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Boson forbidden low-energy E1-transitions in spherical nuclei

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Abstract

Low-energy E1-transitions in spherical nuclei forbidden in the ideal boson picture are considered. For that the internal fermion structure of nuclear excitations is taken into account. Several examples of such transitions calculated within the Quasiparticle Phonon Model are considered and the role of dipole core polarization is discussed. It is shown that transition probabilities of an order of 10^{-3} W.u. observed experimentally are well described by this model. © 1998 Elsevier Science B.V.

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1. Introduction

The electric dipole response of the nucleus at low excitation energies occupies a special place in nuclear structure. There is no collective one-particle–one-hole (1p1h) isoscalar 1^- state, as the well-known 2^+_1 and 3^-_1 states. This kind of a state corresponds to the spurious center of mass motion [1] and several approaches have been developed to exclude it in a realistic calculation [2]. The lowest known 1^- state has a two-phonon $[2^+_1 \otimes 3^-_1]_{1^-}$ nature [3] while the first one-phonon 1^- states in the model calculations appear above 5 MeV in heavy spherical nuclei. Direct excitation of two-phonon states from the ground state by the electromagnetic field is possible only due to the internal fermion structure of phonons [4] but it is strongly hindered as compared to the excitation of one-phonon states. The excitation of low-lying two-phonon (isoscalar) 1^- states is additionally hindered by the isovector nature of the E1-operator (it is

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purely isovector only when $N = Z$). The same is valid for the E1-transitions between isoscalar low-lying excited states because the transitions between their main one-phonon components are also forbidden if one treats phonons as ideal bosons. It is confirmed by the systematics [5,6] that the $B(\mathrm{E1})$ value varies from 10^{-4} to 10^{-6} W.u. for low-energy E1-transitions. This should be compared to the values from 1 to 100 W.u. for the low-lying transitions of other multipolarities. Nevertheless, a new generation of HPGe detectors with great improvement of the detection limits [7] makes it possible to investigate such weak E1-transitions [3].

From a theoretical point of view microscopic studies of these hindered low-energy E1-transitions are of interest because they allow us to look inside the fermion structure of nuclear vibrations and provide a sensitive test of existing nuclear models. The $B(\mathrm{E1})$ values, obtained recently, are the result of a delicate interplay between isoscalar and isovector modes in the structure of low-lying excited states. A study carried out within the Quasiparticle Phonon Model (QPM) [8] will be summarized in the present paper. In Section 2 the main features of this theoretical approach are presented. The analysis of the numerical results is given in Section 3.

2. Theoretical treatment of low-energy E1-transitions

Let us follow the so-called boson mapping procedure and introduce 1p1h-creation operator $Q^+_{\lambda\mu i}$ as a superposition of a bi-linear forms of the quasiparticle creation α^+_{jm} and annihilation α_{jm} operators

$$Q^+_{\lambda\mu i} = \tfrac{1}{2}\sum_{\tau}^{n,p}\sum_{jj'}\left\{\psi^{\lambda i}_{jj'}[\alpha^+_j\alpha^+_{j'}]_{\lambda\mu} - (-1)^{\lambda-\mu}\varphi^{\lambda i}_{jj'}[\alpha_{j'}\alpha_j]_{\lambda-\mu}\right\}, \tag{1}$$

where jm denote a single-particle level of the average field for neutrons (or protons) and the notation $[\ldots]_{\lambda\mu}$ means coupling to the total momentum λ with projection μ: $[\alpha^+_j\alpha^+_{j'}]_{\lambda\mu} = \sum_{mm'} C^{\lambda\mu}_{jmj'm'}\alpha^+_{jm}\alpha^+_{j'm'}$; the quantity $C^{\lambda\mu}_{jmj'm'}$ is the Clebsch–Gordan coefficient. Quasiparticles themselves are the result of the linear Bogoliubov transformation from the particle creation a^+_{jm} and annihilation a_{jm} operators: $a^+_{jm} = u_j\alpha^+_{jm} + (-1)^{j-m}v_j\alpha_{j-m}$. In the QPM, quasiparticle energies and Bogoliubov's coefficients u_j and v_j are obtained by solving the BSC equations. The following notation for the combinations of Bogoliubov's coefficients will be used: $u^{(\pm)}_{jj'} = (u_jv_{j'} \pm v_ju_{j'})$; $v^{(\pm)}_{jj'} = (u_ju_{j'} \pm v_jv_{j'})$.

Taking into account the fermion structure of the operator (1) their commutation relations read

$$[Q_{\lambda\mu i}, Q^+_{\lambda'\mu' i'}]_- = \frac{\delta_{\lambda,\lambda'}\delta_{\mu,\mu'}\delta_{i,i'}}{2}\sum_{jj'}[\psi^{\lambda i}_{jj'}\psi^{\lambda i'}_{jj'} - \varphi^{\lambda i}_{jj'}\varphi^{\lambda i'}_{jj'}]$$
$$- \sum_{\substack{jj'j_2\\ mm'm_2}}\alpha^+_{jm}\alpha_{j'm'}\left\{\psi^{\lambda i}_{j'j_2}\psi^{\lambda' i'}_{jj_2}C^{\lambda\mu}_{j'm'j_2m_2}C^{\lambda'\mu'}_{jmj_2m_2}\right.$$

$$-(-)^{\lambda+\lambda'+\mu+\mu'}\varphi^{\lambda i}_{jj_2}\varphi^{\lambda' i'}_{j' j_2}C^{\lambda-\mu}_{jmj_2m_2}C^{\lambda'-\mu'}_{j'm'j_2m_2}\Big\}. \quad (2)$$

