Eur. Phys. J. A **6**, 243–245 (1999)

THE EUROPEAN PHYSICAL JOURNAL A

Short note

Microscopic analysis of a correlation between dipole transitions $1^-_1 \rightarrow 0^+_{g.s.}$ and $3^-_1 \rightarrow 2^+_1$ in spherical nuclei

V.Yu. Ponomarev[a]

University of Gent, Department of Subatomic and Radiation Physics, Proeftuinstraat 86, 9000 Gent, Belgium

Received: 4 October 1999
Communicated by B. Povh

Abstract. Correlation between B$(E1, 1^-_1 \rightarrow 0^+_{g.s.})$ and B$(E1, 3^-_1 \rightarrow 2^+_1)$ values are considered within microscopic QRPA approach. General arguments for a dependence of a ratio between these values on a collectivity of the 2^+_1 and 3^-_1 phonons and ground state correlations are provided.

PACS. 21.10.Re Collective levels – 21.60.-n Nuclear-structure models and methods – 23.20.Js Multipole matrix elements

Recently, an impressive correlation between B$(E1, 1^-_1 \rightarrow 0^+_{g.s.})$ and B$(E1, 3^-_1 \rightarrow 2^+_1)$ values have been reported by Pietralla [1] from an experimental systematic. Although the measured B$(E1)$ values themselves vary within two orders of magnitude for different medium-heavy spherical nuclei, a ratio

$$R = \frac{\mathrm{B}(E1, 1^-_1 \rightarrow 0^+_{g.s.})}{\mathrm{B}(E1, 3^-_1 \rightarrow 2^+_1)} \qquad (1)$$

keeps practically constant and is close to one. In the same paper it is shown that one expects R equal 7/3 if a simple bosonic phonon model is applied to describe the states involved. In this Short note we present a microscopic analysis of transition matrix elements under consideration and demonstrate that $0 < R < 7/3$ in any nuclear model based on a quasiparticle random phase approximation (QRPA) approach if an internal fermion structure of phonons is accounted for.

Let us introduce a phonon operator $Q^+_{\lambda\mu}$ with a multipolarity λ and projection μ to describe excited states in nuclei as a superposition of different two-quasiparticle configurations:

$$Q^+_{\lambda\mu} = \frac{1}{2}\sum_{\tau}^{n,p}\sum_{jj'}\Big\{C^{\lambda\mu}_{jmj'm'}X^{\lambda}_{jj'}\alpha^+_{jm}\alpha^+_{j'm'} - (-1)^{\lambda-\mu}C^{\lambda-\mu}_{j'm'jm}Y^{\lambda}_{jj'}\alpha_{j'm'}\alpha_{jm}\Big\} . \qquad (2)$$

The quantity $jm \equiv |nljm>$ denotes a single-particle level of an average field and C is a Clebsh-Gordon coefficient. Quasiparticle operators, α^+_{jm}, are obtained from a linear Bogoliubov transformation from the particle creation a^+_{jm} and annihilation a_{jm} operators:

$$a^+_{jm} = u_j\alpha^+_{jm} + (-1)^{j-m}v_j\alpha_{j-m}$$
$$a_{jm} = u_j\alpha_{jm} + (-1)^{j-m}v_j\alpha^+_{j-m}$$

where u^2_j and v^2_j are occupation numbers.

The properties of phonons (2), i.e. their excitation energies, E_λ, and the values of forward, $X^\lambda_{jj'}$, and backward, $Y^\lambda_{jj'}$, amplitudes are obtained by solving QRPA equations. We are interested here only in the first collective QRPA solution, thus, the index i which is often used to distinguish phonons with a different excitation energy, is dropped.

It is well-known that in spherical nuclei the lowest 2^+ and 3^- excited states have a practically pure one-phonon nature while the lowest 1^- state is a two-phonon configuration which we describe by a wave function:

$$|1^-_1\mu_1> = \sum_{\mu_2\mu_3} C^{1\mu_1}_{2\mu_2 3\mu_3} Q^+_{2^+\mu_2}Q^+_{3^-\mu_3}| >_{ph}$$

where $| >_{ph}$ is a wave function of a ground state of an even-even nucleus, a phonon vacuum.

