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Gamow–Teller Resonance in Hot Nuclei and Astrophysical Applications

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Abstract—A formalism based on thermo field dynamics is described. It allows the effect of the temperature on the strength distribution of charge-exchange transitions in hot nuclei to be taken into account. Numerical calculations with the pair correlations in the BCS approximation and the schematic $\sigma\tau$ interaction are carried out for Gamow–Teller-like transitions in the ^{56}Fe nucleus. The electron capture and β^- decay rates are calculated for this nucleus at temperatures and densities corresponding to the late stage of the evolution of massive stars.

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INTRODUCTION

Collective excitation properties of hot nuclei have been investigated since the 1980s. Both experimental and theoretical studies are mainly focused on the behavior of the giant $E1$ resonance (see [1, 2]). Despite the progress made, there still remain unsolved problems. For example, the total width of the $E1$ resonance at nonzero temperature cannot be quantitatively described so far, the cause for its increase with increasing temperature is not quite clear either (see, for example, [3]).

In the meantime, of great interest is the behavior of charge-exchange resonances, first of all the Fermi and Gamow–Teller resonances, in hot nuclei. These resonances are involved in many weak processes occurring in nuclei, such as nucleosynthesis or stellar core collapse preceding the supernova explosion. Experimental studies of charge-exchange processes in hot nuclei are hardly possible and thus it is theory that must answer the arising questions.

A few approaches have been developed in the theory of collective excitations of hot nuclei (see [1–3]). One of them [4, 5], based on the quasiparticle–phonon model of the nucleus [6] and the thermo field dynamics (TFD) formalism [7], was already used to analyze the temperature behavior of the $E1$ resonance width [8]. An important advantage of this approach is that it allows the relation of the giant resonance to other collective excitations and thus its fragmentation to be taken into account.

In this study the above approach is extended in order to analyze the behavior of charge-exchange resonances in hot nuclei. Particular calculations are carried out for the Gamow–Teller resonance in the ^{56}Fe nucleus because this nucleus is an important component of stellar evolution.

DESCRIPTION OF CHARGE-EXCHANGE EXCITATIONS IN A HOT NUCLEUS

The TFD formalism was used to study properties of hot nuclei not only in [4, 5, 8] but also in [9–11]. In our description of the main TFD points below, brief as it has to be, we follow [7] and the above-mentioned papers.

Let us assume that a hot nuclear system in the state of thermal equilibrium is described by the distribution function of a large canonical ensemble. The main idea of the TFD is that instead of the standard statistical average over the ensemble of an operator A

$$\langle\langle A\rangle\rangle = \frac{1}{\mathrm{Tr}(\exp(-H/T))}\mathrm{Tr}[A\exp(-H/T)]$$

one calculates the average of the state $|0(T)\rangle$ depending on the temperature T

$$\langle\langle A\rangle\rangle = \langle 0(T)|A|0(T)\rangle.$$

Here H is the Hamiltonian of the system; we denote its eigenstates by $|n\rangle$ and eigenvalues by E_n.

The state $|0(T)\rangle$, which is called the thermal vacuum, cannot be constructed using only functions of the ordinary Hilbert space of states of the system in question [7]. The thermal vacuum can only be constructed by formally doubling the number of the system's degrees of freedom, which is achieved by introducing space of so-called tilde states $|\tilde{n}\rangle$, which are eigenstates of the tilde Hamiltonian $\tilde{H}$ with the same eigenvalues E_n as the ordinary eigenstates. With a certain degree of arbitrariness, tilde states may be identified with thermostat states. The direct product of spaces of ordinary and tilde states makes up the total Hilbert space of states of the hot system. In this space the operator for the shift of the system along the time axis is the thermal Hamiltonian $\mathcal{H} = H - \tilde{H}$. This means that $\mathcal{H}$ should be diagonalized for finding the spectrum of excitations of the hot system and that its thermal behavior is governed by the thermal vacuum.

The thermal vacuum $|0(T)\rangle$ is the eigenstate of the zero-energy thermal Hamiltonian. At the same time the thermal vacuum is a vacuum for the annihilation operators of the so-called thermal quasiparticles β_{jm} and $\tilde{\beta}_{jm}$. The latter are related to the creation and annihilation operators of quasiparticles diagonalizing H by the thermal rotational transformation (also called Bogolyubov rotational transformation) [7].