The first term of Eq. (2) corresponds to the ideal boson approximation while the second one takes into account the internal fermion structure of operator (1). The second term is important in considering multi-phonon states which violate the Pauli principle. The basis generated by the operator (1) is constructed by diagonalizing the QPM Hamiltonian which includes an average field, monopole pairing and a residual interaction in a separable form on the set of one-phonon states [8]. This procedure yields the RPA equations, and solving these equations one obtains the energy spectrum and the internal structure of the operator (1), i.e. the coefficients $\psi^{\lambda i}_{jj'}$ and $\varphi^{\lambda i}_{jj'}$ for any multipolarity λ under consideration. The index i in the definition of the operator (1) gets the meaning of the RPA root number. The majority of the RPA eigenstates are practically pure two-quasiparticle excitations. But some of them posses the properties of collective vibrations. The last are usually called as phonons. Indeed, there is no definite criteria in nuclear structure for a boarder between collective, weakly collective and non-collective excitations. To unify all types of 1p1h excitations we will refer below to all of them as phonons. [1]

The main advantage of the QPM is that making use of a separable form of the residual interaction it allows easily to go beyond the one-phonon approximation and take into account a coupling between one- and multi-phonon configurations. It is well known that the boson mapping gives rise to two main problems in considering multi-phonon states as compared to the second-RPA treatment [9,10] of the same problem. The first is an admixture of spurious states which violate the Pauli principle. The second is related to the fact that the set of pure n-phonon states is mathematically non-orthonormal if the internal fermion structure of phonons is taken into account (see Refs. [11,12] for more details). To avoid these problems we introduce an orthonormal set of excited states with angular momentum J and projection M in even–even nuclei as a mixture of two-, four-, six-quasiparticle, etc configurations keeping the phonon's imaging in the following way: [2]

$$\Psi_\nu(JM) = \Bigg\{ \sum_i R_i(J\nu)Q^+_{JMi} + \sum_{\substack{\lambda_1 i_1 \\ \lambda_2 i_2}} P^{\lambda_1 i_1}_{\lambda_2 i_2}(J\nu)\left[Q^+_{\lambda_1\mu_1 i_1}\times Q^+_{\lambda_2\mu_2 i_2}\right]_{JM} + \sum_{\substack{\lambda_1 i_1 \lambda_2 i_2 \\ \lambda_3 i_3 I}} T^{\lambda_1 i_1 \lambda_2 i_2 I}_{\lambda_3 i_3}(J\nu)\left[\left[Q^+_{\lambda_1\mu_1 i_1}\times Q^+_{\lambda_2\mu_2 i_2}\right]_{IK}\times Q^+_{\lambda_3\mu_3 i_3}\right]_{JM} + \ldots \Bigg\}\Psi_0\,, \quad (3)$$

[1] We understand that some authors may not agree with this evolution of terminology. In fact, there is no big confrontation in terminology since the properties of the states under consideration in the present paper are mainly determined by collective RPA states.

[2] We limited Eq. (3) to three-phonon terms as would be used in realistic calculations presented below. The wave function of the ground state Ψ_0 is considered as a phonon vacuum.

and use the internal fermion structure of phonons in the calculation of the norm of the wave function (3). This means that the exact commutation relations (2) between phonon operators and exact commutation relations between phonon and quasiparticle operators

$$\left[\alpha_{jm}, Q^{+}_{\lambda\mu i}\right]_{-} = \sum_{j'm'} \psi^{\lambda i}_{jj'} C^{\lambda\mu}_{jmj'm'} \alpha^{+}_{j'm'} ,$$
$$\left[\alpha^{+}_{jm}, Q^{+}_{\lambda\mu i}\right]_{-} = (-1)^{\lambda-\mu} \sum_{j'm'} \varphi^{\lambda i}_{jj'} C^{\lambda-\mu}_{jmj'm'} \alpha_{j'm'} \qquad (4)$$

are applied. The explicit form for the norm of the wave function of excited states in the case when the coupling between one- and two-phonon configurations is taken into account can be found in Ref. [8]. It includes overlap matrix elements between different two-phonon components

$$\left\langle \left[Q_{\lambda'_2 i'_2} \times Q_{\lambda'_1 i'_1}\right]_J \middle| \left[Q^{+}_{\lambda_1 i_1} \times Q^{+}_{\lambda_2 i_2}\right]_J \right\rangle$$
$$= \left\langle \left[b_{\lambda'_2 i'_2} \times b_{\lambda'_1 i'_1}\right]_J \middle| \left[b^{+}_{\lambda_1 i_1} \times b^{+}_{\lambda_2 i_2}\right]_J \right\rangle + K^J(\lambda'_2 i'_2 \lambda'_1 i'_1 | \lambda_1 i_1 \lambda_2 i_2) ,$$

where $b^{+}_{\lambda i}$ is the ideal boson operator. The quantities K can also be found in Ref. [8]. The experience of realistic calculations shows that the so-called diagonal approximation, $K^J(\lambda'_2 i'_2 \lambda'_1 i'_1 | \lambda_1 i_1 \lambda_2 i_2) = K^J(\lambda_1 i_1 \lambda_2 i_2)\delta_{\lambda_1 i_1, \lambda'_1 i'_1}\delta_{\lambda_2 i_2, \lambda'_2 i'_2}$, provides good accuracy. Within this approximation the normalization condition for states (3) reads [13]

