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On single and double resonances in spherical nuclei

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Microscopic calculations of the fine structure of single and double giant dipole resonances (GDR) in ^{136}Xe are presented. The single GDR is treated by a wave function that takes into account a coupling of one-phonon with two-phonon configurations while for the double GDR we mix two-phonon and three-phonon configurations.

1. INTRODUCTION

Experimental observation of two-phonon resonances in heavy ion collisions at intermediate and relativistic energies (for a review see refs. [1, 2] and contributions by S. Mordechai, Y. Blumenfeld and J. Stroth in the proceedings of this conference) by different groups and in different nuclei allow one to speak about systematic investigation of this phenomenon. The properties of these states may be summarized as follows. They are located at an energy close to the double energy of the corresponding single resonances and have a width somewhat between $\sqrt{2}$ and 2 of the width of single ones. A disturbing feature of the results at relativistic energies is that the measured cross sections is about 3-4 times larger than predicted by theory [3, 4, 5], although the latest results from LAND collaboration for ^{208}Pb reveal no discrepancy with theory in excitation cross section [6]. In this paper we report on microscopic calculations of the fine structure of the observed single and double giant dipole resonances (GDR) in ^{136}Xe [7].

2. SINGLE GDR

Before proceeding with the double GDR we briefly present the formalism used in the calculation of one phonon state. A microscopic Hamiltonian (H) is considered, which includes an average field, a monopole pairing and multipole-multipole interactions of separable form. For more details see refs. [8, 9]. Solving the quasiparticle-RPA equations, we a obtain phonon basis of different multipolarities λ^π among which we have collective excitations as well as almost pure two-quasiparticle ones. All model parameters are fixed at this stage of calculations.

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To reproduce the width of giant resonances we couple simple one-phonon configurations with more complex ones. Thus, for the states forming a single GDR we use the following wave function

$$\Psi^{\nu_1}_{1^-} = \left\{ \sum_{i_1}^{n} S^{\nu_1}_{i_1} Q^+_{1^- i_1} + \sum_{\alpha_2\beta_2} \frac{D^{\nu_1}_{\alpha_2\beta_2} Q^+_{\alpha_2} Q^+_{\beta_2}}{\sqrt{1+\delta_{\alpha_2,\beta_2}}} \right\} |\rangle_{ph} \tag{1}$$

where Q^+ is the phonon creation operator and $|\rangle_{ph}$ is the phonon vacuum. With greek letters we denote the set of quantum numbers $\{\lambda, i\}$, where i is the RPA root number while λ is the multipolarity of the mode. The ansatz is made that any combination $\alpha_n, \beta_n, \gamma_n$ appears in the sums only once. The second term in eq. (1) includes phonons of different multipolarities coupled to angular momentum and parity to 1^-.

Diagonalization of the model Hamiltonian in the set (1) of wave functions, yields the secular equation for the corresponding eigenenergies, which in the space of one-phonon configurations has the form

$$F_1(E_x) = det \left\| (\omega_{1^- i_1} - E_x)\delta_{i_1,i'_1} - \sum_{\alpha_2\beta_2} \frac{(1+\delta_{\alpha_2,\beta_2}) U^{1^- i_1}_{\alpha_2\beta_2} U^{1^- i'_1}_{\alpha_2\beta_2}}{\omega_{\alpha_2} + \omega_{\beta_2} - E_x} \right\| = 0. \tag{2}$$

Here $U^{\alpha_1}_{\alpha_2\beta_2} = \frac{1}{2}\langle Q_{\alpha_1}|H|Q^+_{\alpha_2}Q^+_{\beta_2}\rangle$ is the matrix element of the interaction between one- and two-phonon configurations. It can be expressed in terms of forward going and backward going phonon amplitudes. The quantity ω_α is the RPA energy for the i^{th} state of multipolarity λ. The coefficients $S^{\nu_1}_{i_1}$ are minors of the matrix given in eq. (2) calculated at the eigenvalues. The coefficients $D^{\nu_1}_{\alpha_2\beta_2}$ are related to $S^{\nu_1}_{i_1}$ [9]. In the present calculation we assume a boson picture for nuclei excitation although the structure of bosons (phonons) and interaction between them are calculated microscopically. Following this approach one can easily include the Pauli principle corrections [9] but our experience shows that the role of the Pauli principle in the resonance region is marginal.