In this study we use a simple model Hamiltonian consisting of a mean-field phenomenological potential, pairing interaction in the BCS form with the constants G_n and G_p, and isovector spin-isospin interaction in the particle–hole channel characterized by one constant χ (see, for example, [6, 12]).

We begin diagonalization of the thermal Hamiltonian $\mathcal{H}$ with defining the basis of Bogolyubov quasiparticles in which the thermal effect is also taken into account [11, 5]. In this basis the part of the thermal Hamiltonian which involves the mean field and the pairing interaction becomes diagonal.

The initial thermal Hamiltonian is expressed in terms of fermion operators of thermal quasiparticles by means of two successive unitary transformations. The first of them is the standard Bogolyubov transformation from the particle operators a^+_{jm} and a_{jm} to the quasiparticle operators α^+_{jm} and α_{jm}

$$\begin{aligned} a_{jm} &= u_j\alpha_{jm} + \upsilon_j\alpha^+_{\overline{jm}}, \\ a^+_{jm} &= u_j\alpha^+_{jm} + \upsilon_j\alpha_{\overline{jm}}, \quad (u_j^2 + \upsilon_j^2 = 1) \end{aligned} \tag{1}$$

(similarly for the corresponding tilde operators). The second transformation is thermal rotation mixing ordinary and tilde quasiparticle operators

$$\begin{aligned} \alpha^+_{jm} &= x_j\beta^+_{jm} + y_j\tilde{\beta}_{jm}, \\ \tilde{\alpha}^+_{jm} &= x_j\tilde{\beta}^+_{jm} - y_j\beta_{jm}, \quad (x_j^2 + y_j^2 = 1). \end{aligned} \tag{2}$$

The coefficients (u_j, υ_j) and (x_j, y_j) are defined from the condition of the minimum free energy of the system of independent quasiparticles. Thus, there arise a system of equations of superfluidity at nonzero temperature [13, 11, 5], solutions to which govern the thermal quasiparticle energies ε_{j_τ}, values of the energy gap Δ_τ and chemical potential λ_τ ($\tau = n, p$ is the isotopic index) as functions of single-particle energies, interaction constants, and temperature. Note that the coefficients (x_j, y_j) are expressed in terms of the Fermi–Dirac thermal occupation factors of Bogolyubov quasiparticles.

The remaining part of the Hamiltonian is diagonalized in the random phase approximation, i.e., on the assumption that the wave function of the excited state of the Gamow–Teller (GT) type is expressed in terms of the thermal GT phonon creation operator $Q^+_{\mu i}$ acting on the thermal phonon vacuum $|\Psi_0(T)\rangle$

$$\begin{aligned} Q^+_{\mu i} = \frac{1}{\sqrt{2}}\sum_{j_p j_n} & (\psi^i_{j_p j_n}[\beta^+_{j_p}\beta^+_{j_n}]_\mu + \tilde{\psi}^i_{j_p j_n}[\tilde{\beta}^+_{\bar{j}_p}\tilde{\beta}^+_{\bar{j}_n}]_\mu \\ & + \eta^i_{j_p j_n}[\beta^+_{j_p}\tilde{\beta}^+_{\bar{j}_n}]_\mu + \tilde{\eta}^i_{j_p j_n}[\tilde{\beta}^+_{\bar{j}_p}\beta^+_{j_n}]_\mu) \\ & + (\phi^i_{j_p j_n}[\beta_{\bar{j}_p}\beta_{\bar{j}_n}]_\mu + \tilde{\phi}^i_{j_p j_n}[\tilde{\beta}_{j_p}\tilde{\beta}_{j_n}]_\mu \\ & - \xi^i_{j_p j_n}[\beta_{\bar{j}_p}\tilde{\beta}_{j_n}]_\mu - \tilde{\xi}^i_{j_p j_n}[\tilde{\beta}_{j_p}\beta_{\bar{j}_n}]_\mu), \end{aligned} \tag{3}$$

where $[\ ]_\mu$ denotes coupling of the single-particle angular momenta j and j' to the total angular momentum l with the projection μ. The phonon operators should satisfy the boson commutation relations, which imposes limitations on the phonon amplitudes [5].