$$\langle \Psi_\nu(JM) \mid \Psi_\nu(JM) \rangle$$
$$= \sum_i [R_i(J\nu)]^2 + 2 \sum_{\substack{\lambda_1 i_1 \\ \lambda_2 i_2}} \left[P^{\lambda_1 i_1}_{\lambda_2 i_2}(J\nu)\right]^2 \left[1 + \tfrac{1}{2} K^J(\lambda_1 i_1 \lambda_2 i_2)\right]$$
$$+ 6 \sum_{\substack{\lambda_1 i_1 \lambda_2 i_2 \\ \lambda_3 i_3 I}} \left[T^{\lambda_1 i_1 \lambda_2 i_2 I}_{\lambda_3 i_3}(J\nu)\right]^2 C^J_I(\lambda_1 i_1, \lambda_2 i_2, \lambda_3 i_3) = 1 ,$$

where

$$C^J_I(\lambda_1 i_1, \lambda_2 i_2, \lambda_3 i_3) = 1 + \tfrac{3}{2} K^I(\lambda_1 i_1 \lambda_2 i_2)$$
$$+ \tfrac{1}{2} K^I(\lambda_1 i_1 \lambda_2 i_2) \sum_{I'} \overline{U^2}(\lambda_1 \lambda_2 J \lambda_3; I, I') K^{I'}(\lambda_2 i_2 \lambda_3 i_3) .$$

The quantities $\overline{U}$ stand for the Jahn coefficients [14].

The matrix elements of the coupling between different configurations included in the wave function in Eq. (3) are calculated within QPM on a microscopic footing without any free parameters making use the internal fermion structure of phonons in Eq. (1) and the model Hamiltonian. Using the exact phonon–phonon (2) and quasiparticle–phonon (4) commutation relations in calculation of the energy, $\langle \Psi_\nu(JM)|H|\Psi_\nu(JM)\rangle$, of excited states (3) results in two types of the Pauli principle corrections. The first effect means some renormalization of the matrix elements of interaction between n- and

$(n+1)$-phonon configurations as compared to the technique when the pair of operators $\alpha^+\alpha$ is projected onto the space of the phonon operators and then the boson algebra is applied. Usually, this effect is marginal for collective configurations. Another, more important, role of the Pauli principle corrections is related to the interaction between n- and n-phonon configurations (with $n \geqslant 2$), known as anharmonicity effect. It means a shift by energy of the n-phonon state from the sum of unperturbed phonon energies. The last effect is allowed for consistently in the present approach as a result of keeping the information on the internal fermion structure of phonons. In realistic calculation presented below these ingredients are taken into account in the diagonal approximation as discussed above for the norm of the wave function for the same reasons.

It may be argued that the boson mapping with keeping the fermion information of the phonon images at all stages of transformations gives no advantage as compared to the n-particle–n-hole (npnh) approach since, mathematically, a direct correspondence between two methods can be established only if the full basis of n-phonon states is used. This is not true because many npnh configurations interact very weakly with others and as a result practically do not mix with them. It allows a sufficient truncation of multi-phonon configurations in the wave functions (3) based on their physical properties with good accuracy for the components important for the subject of research.

Diagonalizing the model Hamiltonian on the set of wave functions (3) we obtain the spectrum of nuclear excitation with a definite value J (the index ν gets the meaning of the order number) and the structure of each excited state, i.e. the coefficients R, P and T. Below we will also use the following notation for the states of Eq. (3): *we will denote by* λ_i^π *the state with the main one-phonon component,* $\lambda^\pi i \equiv Q^+_{\lambda^\pi i}$, (the last is degenerated by energy for different μ in spherical nuclei) *and by* $[\lambda_i^\pi \otimes \lambda_{i'}^{\prime\pi'}]_L$ *the state with the main two-phonon component* $[\lambda^\pi i \times \lambda^{\prime\pi'} i']_L$.

The one-body operator of electric transition has the form

$$M(\mathrm{E}\lambda\mu) = \sum_\tau^{n,p} e_\tau^{(\lambda)} \sum_{\substack{jj' \\ mm'}} (-1)^{j'+m'} \frac{\langle j||\mathrm{E}\lambda||j'\rangle}{\sqrt{2\lambda+1}} C^{\lambda\mu}_{jmj'm'} a^+_{jm} a_{j'-m'}\,, \tag{5}$$

where $\langle j||\mathrm{E}\lambda||j'\rangle \equiv \langle j||i^\lambda Y_\lambda r^\lambda||j'\rangle$ is a reduced 1p1h transition matrix element and $e_\tau^{(\lambda)}$ are effective charges for neutrons and protons. It can be written in terms of quasiparticle and phonon operators as follows:

$$M(\mathrm{E}\lambda\mu) = \sum_\tau^{n,p} e_\tau^{(\lambda)} \sum_{jj'} \frac{\langle j||\mathrm{E}\lambda||j'\rangle}{\sqrt{2\lambda+1}} \left\{ \frac{u^{(+)}_{jj'}}{2} \sum_i (\psi^{\lambda i}_{jj'} + \varphi^{\lambda i}_{jj'}) \right.$$
$$\left. \times (Q^+_{\lambda\mu i} + (-)^{\lambda-\mu} Q_{\lambda-\mu i}) + v^{(-)}_{jj'} \sum_{mm'} C^{\lambda\mu}_{jmj'm'} (-)^{j'+m'} \alpha^+_{j'm'} \alpha_{j'-m'} \right\}. \tag{6}$$

