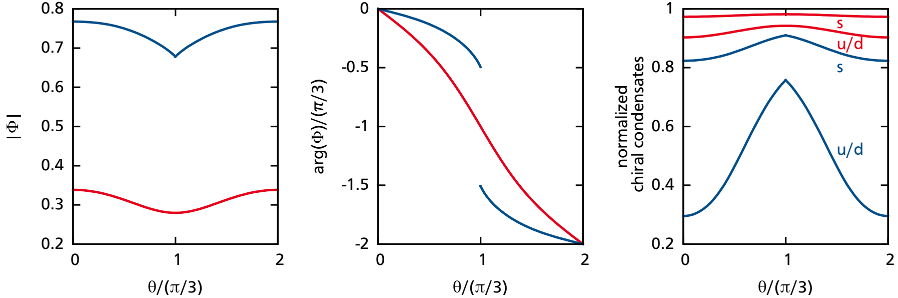
$$\frac{\mathcal{U}_{\log}}{T^4} = -\frac{a(T)}{2} \Phi \Phi^* + b(T) \log \left[1 - 6 \Phi \Phi^* + 4 (\Phi^3 + \Phi^{*3}) - 3 (\Phi \Phi^*)^2 \right]$$

- [Fukushima (2008)]:

$$\frac{\mathcal{U}_{\text{Fuku}}}{T^4} = -bT \left(54 e^{-a/T} \Phi \Phi^* + \log \left[1 - 6 \Phi \Phi^* + 4 (\Phi^3 + \Phi^{*3}) - 3 (\Phi \Phi^*)^2 \right] \right)$$

Order parameters
at imaginary chemical potential

$T_1 < T_{RW} < T_2$



RW transition revisited (II)

T = 185, 188, 191 MeV