The energy of the single-phonon state $Q^+_{\mu i}|\Psi_0(T)\rangle$ is found from the condition of the minimum energy of the thermal Hamiltonian in the single-phonon state at the above-mentioned additional limitations on the amplitudes ψ, ϕ, η, and ξ and their tilde partners. The corresponding variational equation has the form

$$\begin{aligned} \delta\Big\{ & \langle\Psi_0(T)|Q_{\mu i}\mathcal{H}Q^+_{\mu i}|\Psi_0(T)\rangle \\ & - \frac{\omega_i}{2}[\langle\Psi_0(T)|[Q_{\mu i}, Q^+_{\mu i}]|\Psi_0(T)\rangle - 1]\Big\} = 0. \end{aligned} \tag{4}$$

The Lagrange multiplier ω_i is the energy of the single-phonon state. Energies of thermal phonons are solutions to the following secular equation[1]:

$$[\chi X^{(+)}(\omega_i) - 1][\chi X^{(-)}(\omega_i) - 1] - [\chi X^{(0)}(\omega_i)]^2 = 0. \tag{5}$$

The functions $X^{(\pm)}(\omega_i)$ and $X^{(0)}(\omega_i)$ are defined by the expressions

$$\begin{aligned} X^{(\pm)}(\omega_i) = \frac{2}{3}\sum_{j_p j_n} & \frac{\varepsilon^{(+)}_{j_p j_n}(f_{j_p j_n})^2(u^{(\pm)}_{j_p j_n})^2}{(\varepsilon^{(+)}_{j_p j_n})^2 - \omega_i^2}(1 - y^2_{j_p} - y^2_{j_n}) \\ & - \frac{\varepsilon^{(-)}_{j_p j_n}(f_{j_p j_n})^2(\upsilon^{(\mp)}_{j_p j_n})^2}{(\varepsilon^{(-)}_{j_p j_n})^2 - \omega_i^2}(y^2_{j_p} - y^2_{j_n}), \end{aligned}$$

[1] Our particle–hole $\sigma\tau$ interaction is separable.

$$X^{(0)}(\omega_i) = \frac{2}{3}\omega_i \sum_{j_p j_n} \frac{(f_{j_p j_n})^2 u^{(+)}_{j_p j_n} u^{(-)}_{j_p j_n}}{(\varepsilon^{(+)}_{j_p j_n})^2 - \omega_i^2}(1 - y^2_{j_p} - y^2_{j_n}) - \frac{(f_{j_p j_n})^2 \upsilon^{(-)}_{j_p j_n} \upsilon^{(+)}_{j_p j_n}}{(\varepsilon^{(-)}_{j_p j_n})^2 - \omega_i^2}(y^2_{j_p} - y^2_{j_n}),$$

where $f_{j_p j_n}$ is the reduced single-particle matrix element of the operator $\sigma\tau$, a $u^{(\pm)}_{j_p j_n} = u_{j_p}\upsilon_{j_n} \pm \upsilon_{j_p}u_{j_n}$, $\upsilon^{(\pm)}_{j_p j_n} = u_{j_p}u_{j_n} \pm \upsilon_{j_p}\upsilon_{j_n}$, $\varepsilon^{(\pm)}_{j_p j_n} = \varepsilon_{j_p} \pm \varepsilon_{j_n}$.

The excitation spectrum of the hot nucleus is enriched because there are a number of excited quasiparticles in the thermal phonon vacuum. This is why poles $\varepsilon^{(-)}_{j_p j_n}$ appear in secular equation (5) and low-energy transitions of the hole–hole and particle–particle type become possible. Note that solutions to (5) with the negative energy also have a physical meaning: they correspond to the excitation of tilde phonons $\tilde{Q}^{+}_{\mu i}|\Psi_0(T)\rangle$.