The first term of Eq. (6) corresponds to one-phonon exchange between initial and final states and only transitions described by this term are allowed when phonons are

treated as ideal bosons. Since some of the phonons are collective ones, this term is responsible for enhanced transitions between the ground and the collective one-phonon states, e.g. $0^+_{\rm g.s.} \to 2^+1$ and $0^+_{\rm g.s.} \to 3^-1$. The same is true for the decay of the collective two-phonon states into the one-phonon states: $[2^+1 \times 2^+1]_{0^+,2^+,4^+} \to 2^+1$, etc. The reduced probability for the one-phonon exchange transition is a function of the phonon amplitudes ψ and φ

$$B(\mathrm{E}\lambda, 0^+_{\rm g.s.} \to \lambda^\pi i) = |\langle Q_{\lambda i}|M(\mathrm{E}\lambda)|0^+_{\rm g.s.}\rangle|^2 = \left|\sum_{\tau}^{n,p} e_\tau^{(\lambda)} \sum_{j_1 j_2} \tfrac{1}{2}\langle j_1||\mathrm{E}\lambda||j_2\rangle u^{(+)}_{j_1 j_2}\left(\psi^{\lambda i}_{j_1 j_2} + \varphi^{\lambda i}_{j_1 j_2}\right)\right|^2 . \tag{7}$$

The amplitudes $\psi^{\lambda i}_{j_1 j_2}$ and $\varphi^{\lambda i}_{j_1 j_2}$ are proportional to $\langle j_1||i^\lambda Y_\lambda R_\lambda(r)||j_2\rangle$ where $R_\lambda(r)$ is the radial formfactor of our residual interaction. If one uses $R_\lambda(r) = r^\lambda$, Eq. (7) indicates that all elements of the sum have the same signs for excitation of the 2^+1 and 3^-1 states, i.e. collective excitation with coherent contribution of all 1p1h-configurations takes place.

The Eλ-transitions between multiphonon initial J_i and final J_f states are allowed in the ideal boson picture only when a more complex state includes a phonon of multipolarity λ and other phonons are the same in the both states. Then, the transition matrix element

$$\langle Q_{\lambda_n i_n} \ldots Q_{\lambda_2 i_2} Q_{\lambda_1 i_1}||M(\mathrm{E}\lambda)||Q^+_{\lambda i} Q^+_{\lambda_1 i_1} Q^+_{\lambda_2 i_2} \ldots Q^+_{\lambda_n i_n}\rangle \neq 0 ,$$

while for transitions between other states, although allowed by spin and parity, the transition matrix element equals zero within this approximation. To describe these "forbidden" transitions, we need to go beyond the ideal boson picture and take into account the internal fermion structure of phonons and the fermion nature of the electromagnetic operator in Eq. (5).

The second term in Eq. (6) allows Eλ-transitions between configurations with the same number of phonons or between the ones which differ by an even number of phonons. For example, the reduced probability of direct excitation of the two-phonon state $[\lambda_1^{\pi_1} i_1 \times \lambda_2^{\pi_2} i_2]_{\lambda^\pi}$ from the ground state (the phonon vacuum) has the form

$$B(\mathrm{E}\lambda; 0^+_{\rm g.s.} \to [\lambda_1^{\pi_1} i_1 \times \lambda_2^{\pi_2} i_2]_{\lambda^\pi}) = (2\lambda_1+1)(2\lambda_2+1) \times \left|\sum_{\tau}^{n,p} e_\tau^{(\lambda)} \sum_{j_1 j_2 j_3} v^{(-)}_{j_1 j_2}\langle j_1||\mathrm{E}\lambda||j_2\rangle \begin{Bmatrix} \lambda_2 & \lambda_1 & \lambda \\ j_1 & j_2 & j_3 \end{Bmatrix} \left(\psi^{\lambda_2 i_2}_{j_2 j_3}\varphi^{\lambda_1 i_1}_{j_3 j_1} + \psi^{\lambda_1 i_1}_{j_3 j_1}\varphi^{\lambda_2 i_2}_{j_2 j_3}\right)\right|^2 . \tag{8}$$

and for transitions between the one-phonon states we get

$$B(\mathrm{E}\lambda, \lambda_1^{\pi_1} i_1 \to \lambda_2^{\pi_2} i_2) = (2\lambda_2+1) \times \left|\sum_{\tau}^{n,p} e_\tau^{(\lambda)} \sum_{j_1 j_2 j_3} v^{(-)}_{j_1 j_2}\langle j_1||\mathrm{E}\lambda||j_2\rangle \begin{Bmatrix} \lambda_1 & \lambda_2 & \lambda \\ j_1 & j_2 & j_3 \end{Bmatrix} \left(\psi^{\lambda_1 i_1}_{j_2 j_3}\psi^{\lambda_2 i_2}_{j_3 j_1} + \varphi^{\lambda_1 i_1}_{j_2 j_3}\varphi^{\lambda_2 i_2}_{j_3 j_1}\right)\right|^2 . \tag{9}$$

Eqs. (8) and (9) are obtained by making use of the commutation relations between the quasiparticle and phonon operators (4) and their hermitian conjugates. The matrix element for transitions between the two-phonon states $[\lambda_1^{\pi_1} i_1 \times \lambda_2^{\pi_2} i_2]_{\lambda'^{\pi'}}$ and $[\lambda_3^{\pi_3} i_3 \times \lambda_4^{\pi_4} i_4]_{\lambda''^{\pi''}}$ is very complex and not presented here. Its first order term is very similar to the one for transitions between the one-phonon states $\lambda_1^{\pi_1} i_1$ and $\lambda_4^{\pi_4} i_4$ and may be obtained by assuming that the fermion structure of one phonon is "frozen", i.e., assuming that $\lambda_2^{\pi_2} i_2 \equiv \lambda_3^{\pi_3} i_3$. The next second order term is smaller and it is not taken into account in the present calculations.