In terms of quasiparticles and phonons, a one-body operator of an electromagnetic $E\lambda$ transition has the form:

[a] *NATO fellow. Permanent address:* Bogoliubov Laboratory of Theoretical Physics, Joint Institute for Nuclear Research, 141980, Moscow region, Dubna, Russia, E-mail: vlad@thsun1.jinr.ru

$$\mathcal{M}(E\lambda\mu) = \sum_{\tau}^{n,p} e_{\tau}^{(\lambda)} \sum_{jj'} \frac{< j||E\lambda||j' >}{\sqrt{2\lambda+1}}$$
$$\times \left\{ \frac{(u_j v_{j'} + v_j u_{j'})}{2} (X_{jj'}^{\lambda} + Y_{jj'}^{\lambda})(Q_{\lambda\mu}^{+} + (-)^{\lambda-\mu} Q_{\lambda-\mu}) \right.$$
$$\left. + \, (u_j u_{j'} - v_j v_{j'}) \sum_{mm'} C_{jmj'm'}^{\lambda\mu} (-)^{j'+m'} \alpha_{jm}^{+} \alpha_{j'-m'} \right\} \quad (3)$$

where $< j||\mathrm{E}\lambda||j' > \equiv < j||i^{\lambda} Y_{\lambda} r^{\lambda}||j' >$ is a single particle transition matrix element and $e_{\tau}^{(\lambda)}$ are effective charges for neutrons and protons. The first term of (3) corresponds to an one-phonon exchange and it does not contribute to transitions between the one-phonon 3_1^- and 2_1^+ excited states and to a decay of the two-phonon 1_1^- state into the ground state.

Applying exact commutation relations between phonon and quasiparticle operators:

$$[\alpha_{jm}, Q_{\lambda\mu}^{+}]_{-} = \sum_{j'm'} X_{jj'}^{\lambda} C_{jmj'm'}^{\lambda\mu} \alpha_{j'm'}^{+} \ ,$$
$$[\alpha_{jm}^{+}, Q_{\lambda\mu}^{+}]_{-} = (-1)^{\lambda-\mu} \sum_{j'm'} Y_{jj'}^{\lambda} C_{jmj'm'}^{\lambda-\mu} \alpha_{j'm'}$$

we obtain

$$\mathrm{B}(E\lambda_1; [\lambda_2 \times \lambda_3]_{\lambda_1} \to 0_{g.s.}^{+}) = \frac{(2\lambda_2+1)(2\lambda_3+1)}{(2\lambda_1+1)}$$
$$\times \left| \sum_{\tau}^{n,p} e_{\tau}^{(\lambda_1)} \sum_{j_1 j_2 j_3} (u_{j_1} u_{j_2} - v_{j_1} v_{j_2}) < j_1||\mathrm{E}\lambda_1||j_2 > \right.$$
$$\left. \times \begin{Bmatrix} \lambda_3 & \lambda_2 & \lambda_1 \\ j_1 & j_2 & j_3 \end{Bmatrix} \left(X_{j_2 j_3}^{\lambda_3} Y_{j_3 j_1}^{\lambda_2} + Y_{j_2 j_3}^{\lambda_3} X_{j_3 j_1}^{\lambda_2} \right) \right|^2 \quad (4)$$