Variational equation (4) defines the structure of thermal phonon (3) to within the accuracy of the transformation equivalent to thermal rotation of the phonon operators[2]:

$$Q^{+}_{\mu i} \to X_i Q^{+}_{\mu i} - Y_i \tilde{Q}_{\mu i},$$
$$\tilde{Q}^{+}_{\mu i} \to X_i \tilde{Q}^{+}_{\mu i} - Y_i Q_{\mu i} \quad (X_i^2 - Y_i^2 = 1).$$

As in the case of thermal quasiparticles, the thermal rotation coefficients are found from the condition of the minimum free energy (of the system of noninteracting phonons in this case)

$$\Omega_B(T) = \langle\Psi_0(T)| \sum_{i,\,\mu\,=\,0,\,\pm 1} \omega_i q^{+}_{\mu i} q_{\mu i} - T\hat{S}_B|\Psi_0(T)\rangle. \quad (6)$$

The symbols $q^{+}_{\mu i}$ and $q_{\mu i}$ stand for the operators of the phonons which can be called "nonhot" (though they consist of thermal quasiparticles) because $Y_i = 0$ for them; $\hat{S}_B$ is the entropy operator for the system of
1 bosons [7],

$$\hat{S}_B = -\sum_{i,\,\mu\,=\,0,\,\pm 1} \{q^{+}_{\mu i} q_{\mu i} \ln Y_i^2 - q_{\mu i} q^{+}_{\mu i} \ln X_i^2\}. \quad (7)$$

Varying $\Omega_B(T)$ with respect to X_i and Y_i, we get

$$Y_j^2 = \left[\exp\left(\frac{\omega_i}{T}\right) - 1\right]^{-1}. \quad (8)$$

[2] This point was ignored in [4, 5, 8].

Thus, the coefficients Y_j^2 are Bose–Einstein thermal occupation factors. They define the number of thermal phonons $q_{\mu i}$ in the thermal vacuum: $\langle\Psi_0(T)|q^{+}_{\mu i} q_{\mu i}|\Psi_0(T)\rangle = Y_j^2$.

Each single-phonon state $Q^{+}_{\mu i}|\Psi_0(T)\rangle$ and $\tilde{Q}^{+}_{\mu i}|\Psi_0(T)\rangle$ is a superposition of the excited states of the $(Z + 1, N - 1)$ and $(Z - 1, N + 1)$ nuclei. This mixing results from pair correlations and correlations that are due to residual proton–neutron interaction. The probability $\Phi_i^{(\mp)}$ and energy $E_i^{(\mp)}$ of the $\Gamma\mathrm{T}^{(\mp)}$ transition to the $Q^{+}_{\mu i}|\Psi_0(T)\rangle$ state are given by the expressions

$$\Phi_i^{(\mp)} = \left|\langle\Psi_0(T)||S^{(\mp)}_{\mu} Q^{+}_{\mu i}||\Psi_0(T)\rangle\right|^2 = \frac{9}{4}\frac{(1 \pm \mathcal{Y}_i)^2}{\chi^2 \mathcal{N}_i} X_i^2,$$
$$E_i^{(\mp)} = \omega_i \mp (\Delta\lambda_{np} + \Delta m_{np}), \quad (9)$$

where $\Delta\lambda_{np} = \lambda_n - \lambda_p$ is the difference in value between the neutron and proton chemical potentials and $\Delta m_{np} = m_n - m_p = 1.29$ MeV is the mass difference between the neutron and the proton. For the probability and energy of the transition to the $\tilde{Q}^{+}_{\mu i}\Psi_0(T)\rangle$ state we get

$$\tilde{\Phi}_i^{(\mp)} = \left|\langle\Psi_0(T)||S^{(\mp)}_{\mu} Q^{+}_{\mu i}||\Psi_0(T)\rangle\right|^2 = \frac{9}{4}\frac{(1 \mp \mathcal{Y}_i)^2}{\chi^2 \mathcal{N}_i} Y_i^2,$$
$$\tilde{E}_i^{(\mp)} = -\omega_i \pm (\Delta\lambda_{np} + \Delta m_{np}). \quad (10)$$

Thus, to each neutron–proton (proton– neutron) transition from the compound state of the hot nucleus (i.e., thermal phonon vacuum) to the $Q^{+}_{\mu i}|\Psi_0(T)\rangle$ state there corresponds a proton–neutron (neutron– proton) transition from the $|\Psi_0(T)\rangle$ state to the $\tilde{Q}^{+}_{\mu i}|\Psi_0(T)\rangle$ state. Probabilities of these transitions differ in multipliers related to the boson occupation factors and their energies are equal in absolute value and differ in sign.