The B(E1) strength distribution of the one-phonon 1^- states in ^{136}Xe are presented in fig. 1a. Due to the coupling to complex configurations, the one-phonon 1^- states become fragmented over a great number of states of the type given in eq. (1) (cfr. figs. 1b-c). The comparison shows that the truncation of two-phonon basis leads to underestimated fragmentation of the GDR especially at higher energies.

3. DOUBLE GDR

The most direct way to use the fine structure of the single GDR in calculating properties of the double GDR is to build on top of each 1^- state described by eq. (1), the full set of 1^- states, following the Axel-Brink hypothesis. Making use of the results one can calculate the Coulomb excitation cross section of the double GDR excitation in the second order perturbation theory [10] since direct excitation of two-phonon configurations from the ground state in the first order is very weak [11]. This has been done in ref. [5]. For the double GDR in ^{136}Xe we obtained a position very close to twice energy of the

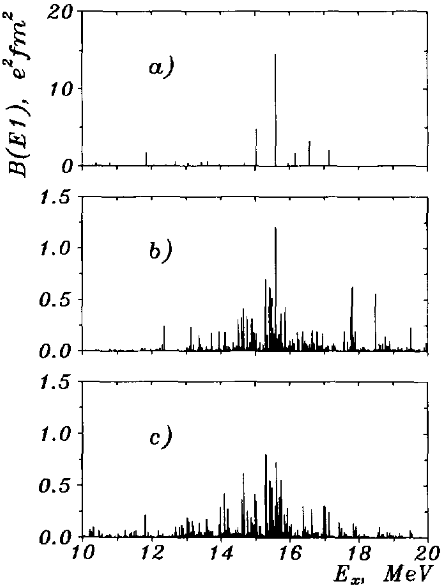

Fig. 1 Distribution of the B(E1) strength in ^{136}Xe: a) over one-phonon states and b,c) over the states described by eq. (1). The cases b) and c) correspond to the calculation with 5(15) strongest one-phonon configurations in the GDR region and two-phonon configurations up to 20(22) MeV, respectively.

single GDR. The width was about 1.5 the width of the single phonon resonance. Such an approach has two main shortcomings. First, the wave functions of states for the double GDR are not eigenfunctions of our Hamiltonian. Second, although we use 'anharmonic' phonons to describe the single GDR, we have a harmonic picture for the double GDR.

To overcome these shortcomings, we follow the same scheme used for the single GDR. Namely, we write the wave function for the double GDR states as a mixture of 'simple' (two-phonon) and complex (three-phonon) configurations:

$$\Psi^{\nu_2}_{[1^-\otimes 1^-]} = \left\{ \sum_{i_2,i'_2} \frac{D^{\nu_2}_{i_2 i'_2} Q^+_{1^- i_2} Q^+_{1^- i'_2}}{\sqrt{1+\delta_{i_2,i'_2}}} + \sum_{\alpha_3\beta_3\gamma_3} \frac{T^{\nu_2}_{\alpha_3\beta_3\gamma_3} Q^+_{\alpha_3} Q^+_{\beta_3} Q^+_{\gamma_3}}{\sqrt{1+\delta_{\alpha_3,\beta_3}+\delta_{\alpha_3,\gamma_3}+\delta_{\beta_3,\gamma_3}+2\delta_{\alpha_3,\beta_3,\gamma_3}}} \right\} |\rangle_{ph} \quad (3)$$

and diagonalize the Hamiltonian within the set of these wave functions. We do not include a one-phonon term in eq. (3). Thus the states $[1^- \otimes 1^-]_{2^+}$ and $[1^- \otimes 1^-]_{0^+}$ are degenerate within this approximation. As recently shown, the anharmonicity effects due to the coupling of 1p1h- to 2p2h-configurations are very small in the case of the double GDR [12]. The secular equation then reads

$$F_2(E_x) = det \left\| (\omega_{1^- i_2} + \omega_{1^- i'_2} - E_x)\delta_{\{i_2 i'_2\},\{i_2 i'_2\}'} - \sum_{\alpha_3\beta_3\gamma_3} \frac{U^{\{1^- i_2 1^- i'_2\}}_{\alpha_3\beta_3\gamma_3} U^{\{1^- i_2 1^- i'_2\}'}_{\alpha_3\beta_3\gamma_3}}{\omega_{\alpha_3}+\omega_{\beta_3}+\omega_{\gamma_3}-E_x} \right\| = 0 \quad (4)$$

