QCD phase diagram from the PNJL model at imaginary chemical potential

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Motivation

Lattice QCD

- Lattice QCD has sign problem for $Re(\mu) \neq 0$, but not for $Im(\mu) \neq 0$
- imaginary chemical potential $\mu = i\theta T$
- use analytic continuation from $\mu^2 < 0$ to $\mu^2 > 0$

Polyakov loop extended Nambu–Jona-Lasinio model

- calculations possible for real and imaginary chemical potential
- use analytic continuation from $\mu^2 < 0$ to $\mu^2 > 0$ and compare to ordinary calculations
- 2 flavors [Sakai, Kashiwa, Kouno, Matsuzaki, Yahiro (2008&2009)]
- here: 2+1 flavors

PNJL Model

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- Model Lagrangian $\mathcal{L}_{PNJL} = \bar{\psi}\left(i\gamma_\mu D^\mu - m_f\right)\psi$
$$+ \frac{g_S}{2}\left[(\bar{\psi}\tau_a\psi)^2 + (\bar{\psi}i\gamma_5\tau_a\psi)^2\right]$$
$$+ g_D\left[\det\left(\bar{\psi}(1-\gamma_5)\psi\right) + \det\left(\bar{\psi}(1+\gamma_5)\psi\right)\right]$$
$$+ \mathcal{U}_{Polyakov}(\Phi[A], \bar{\Phi}[A], T)$$
- Mean field approximation
$\mathcal{L}_{MF} = \bar{\psi}\left(i\gamma_\mu D^\mu - M_f\right)\psi + g_S\left(\sigma_u^2 + \sigma_d^2 + \sigma_s^2\right) + 4g_D\sigma_u\sigma_d\sigma_s + \mathcal{U}_{Polyakov}(\Phi, \bar{\Phi}, T)$
- chiral condensates σ_f: order parameters for chiral transition
- thermodynamic potential $\Omega(T, \mu; \sigma_f, \Phi, \bar{\Phi})$
- stationary conditions ("gap equations"): $\frac{\partial\Omega}{\partial X} = 0 \qquad X = \left\{\sigma_{u,d,s}, \Phi, \bar{\Phi}\right\}$
- regularized by three-momentum cut-off

Polyakov loop

- $L(\vec{x}) = \mathcal{P} \exp \left[i \int_0^{1/T} d\tau \; A_4(\vec{x}, \tau) \right]$
- expectation values $\Phi = \frac{1}{N_c} \langle \mathrm{tr}\, L \rangle \qquad \bar{\Phi} = \frac{1}{N_c} \langle \mathrm{tr}\, L^\dagger \rangle$
- approximate order parameters for confinement-deconfinement transition
- polynomial Polyakov loop potential

$$\frac{\mathcal{U}_{Polyakov}}{T^4} = -\frac{b_2(T)}{2} \Phi\Phi^* - 6\frac{b_3}{6}(\Phi^3 + \Phi^{*3}) + \frac{b_4}{4}(\Phi\Phi^*)^2$$

[Ratti, Thaler, Weise (2006)]

extended $\mathbb{Z}_3$ symmetry

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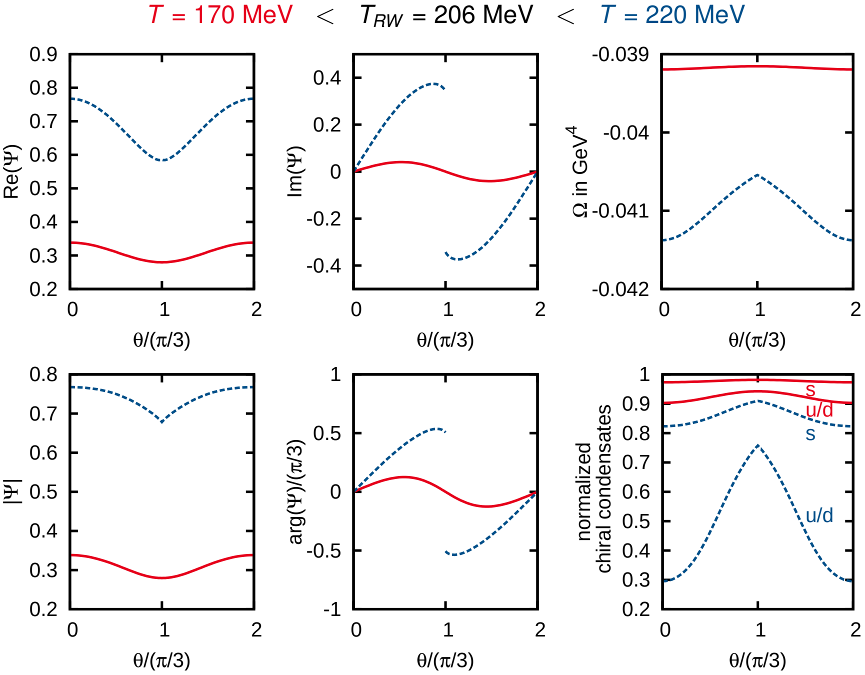
- Roberge and Weiss (1986): QCD has a periodicity $\Omega_{QCD}(\theta) = \Omega_{QCD}(\theta + 2\pi k/3)$, connected by a $\mathbb{Z}_3$ transformation.
- "extended $\mathbb{Z}_3$ transformation":

$$\Phi \longrightarrow \Phi e^{-i2\pi k/3} \text{ and } \theta \longrightarrow \theta + 2\pi k/3$$

- $\Omega_{QCD}(\theta)$ and related quantities $(\sigma_f(\theta), \ldots)$ invariant under the extended $\mathbb{Z}_3$ transformation, have RW periodicity: period $\frac{2\pi}{3}$
- for convenience: modified Polyakov loop: $\Psi = \Phi e^{i\theta}$ (also invariant, complex!)