NUMERICAL CALCULATIONS: DECAY OF THE HOT ^{56}Fe NUCLEUS

Numerical calculations were carried out for the ^{56}Fe nucleus. Single-particle wave functions and single-particle energies were calculated in the spherically symmetrical Woods–Saxon potential with the parameters from [14]. Energies of the $1f$ and $2p$ shells were taken from the processed experimental data on nucleon stripping and pickup reactions [15, 16]. The constants G_n and G_p were determined from the experimental values of pair energies. The pairing energy gaps for the ^{56}Fe were found to be $\Delta_n = 1.27$ MeV and $\Delta_p = 1.25$ MeV. The constant χ of the isovector spin-dipole interaction was determined from the position of the experimental peak of the Gamow–Teller resonance [17, 18]. The

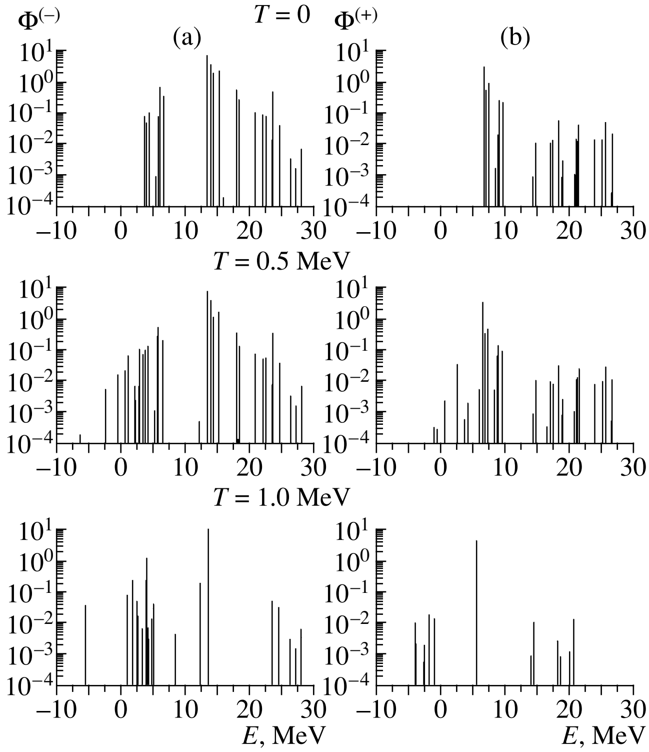

Fig. 1. Strength distribution of the GT^- (a) and GT^+ resonance (b) in the ^{56}Fe nucleus at the temperatures $T = 0$, 0.5, and 1.0 MeV. E is the energy transferred to the parent nucleus.

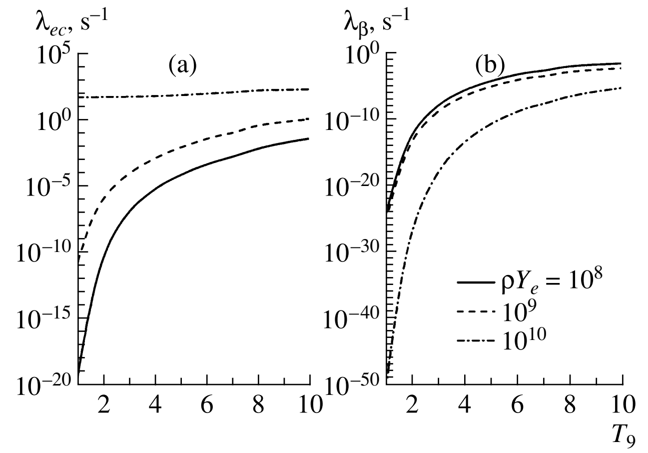

Fig. 2. Rates of electron capture (a) and β^- decay for the ^{56}Fe nucleus as a function of temperature at different densities ρY_e of the stellar material. Temperature T_9 is in units of 10^9 K; density ρY_e is in units of mol cm^{-3}.

value obtained agrees with the estimate $\chi = 23/A$ MeV [12].

The strength distribution of the GT resonance built on the ^{56}Fe nucleus as a function of the energy transferred to the nucleus is shown in Fig. 1 for three temperature values $T = 0$, 0.5, and 1.0 MeV (remember that 0.1 MeV $\approx 1.2 \times 10^9$ K).