The brackets $\{i_2 i'_2\}$ indicate that eq. (4) is solved in the space of two-phonon configurations. The matrix element $U^{\alpha_2\beta_2}_{\alpha_3\beta_3\gamma_3}$ of the interaction between two- and three-phonon configurations can be expressed in terms of the matrix elements $U^{\alpha_1}_{\alpha_2\beta_2}$ of the interaction between one- and two-phonon configurations. The $D^{\nu_2}_{i_2 i'_2}$ coefficients in eq. (3), are minors of the matrix, eq. (4), calculated at $E_x = E_x^{\nu_2}$. The $T^{\nu_2}_{\alpha_3\beta_3\gamma_3}$ coefficients are given by

$$T^{\nu_2}_{\alpha_3\beta_3\gamma_3} = -\frac{\sum_{i_2 i'_2} D^{\nu_2}_{i_2 i'_2} U^{1^- i_2 1^- i'_2}_{\alpha_3\beta_3\gamma_3}}{\omega_{\alpha_3} + \omega_{\beta_3} + \omega_{\gamma_3} - E_x^{\nu_2}}. \tag{5}$$

While eqs. (2) and (4) look very similar, the numerical solving of the last one is much more time consuming because of much higher density of two- and three-phonon configurations in the double GDR region in comparison to the density of one- and two-phonon configurations in the case of the single GDR. To make the problem tractable, we have chosen for the first term of eq. (3) only two-phonon configurations built of the strongest five 1^- phonons ($n = 5$). Truncation to the same phonons for the single GDR produced the strength distribution presented in fig. 1b. Under this truncation we have $n(n+1)/2 = 15$ two-phonon configurations for the first term of eq. (3) and about 6500 three-phonon configurations were used.

The most collective 1^- one-phonon configuration has excitation energy equal to 15.6 MeV (see fig. 1a). Fragmentation of this configuration over 1^- states described by eq. (1) is presented in fig. 2a (the calculation was performed with $n = 5$). The results have been averaged with the aid of a Breit-Wigner distribution of width 0.2 MeV. For the double GDR, fragmentation of the two-phonon configuration $[1^- i \otimes 1^- i]$ made of two most collective 1^- phonons, over states described by eq. (3) is presented in fig. 2b.

Let us consider the excitation probability of the double GDR by the Coulomb field. We assume that there is no energy dependence in the reaction amplitude, which is actually very weak at relativistic energies [5, 10, 13]. Then, the cross section of the single GDR excitation can be written as

$$\sigma_{1^-}(E_x^{\nu_1}) = A \cdot |\langle \Psi^{\nu_1}_{1^-} | M(E1) | \Psi_{g.s.} \rangle|^2 = A \cdot | \sum_{i_1} S^{\nu_1}_{i_1} M_{i_1} |^2, \tag{6}$$

where $M_{i_1} = \langle Q_{1^- i_1} | M(E1) | 0^+_{g.s.} \rangle$ is the matrix element of $E1$ excitation of the i_1 one-phonon configuration from the ground state. For the two-step process of excitation of the double GDR the cross section in the second order perturbation theory has the form

$$\sigma_{[1^- \otimes 1^-]}(E_x^{\nu_2}) = A^2 \cdot | \sum_{\nu_1} \langle \Psi^{\nu_2}_{[1^- \otimes 1^-]} | M(E1) | \Psi^{\nu_1}_{1^-} \rangle \cdot \langle \Psi^{\nu_1}_{1^-} | M(E1) | \Psi_{g.s.} \rangle |^2 \tag{7}$$

The matrix element of transition between single, eq. (1), and double, eq. (3), resonances is

$$\langle \Psi^{\nu_2}_{[1^- \otimes 1^-]} | M(E1) | \Psi^{\nu_1}_{1^-} \rangle = \sum_{i_2, i'_2} \frac{D^{\nu_2}_{i_2, i'_2}}{\sqrt{1 + \delta_{i_2, i'_2}}} \sum_{i_1} S^{\nu_1}_{i_1} \sum_i \langle Q_{1^- i'_2} Q_{1^- i_2} | Q^+_{1^-_i} | Q^+_{1^- i_1} \rangle M_i +$$

$$+ \sum_{\substack{\alpha_2 \beta_2 \\ \alpha_3 \beta_3 \gamma_3}} D^{\nu_1}_{\alpha_2 \beta_2} T^{\nu_2}_{\alpha_3 \beta_3 \gamma_3} \sum_i \langle Q_{\alpha_3} Q_{\beta_3} Q_{\alpha_3} | Q^+_{1^- i} | Q^+_{\alpha_2} Q^+_{\beta_2} \rangle M_i. \tag{8}$$