for the $E1$-decay $1_1^- \to 0_{g.s.}^+$ and

$$\mathrm{B}(E\lambda_1, \lambda_3 \to \lambda_2) = (2\lambda_2+1)$$
$$\times \left| \sum_{\tau}^{n,p} e_{\tau}^{(\lambda_1)} \sum_{j_1 j_2 j_3} (u_{j_1} u_{j_2} - v_{j_1} v_{j_2}) < j_1||\mathrm{E}\lambda_1||j_2 > \right.$$
$$\left. \times \begin{Bmatrix} \lambda_3 & \lambda_2 & \lambda_1 \\ j_1 & j_2 & j_3 \end{Bmatrix} \left(X_{j_2 j_3}^{\lambda_3} X_{j_3 j_1}^{\lambda_2} + Y_{j_2 j_3}^{\lambda_3} Y_{j_3 j_1}^{\lambda_2} \right) \right|^2 \quad (5)$$

for the $E1$-decay $3_1^- \to 2_1^+$, where $\lambda_1 = 1$, $\lambda_2^{\pi} = 2^+$ and $\lambda_3^{\pi} = 3^-$.

Assuming $X_{jj'}^{\lambda} \equiv Y_{jj'}^{\lambda}$ and equal effective charges, $e_{\tau}^{(\lambda)}$, for both $E1$ transitions under consideration, equations (4,5) yield the value $R = 7/3$, the same as in a simple bosonic phonon model [1]. In fact, the amplitudes $X_{jj'}^{\lambda}$ are always larger than the $Y_{jj'}^{\lambda}$ amplitudes. For example, if a separable form of a residual interaction is used, they have the following analytical expressions:

$$\begin{pmatrix} X \\ Y \end{pmatrix}_{jj'}^{\lambda} (\tau) = \frac{1}{\sqrt{\mathcal{Y}_{\tau}^{\lambda}}} \cdot \frac{f_{jj'}^{\lambda}(\tau)(u_j v_{j'} + u_{j'} v_j)}{\varepsilon_{jj'} \mp E_{\lambda}} \quad (6)$$

where $\varepsilon_{jj'}$ is an energy of a two-quasiparticle configuration $(\alpha_j^{+} \alpha_{j'}^{+})$, $f_{jj'}^{\lambda}$ is a reduced single-particle matrix element of residual forces, and the value $\mathcal{Y}_{\tau}^{\lambda}$ is determined from a normalization condition for phonon operators.

As we notice from (6), $X_{jj'}^{\lambda}$ and $Y_{jj'}^{\lambda}$ amplitudes always have the same sign for the first collective phonon because $E_{\lambda} < \varepsilon_{jj'}$. An approximation $X_{jj'}^{\lambda} \approx Y_{jj'}^{\lambda}$ is valid only when a phonon energy is very small as compared to two-quasiparticle energies, i.e. $E_{\lambda} << \varepsilon_{jj'}$, and corresponds to extremely collective vibrations. Thus, the value 7/3 should be considered as an upper unreachable limit for the quantity R of (1).

Since it is always true that $X_{jj'}^{\lambda} > Y_{jj'}^{\lambda}$, all elements of the sum in (4) are systematically smaller than the corresponding ones in (5) reducing the value of R from the upper 7/3 limit. In spherical nuclei the excitation energies of the lowest vibrational 2^+ and 3^- states approximately equal to 2/3 of an energy of a lowest two-quasiparticle configuration, $\varepsilon_{(jj')_l}$. Keeping only the main term in the sums (4,5), we obtain the value $R \approx 0.9$. In fact, the quantity R should be somewhat larger because the ra-