Note the characteristic features of the distributions obtained. For β-stable nuclei, such as ^{56}Fe, at $T = 0$ all strength of GT transitions is above the ground-state energy of the parent nucleus. Therefore, only GT transitions accompanied by energy absorption, e.g., as a result of electron or positron capture, are possible in these nuclei at zero temperature. As the temperature increases, redistribution of the GT transition strength takes place and part of it finds itself in the energy region below the thermal vacuum corresponding to the compound state of the parent nucleus. As a result, GT transitions accompanied by energy loss become possible. They correspond in particular to the $\beta^{\mp}$ decay of the parent nucleus from the excited state. An increase in the temperature leads to an increase in the fraction of the GT strength lying below the thermal vacuum energy, and thus the probability of $\beta^{\mp}$ decays increases.

As the temperature increases, the centroid of the GT resonance moves downward on the energy scale. This decrease results from two causes. One is weakening and subsequent disappearance of pair correlations with increasing temperature. This mechanism dominates at temperatures below the critical value $T_C \le 0.7$ MeV. The other cause is thermal smearing of the Fermi surface which increases with temperature. The smearing increases the fraction of low-energy transitions in the structure of the strength function. At $T = 0$ these transitions were strongly suppressed or forbidden by the Pauli principle (for nuclei of the *fp* shell beginning these are mainly transitions like $1f_{7/2}(p) \to 1f_{7/2}(n)$). At the same time the contribution from the transitions to the states below the thermal vacuum increases. In addition, smearing of the Fermi surface effectively affects the strength of the particle–hole $\sigma\tau$ interaction: it decreases with increasing temperature.

For the reaction of electron capture by nuclei in the stellar material the lowering of the GT^+ resonance centroid leads to a decrease in the reaction threshold and a substantial increase in the electron capture rate at low densities of the stellar material, when the Fermi energy of the degenerate electron gas surrounding atomic nuclei is small.

The lowering of the GT^- resonance centroid with increasing temperature affects the process of nucleosynthesis during the explosion of the supernova, when neutron-rich nuclei resulting from fast neutron capture (*r*-process) undergo neutron decay from excited states under the effect of the (ν_e, e^-) reaction proceeding just through excitation of the GT^- resonance.

Figure 2 shows calculated electron capture and β^- decay rates for the ^{56}Fe nucleus at temperatures and densities of the stellar material corresponding to the late stage of the evolution of massive stars [19, 20]. Under these conditions the atomic nucleus is fully ionized and electrons are captured only from the surrounding degenerate electron gas, whose Fermi energy increases with increasing density.

The electron capture rate increases both with increasing density and with increasing temperature. This is primarily because the fraction of electrons with energy sufficient for excitation of the GT^+ resonance increases in the electron gas. Another cause for the increase in the electron capture rate is, as was already mentioned, the lowering of the centroid for the GT^+ resonance with increasing nuclear temperature.

The effect of the increase in the density and temperature of the stellar material on the probability of the inverse reaction, β^- decay, is quite opposite. The probability decreases with increasing density because the chemical potential of the electron gas increases and more and more states that could be occupied by the electron emitted from the nucleus turn out to be already occupied. On the contrary, the increase in temperature weakens the Pauli blocking effect and increases the fraction of GT^- transitions from excited states of ^{56}Fe.

Rates for electron capture by the ^{56}Fe nucleus shown in Fig. 2 are in good agreement with the shell-model calculations [19, 20]. However, our results for the β^- decay rates are much larger than in [19, 20] (by a few orders of magnitude at low temperatures). A possible cause for the discrepancy is that the distribution of the GT transition strength calculated in the random phase approximation mostly ignores its fragmentation (see [6, 12]). It is hoped that allowance for the interaction with complicated configurations in the spirit of the [4, 8] will change the situation for the better.

CONCLUSION

In this study, equations of the thermal random-phase approximation for charge-exchange excitations are obtained within the thermo field dynamics. Strength distributions are calculated for GT transitions like $n \to p$ (GT^+) and $p \to n$ (GT^-) built on the ^{56}Fe nucleus at the temperature $T \leq 1$ MeV. Temperature dependences of the electron capture and β^- decay are calculated for this nucleus at different stellar material densities. The presented approach may be further elaborated to include interaction of single-phonon configurations with more complicated two-phonon ones, as was done for charge-exchange resonances in "cold nuclei" in [12]. It is also desirable to use more realistic effective interactions.

SPELL: 1. bosons

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