An expression for the "forbidden" Eλ-transitions between the states which differ by an odd number of phonons can be derived by applying the fermion representation for the Eλ-operator in Eq. (5) and commutation relations of Eq. (4). Then, one gets

$$
\begin{aligned}
B(\mathrm{E}\lambda, [\lambda_1^{\pi_1} i_1 \times \lambda_2^{\pi_2} i_2]_{\lambda_3^{\pi_3}} \to \lambda_4^{\pi_4} i_4) &= (2\lambda_1+1)(2\lambda_2+1)(2\lambda_4+1) \\
&\times \left| \sum_{\tau}^{n,p} e_\tau^{(\lambda)} \sum_{\substack{j_1 j_2 \\ j' j''}} u_{j_1 j_2}^{(+)} \langle j_2 \| M(\mathrm{E}\lambda) \| j_1 \rangle \sum_J (-1)^J \begin{Bmatrix} j_2 & j_1 & \lambda \\ j'' & j' & \lambda_4 \\ \lambda_1 & \lambda_2 & J \end{Bmatrix} \right. \\
&\times \left[\left(\psi_{j_1 j'}^{\lambda_1 i_1} \psi_{j_2 j''}^{\lambda_2 i_2} \psi_{j'' j'}^{\lambda_4 i_4} + \phi_{j_1 j'}^{\lambda_1 i_1} \phi_{j_2 j''}^{\lambda_2 i_2} \phi_{j'' j'}^{\lambda_4 i_4} \right) \delta_{J,\lambda_3} \right. \\
&\left. \left. + \left(\psi_{j_1 j'}^{\lambda_1 i_1} \phi_{j_2 j''}^{\lambda_2 i_2} \phi_{j'' j'}^{\lambda_4 i_4} + \phi_{j_1 j'}^{\lambda_1 i_1} \psi_{j_2 j''}^{\lambda_2 i_2} \psi_{j'' j'}^{\lambda_4 i_4} \right) (2J+1) \begin{Bmatrix} \lambda & \lambda_4 & \lambda_3 \\ \lambda_1 & \lambda_2 & J \end{Bmatrix} \right] \right|^2 . \qquad (10)
\end{aligned}
$$

Examples of such transitions are the E1-transitions $[2^+1 \times 3^-1]_{1^-} \to 2^+1$ or $[3^-1 \times 3^-1]_{2^+,4^+} \to 3^-1$. In the ideal boson approach only collective E3-transitions between these states are allowed while realistic calculations indicate that, although hindered, the E1-decay of these two-phonon states is preferable [15].

Eqs. (8), (9), and (10) can be generalized for the transitions between more complex configurations. The phonon amplitudes and the reduced matrix elements of the Eλ-transitions in these equations belong to different multipolarities making "random" the sign of different elements in the sums. It results in destructive interference between different elements. The calculations indicate [4,13,15–17] that all transitions of Eqs. (8), (9), (10) are of the same order of magnitude and 2–3 orders of magnitude weaker than the collective transitions of Eq. (7). Thus, in the presence of collective electromagnetic transitions of the same multipolarity the "forbidden" transitions under consideration are negligibly weak and can be omitted in the calculation. This is true for E2- or E3-transitions at low energies. But it is not the case for the E1-transitions in the low-energy region where no collective 1^- states exist as discussed above. The number of "forbidden" E1-transitions between low-lying states is numerous. In Fig. 1 we schematically present by thin lines only the ones which take place between the ground state, one-phonon 2^+1 and 3^-1 states and two-phonon multiplets made of 2^+1 and 3^-1 phonons; collective decay of GDR (Giant Dipole Resonance) into the ground state is shown by thick line in this figure. Many of these transitions are not known from experiment at the present time but some of them have been systematically studied for different mass regions and

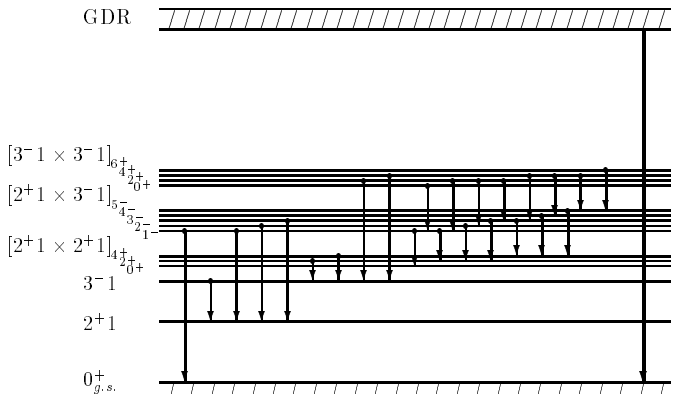

Fig. 1. The E1-transitions between the ground state, one-phonon 2^+1, 3^-1 states and two-phonon multiplets made of 2^+1 and 3^-1 phonons (thin lines). All these transitions are forbidden in the ideal boson picture. Multiplets are degenerated by energy when coupling to other configurations is taken not into account. Allowed within this approach decay of the GDR is shown by thick line.

will be considered below.