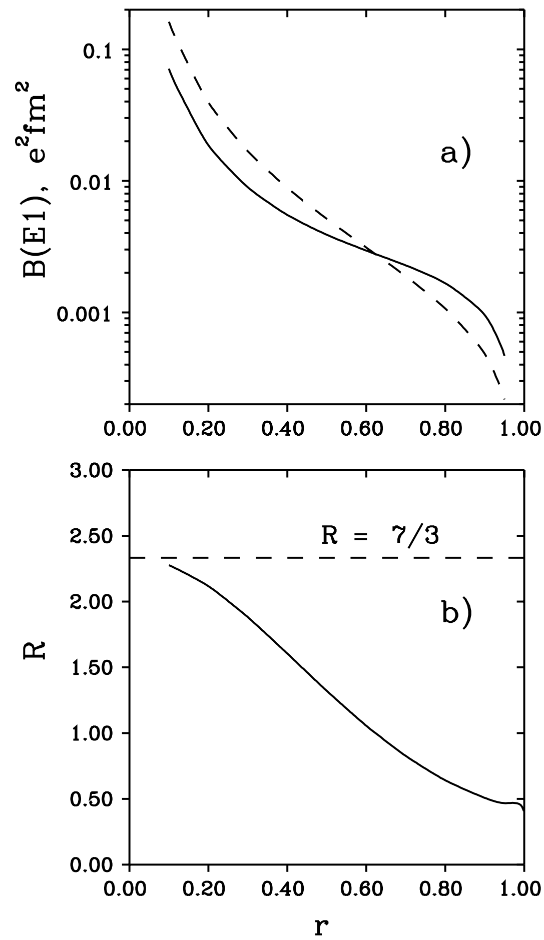

Fig. 1. (**a**) $\mathrm{B}(E1, 1_1^- \to 0_{g.s.}^+)$ (dashed line) and $\mathrm{B}(E1, 3_1^- \to 2_1^+)$ (solid line) values, and b) R value (1) as a function of r which is a ratio between a phonon energy and an energy of a lowest two-quasiparticle configuration. $R = 7/3$ upper limit is shown by dashed line in (**b**)

tios $X_{jj'}^{2(3)}/Y_{jj'}^{2(3)}$ for omitted terms are smaller than for the main, $X_{(jj')_l}^{2(3)}/Y_{(jj')_l}^{2(3)}$, ones.

While $R = 7/3$ should be taken as an upper extreme limit as discussed above, another extreme limit is $R = 0$. The last is approached in a case of non-collective excitations when $E_\lambda \approx \epsilon_{(jj')_l}$ which leads to $X(Y)_{(jj') \neq (jj')_l} \approx 0$ and $X_{(jj')_l} >> Y_{(jj')_l}$.

In Fig. 1a) we present by dashed line B$(E1, 1_1^- \to 0_{g.s.}^+)$ and by solid line B$(E1, 3_1^- \to 2_1^+)$ values as a function of $r = E_\lambda/\epsilon_{(jj')_l}$ in ^{120}Sn. Note a logarithmic scale. Calculations have been performed with a separable form of residual forces and a strength parameter has been varied to obtain an RPA solution at different r values. A smooth evolution of the R value on r from the 7/3 upper limit is shown in Fig. 1b). The curve drops sharply to the 0 lower limit at r very close to 1.

The value R also equals zero when the ground state of a nucleus is considered as a non-correlated vacuum in respect to phonon excitations. It takes place within a Tamm-Dankoff approximation (TDA) approach. In the TDA the nucleus ground state is assumed to be a quasiparticle vacuum and the coefficients $Y_{jj'}^\lambda \equiv 0$. It means that a direct transition $[2_1^+ \times 3_1^-]_{1^-} \to 0_{g.s.}^+$ is totally forbidden (see, (4)). It is different in the QRPA approach because a direct decay of a two-phonon state into the ground state by means of one-body operator of an electromagnetic transition takes place by a simultaneous annihilation of a two-quasiparticle configuration of an excited state and a virtual excitation of another two-quasiparticle configuration in the correlated phonon vacuum. As for the $3_1^- \to 2_1^+$ decay between one-phonon states, one quasiparticle in the 3^- phonon simply re-scatters into the 2^+ phonon by means of the $E1$ operator. It means that the last transition is allowed in second order perturbation theory in both the TDA and QRPA approaches.

Thus, the experimental systematic in [1] for the value of $R \approx 1$ provides an additional good evidence that in spherical nuclei the lowest 2^+ and 3^- states are good collective vibrators built on top of a correlated vacuum.

The present paper was partly supported by the Research Council of the University of Gent.

References

1. N. Pietralla, Phys. Rev. C **59**, 2941 (1999)