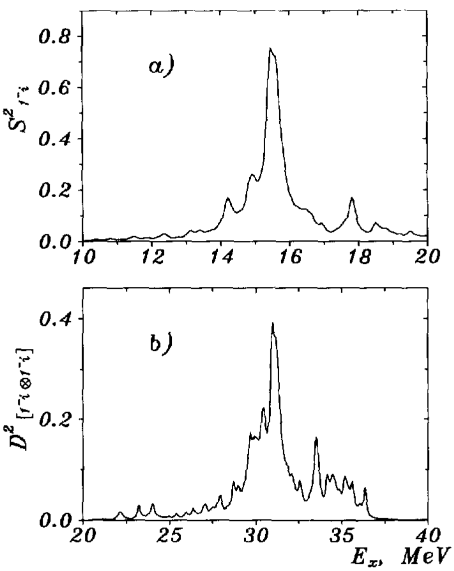

Fig. 2 Fragmentation of the most collective a) one-phonon 1^- and b) two-phonon $[1^-\otimes 1^-]$ configurations in ^{136}Xe due to the coupling to more complex configurations.

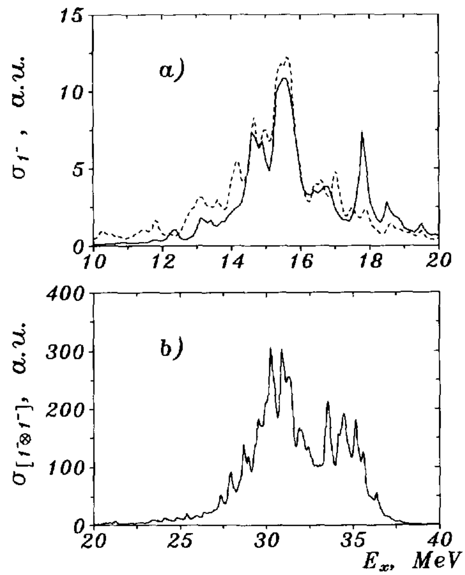

Fig. 3 'Cross section' of excitation of a) the single and b) the double GDR in ^{136}Xe. See the text for details.

Note that the last term does not contribute to $\sigma_{[1^-\otimes 1^-]}$ because of the orthogonality of the wave functions given in eq. (1):

$$\sum_{\nu_1} S^{\nu_1}_{i_1} D^{\nu_1}_{\alpha_2\beta_2} = 0 \tag{9}$$

Summing over ν_1 in eq. (7) and over ν_2 and using the orthogonality relations for the wave functions, eqs. (1) and (3), the total strength of the double GDR excitation is given by the relation

$$\sum_{\nu_2} \sigma_{[1^-\otimes 1^-]}(E_x^{\nu_2}) = \sum_{\nu_2} A^2 \cdot \left| \sum_{i_2 i'_2} \frac{2 \cdot D^{\nu_2}_{i_2,i'_2} M_{i_2} M_{i'_2}}{\sqrt{1+\delta_{i_2,i'_2}}} \right|^2 = 2A^2 \cdot \left(\sum_i M_i^2 \right)^2 . \tag{10}$$

The quantity $\sum_i M_i^2$ is the total strength of the single GDR, a quantity which is independent of the fragmentation. This result again emphasizes the unlikeness that the two-phonon states could have a cross section which deviates markedly from the simple harmonic estimate.

The calculated 'cross sections' for the single and double resonances are presented in fig. 3 in arbitrary units. By the dashed curve in fig. 3a we drew a 'cross section' calculated with a more complete basis that corresponds to the strength distribution in fig. 1c. The calculations of the moment of the distributions presented by the solid curves in fig. 3, lead to $E_{[GDR\otimes GDR]} = 1.98 \cdot E_{GDR}$ and $\Gamma_{[GDR\otimes GDR]} = 1.63 \cdot \Gamma_{GDR}$.

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