We have discussed above the collective and the "forbidden" Eλ-transitions between pure one- and multi-phonon states. If we take into account the coupling between these configurations and describe excited states by the wave function (3), different matrix elements in Eqs. (7), (8), (9), (10) contribute to the transition between the initial J_i and final J_f states with weighting factors R, P and T for the different components. It also means that interference effects are possible and in some cases they play an important role as discussed below. The actual calculations of the properties of low-lying states in nuclei not far from the closed-shell indicates that a contribution of three-phonon configurations to the structure of these states is not large. On the other hand, a coupling to these configurations sufficiently improves the description of the energy of the states with the main two-phonon component. For these reasons, we have included the three-phonon terms in the wave function (3) but omitted transitions from these terms in calculation of the decay properties of the states under consideration.

3. Analysis of some low-energy E1-transitions

Making use of the elements discussed above we have calculated some of these "forbidden" E1-transitions in ^{120}Sn, ^{144}Nd and ^{144}Sm for which experimental data are available. The QPM Hamiltonian includes several fitting parameters. As an average field we have used the Woods–Saxon potential with the parameters from Refs. [16,18]; the value of the constant matrix element for the monopole pairing is also presented in these papers. The parameters of the residual interaction were obtained in the following way. The strength of the residual interaction for $J^\pi = 2^+$ and 3^- was adjusted to reproduce the properties (excitation energy and B(Eλ) value, known from experiment) of the 2^+_1 and 3^-_1 states in the calculation with the wave function in Eq. (3). In this calculation the first

2^+ and 3^- phonons have energies somewhat above the experimental values but due to the coupling to complex configurations, the lowest states are shifted down to the correct position. For the dipole strength the conditions were to exclude the spurious center of mass motion and to achieve the correct position of the GDR centroid. Although several phonons of each multipolarity have been included in the present calculations, the most important are the collective 2^+1 and 3^-1 phonons and also phonons which form the GDR. This is because other (non-collective) phonons have larger excitation energies than 2^+1 and 3^-1 ones, also they carry much less $B(\mathrm{E}\lambda)$-strength and couple weaker to other configurations due to their non-collective nature.

Let us at first consider the direct excitation of the lowest 1^- state from the ground state. This transition is known in many nuclei from the late seventies [19]. Since the energy of the first 1^- state is close to the sum of energies of the 2_1^+ and 3_1^- states it was proposed [20] that the lowest 1^- state was a member of the two-phonon multiplet $[2_1^+ \otimes 3_1^-]$. A renewed interest in the properties of this state has been recently shown up in Nuclear Resonance Fluorescence (NRF) experiments with a new generation of HPGe detectors [3]. Comparing the values of the E2-transitions $2_1^+ \to 0_{\mathrm{g.s.}}^+$ and $1_1^- \to 3_1^-$, the experimental evidence of two-phonon nature of the lowest 1^- has been achieved [21]. This state was also observed in the (n, n$'\gamma$) reaction and inelastic scattering of protons and α-particles [3,22–28].

Theoretically, the properties of the low-lying 1^- states and especially the E1-transitions have been studied in the framework of different nuclear models [4,17,27–30] after the first paper by Vogel and Kocbach [20]. Considering the lowest 1^- state as a pure two-phonon one, the theoretical calculations overpredicted the $B(\mathrm{E1})$ value for this state. To improve the situation it was suggested [1] that renormalized values of effective charges for E1-transitions should be used according to

$$e_{\mathrm{eff}}^{(\mathrm{E1})} = -\tfrac{1}{2}e\left(\tau_z - \frac{N-Z}{A}\right)(1+\chi)\,. \tag{11}$$

This takes phenomenologically into account a core polarization due to the coupling of the first 1^- state to GDR. The parameter χ was estimated in a static limit [31] and its typical value was obtained as $\chi \sim -0.7$ for heavy nuclei. In our approach we are able to avoid the use of this additional parameter by including explicitly the GDR phonons into the model wave function of Eq. (3).

The results of our calculations of the excitation energies and the $B(\mathrm{E1})$ values for the lowest 1^- states along selected nuclei are presented in Table 1 in comparison with experimental data. Since the energies of the first 2^+ and 3^- phonons are above experimental values, as discussed above, the energy of the pure two-phonon configuration $[2^+1 \times 3^-1]_{1^-}$ is also higher than the one from the experiment. The three-phonon configurations in the model wave function sufficiently improve the agreement. These configurations are also responsible for some strength fragmentation of the two-phonon configuration under consideration over several 1^- states. In semi-magic nuclei, ^{120}Sn and ^{144}Sm, this fragmentation is not large and the first 1^- state carries 94% and 95% of this two-phonon configuration, respectively. But the fragmentation increases in the

Table 1
Excitation energies and $B(\mathrm{E1})$ values for first 1^- states in ^{120}Sn, ^{144}Sm and ^{144}Nd. Calculations are performed with wave function (3). Columns "Main" and "Polar." give separate contribution for $B(\mathrm{E1})$-transition to two- and one-phonon part of wave function, respectively; "Total" is the total transition when interference effect between transitions in two previous columns is also taken into account