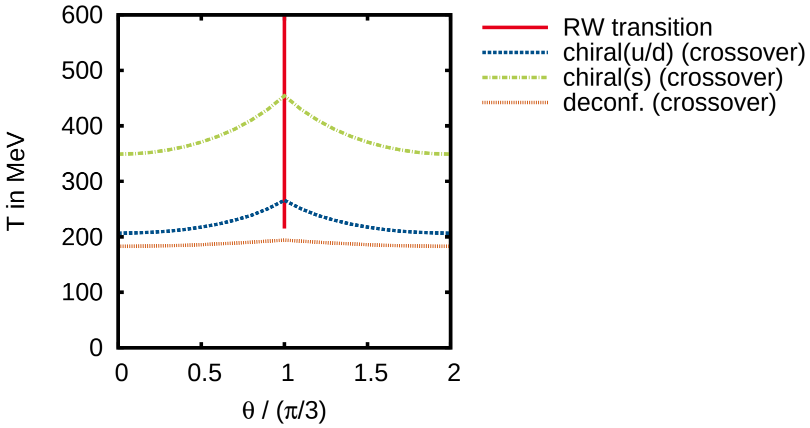
Results

- RW periodicity
- Phase diagram ($\theta - T$ plane)
- Phase diagram ($\mu^2 - T$ plane)
- Analytic continuation of the light quark chiral crossover line



T = 170 MeV < T_{RW} = 206 MeV < T = 220 MeV
Re(Ψ)
Im(Ψ)
Ω in GeV^4
|Ψ|
arg(Ψ)/(π/3)
normalized chiral condensates
θ/(π/3)
s
u/d
s
u/d

Phase diagram ($\theta - T$ plane)



Phase diagram ($\mu^2 - T$ plane)

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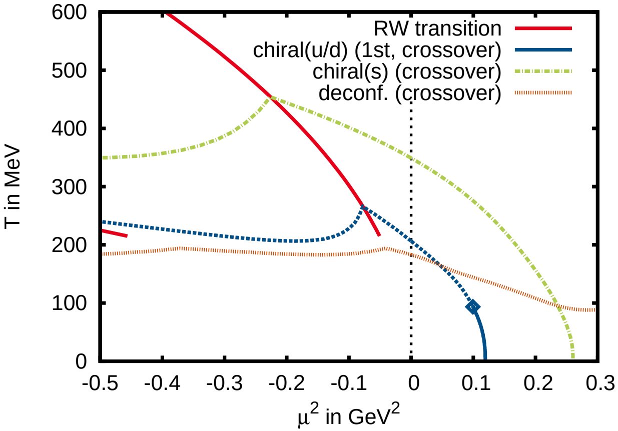
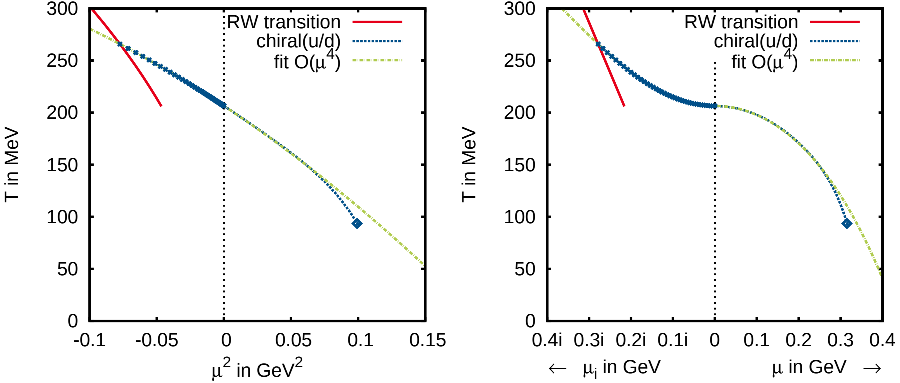


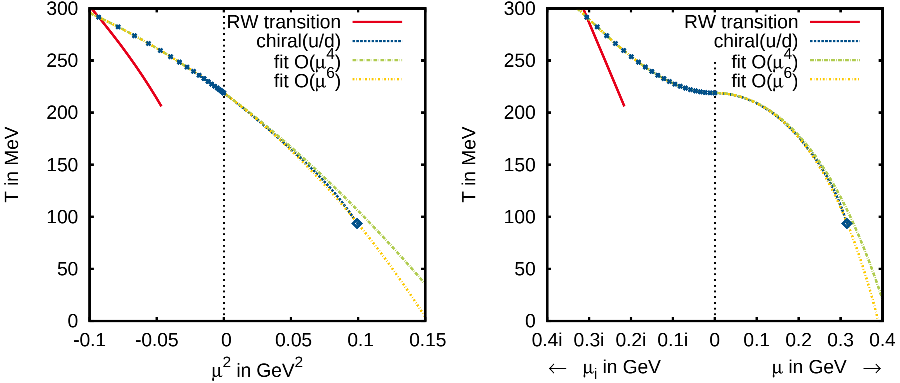
Analytic continuation of the light quark chiral crossover line

Crossover criterion: Half of vacuum value



Analytic continuation of the light quark chiral crossover line

Crossover criterion: Maximum chiral susceptibility



Summary and Outlook

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Summary

- PNJL model has RW symmetry & periodicity
- extrapolation of transition lines from imaginary to real chemical potential possible
- promising tool for crosschecks with Lattice QCD

Outlook

- check different Polyakov loop potentials, parameter sets…
- comparison with lattice data

Thank you!