	E_x (MeV)		Main configuration	$B(\mathrm{E1}, 0^+_{\mathrm{g.s.}} \to [2^+_1 \otimes 3^-_1]_{1^-})$ $(10^{-3}\,\mathrm{e^2fm^2})$			
	Theory	Exp.	$[2^+1 \times 3^-1]_{1^-}$	Theory			Exp.
			(%)	Main	Polar.	Total	
^{120}Sn	3.29	3.271 [32]	94	12.2	2.1	7.2	7.6 [32]
^{144}Sm	3.44	3.225 [25,26]	95	52.2	8.8	18.1	18.9 [25,26]
^{144}Nd	2.57	2.185 [33]	82	51.6	21.8	6.3	7.2 [34]

non-magic nucleus ^{144}Nd, and as a result, the $[2^+1 \times 3^-1]_{1^-}$ configuration contributes to the norm of the first 1^- state with the weight of 82% while the 12% contribution is given by the three-phonon configuration $[[2^+1 \times 2^+1]_{2^+} \times 3^-1]_{1^-}$. There are two main reasons for the enhancement of the fragmentation. First, the collectivity of the lowest phonons is larger in non-magic nuclei, which increases the coupling matrix elements between different configurations. Second, the energies of these phonons are smaller and this decreases the energy gap between the lowest one- and two-; two- and three-phonon configurations and enhances the mixing.

In the column "Main" of Table 1 the $B(\mathrm{E1})$ value for the transition to the two-phonon component $[2^+1 \times 3^-1]_{1^-}$ of the wave function (3) of the first 1^- state is presented. It equals to $|P^{2^+1}_{3^-1}(1^-1) \cdot \langle [2^+1 \times 3^-1]_{1^-} | M(\mathrm{E1}) | 0^+_{\mathrm{g.s.}} \rangle|^2$. As has been mentioned above, it overestimates the experimental value (the last column). The inclusion of the GDR phonons in the model wave function improves the agreement. Since GDR is located more than 10 MeV higher, its admixture to the lowest 1^- state is very weak: $|R_{\mathrm{GDR}}(1^-1)| < 0.5\%$ in the present calculation. But the large value of the collective one-phonon exchange matrix element $\langle \mathrm{GDR} | M(\mathrm{E1}) | 0^+_{\mathrm{g.s.}} \rangle$ leads to the fact that $|P^{2^+1}_{3^-1}(1^-1) \cdot \langle [2^+1 \times 3^-1]_{1^-} | M(\mathrm{E1}) | 0^+_{\mathrm{g.s.}} \rangle|^2$ and $|R_{\mathrm{GDR}}(1^-1) \cdot \langle \mathrm{GDR} | M(\mathrm{E1}) | 0^+_{\mathrm{g.s.}} \rangle|^2$ are the values of the same order of magnitude. The last value, core polarization effect, is presented in the column "Polar." of Table 1. When the interference effect between the transitions to two- and one-phonon components of this state wave function is taken into account, we obtain the final result given in the column "Total". For the lowest 1^- state the interference has a destructive character and brings the final result in good agreement with experimental findings. According to the numerical results in Table 1, it is possible to calculate the parameter χ in Eq. (11) which has to be used if coupling to GDR is not taken into account. We obtain the values -0.34, -0.41 and -0.65 for ^{120}Sn, ^{144}Sm, ^{144}Nd, respectively.

The first 1^- state may decay by E1-transitions not only into the ground state but also into the 2^+_1 state (see, Fig. 1). The results of our calculation for the second transitions are presented in Table 2. Since the lowest 1^- state is mainly of the two-phonon nature

Table 2
B(E1) values for some E1-transitions between excited states in ^{120}Sn, ^{144}Sm and ^{144}Nd. Column "Main" is the contribution of transition between one-phonon components of the 3^-_1 and 2^+_1 states for the $3^-_1 \to 2^+_1$ decay; "Polar." is polarization effect; "Total" is the total transition when interference effect between transitions in two previous columns is also taken into account

	$B(\mathrm{E1}, J_i \to J_f)$ $(10^{-3}\,\mathrm{e^2fm^2})$							
	$[2^+_1 \otimes 3^-_1]_{1^-} \to 2^+_1$		$[3^-_1 \otimes 3^-_1]_{2^+} \to 3^-_1$		$3^-_1 \to 2^+_1$			
	Theory	Exp.	Theory	Exp.	Theory			Exp.
					Main	Polar.	Total	
^{120}Sn	0.08	–	0.93	–	3.7	0.34	1.8	2.02 [35]
^{144}Sm	0.76	0.61 [25,26]	1.71	1.20 [25,26]	24	6.2	5.9	5.0 [36]
^{144}Nd	3.4	4.25 [33]	0.52	–	13.5	3.36	3.38	1.77 [33]

and the 2^+_1 state is practically a one-phonon state, the main term responsible for this transition is the one: $\langle 2^+1|\mathrm{E1}|[2^+1 \times 3^-1]_{1^-}\rangle$ described by Eq. (10). This is the case for the semi-magic nuclei ^{120}Sn and ^{144}Sm. The structure of the lowest 2^+ and 1^- states in ^{144}Nd is more complex than that of the semi-magic nuclei because of the collectivity of the 2^+1 phonon. It results in that the one-phonon component exhausts 88% of the norm of the wave function. The two-phonon components $[2^+1 \times 2^+1]_{2^+}$ and $[2^+1 \times 4^+1]_{2^+}$ largely contribute to the structure of the 2^+_1 state. Also, several configurations contribute the structure of the 1^-_1 state. Because of that the transition probability is influenced by the matrix elements: $\langle [2^+1 \times 2^+1]_{2^+}|E1|[2^+1 \times 3^-1]_{1^-}\rangle$, $\langle [2^+1 \times 4^+1]_{2^+}|E1|[2^+1 \times 3^-1]_{1^-}\rangle$. Experimental values of $\mathrm{B}(E1; 1^-_1 \to 2^+_1)$ are very different for ^{144}Sm and ^{144}Nd. The calculations reproduce the experimental tendency.[3]

The GDR core polarization is marginal for the $B(\mathrm{E1}; [2^+_1 \otimes 3^-_1]_{1^-} \to 2^+_1)$ value, because of the following reason. In principle, an influence of the GDR may occur via some admixture of the two-phonon $[\mathrm{GDR} \times 2^+1]_{1^-}$ configuration in the wave function of the lowest 1^- state. But this admixture is much weaker as compared to the admixture of the GDR itself in the wave function of this state. The energy gap between the $[\mathrm{GDR} \times 2^+1]_{1^-}$ and $[2^+1 \times 3^-1]_{1^-}$ configurations is very big while the coupling matrix between them is small. In general, the coupling between different two-phonon configurations is much weaker as compared to their coupling to one- and three-phonon configurations. The first coupling is responsible for an anharmonicity effect which produces a shift of these configurations of about 100 keV in heavy nuclei [37]. On the other hand, the coupling between one- and two-phonon configurations leads to fragmentation of the one-phonon configuration strength over a few MeV.

Similar arguments for the GDR core polarization are also valid for the E1-decay of the $[3^-_1 \otimes 3^-_1]_{2^+,4^+}$ states into the 3^-_1 state. Our calculations for the E1-transition $[3^-_1 \otimes 3^-_1]_{2^+} \to 3^-_1$ are presented in Table 2. For this transition in addition to the term

[3] The contribution of the matrix element of Eq. (10) to B(E1) value was not taken into account in the calculations presented in Refs. [17,34].

$\langle 3^-1|\mathrm{E1}|[3^-1\times 3^-1]_{2^+}\rangle$ in Eq. (10) there are also some admixtures from the transition matrix elements $\langle 3^-1|\mathrm{E1}|2^+1\rangle$ in Eq. (9) and $\langle [2^+1\times 3^-1]_{3^-}|\mathrm{E1}|[3^-1\times 3^-1]_{2^+}\rangle$. The second matrix element contributes due to an admixture of the one-phonon 2^+1 component in the wave function of the $[3^-_1\otimes 3^-_1]_{2^+}$ state; and the third one, to two-phonon $[2^+1\times 3^-1]_{3^-}$ configuration in the wave function of the 3^-_1 state. All three matrix elements in this paragraph are of the same order of magnitude. Thus, their contribution to the transition probability depends on the value of the amplitudes R and P of the wave function (3). In the case of ^{120}Sn the first matrix element has a weighting factor $R_1(3^-1)\cdot P^{3^-1}_{3^-1}(2^+\nu)\sim 1$ while two other matrix elements are weighted with factors $R_1(3^-1)\cdot R_1(2^+\nu)\ll 1$ and $P^{3^-1}_{2^+1}(3^-1)\cdot P^{3^-1}_{3^-1}(2^+\nu)\ll 1$, respectively. For ^{144}Sm and ^{144}Nd the two-phonon component $\left[3^-1\times 3^-1\right]_{2^+}$ is shared between several states [17]. In Table 2 we have presented the transition for the state with the maximum contribution of the $\left[3^-1\times 3^-1\right]_{2^+}$ component in the structure. It has recently been suggested that the transition $[3^-_1\otimes 3^-_1]_{2^+}\to 3^-_1$ should be used to identify the two-phonon octupole vibrations in ^{208}Pb in the NRF experiment [15].

The GDR core polarization may play an important role for E1-transitions between two excited states with the main one-phonon configurations. An example of such a transition is the decay of the 3^-_1 state into the 2^+_1 state. As for the decay of the $[2^+_1\otimes 3^-_1]_{1^-}$ state into the ground state, we have for this transition a competition between weak "forbidden" $\langle 2^+1|\mathrm{E1}|3^-1\rangle$ matrix element, which has a weighting factor $R_1(3^-1)\cdot R_1(2^+1)\sim 1$, and two collective $\langle [\mathrm{GDR}\times 3^-1]_{2^+}|\mathrm{E1}|3^-1\rangle$ and $\langle 2^+1|\mathrm{E1}|[\mathrm{GDR}\times 2^+1]_{3^-}\rangle$ matrix elements with small weighting factors $R_1(3^-1)\cdot P^{3^-1}_{\mathrm{GDR}}(2^+1)\ll 1$ and $R_1(2^+1)\cdot P^{2^+1}_{\mathrm{GDR}}(3^-1)\ll 1$, respectively. The results of our calculations of the $3^-_1\to 2^+_1$ decay are presented in Table 2. Separate contributions of transitions between one-phonon components of the 3^-_1 and 2^+_1 states and between one- and two-phonon configurations, [GDR×low-lying state], are shown in columns "Main" and "Polar.", respectively.

4. Summary

The low-energy E1-transitions in spherical nuclei between the ground and excited states and between excited states are considered. The transitions are allowed by spin and parity but forbidden in the ideal boson picture. It is shown that the transitions can be quantitatively described making use of the internal fermion structure of phonons. The GDR core polarization is taken into account and its influence on the transitions is calculated on microscopic footing. The contribution of the GDR leads to a sufficient reduction of the transition probability. It is in agreement with the phenomenological renormalization of effective charge for the quantitative description of low-energy E1-transitions.

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