

## Microscopic studies of two-phonon giant resonances \*

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Position, width and excitation probability in relativistic heavy ion collisions of two-phonon giant dipole resonances in spherical and deformed nuclei are considered.

### 1. Introduction

Investigation on the properties of two-phonon giant resonances together with similar studies on low-lying two-phonon states should give an answer on how far the harmonic picture of boson-type excitations holds in the finite fermion systems like atomic nuclei. These resonances were experimentally observed in heavy ion collision at relativistic [1] and intermediate [2] energies and in pion-induced double-charge exchange reactions. The recent theoretical studies [3–7] of the two-phonon giant dipole resonance (DGDR) carried out within the Quasiparticle-Phonon Model (QPM) [8] will be presented in this talk.

### 2. Position and width of the DGDR

Two GDR may couple to the total angular momentum  $J^\pi = 0^+, 1^+, 2^+$ . If the GDR is considered as a single state, the  $J^\pi = 1^+$  component is forbidden by symmetry properties. When Landau damping is taken into account and several RPA-states form the GDR,  $1^+$  two-phonon states made of two different  $1^-$  phonons exist but their excitation is quenched [3] in relativistic heavy ion collision (RHIC). For this reason only  $0^+$  and  $2^+$  components of the DGDR will be considered below.

To describe the position and the width of the DGDR we will use the following set of wave functions for the DGDR states [4,5,9]:

$$\Psi^\nu(J) = \left\{ \sum_{\alpha_1} S_{\alpha_1}^\nu(J) Q_{\alpha_1}^+ + \sum_{\alpha_2 \beta_2} \frac{D_{\alpha_2 \beta_2}^\nu(J) Q_{\alpha_2}^+ Q_{\beta_2}^+}{\sqrt{1 + \delta_{\alpha_2, \beta_2}}} + \sum_{\alpha_3 \beta_3 \gamma_3} \frac{T_{\alpha_3 \beta_3 \gamma_3}^\nu(J) Q_{\alpha_3}^+ Q_{\beta_3}^+ Q_{\gamma_3}^+}{\sqrt{1 + \delta_{\alpha_3, \beta_3} + \delta_{\alpha_3, \gamma_3} + \delta_{\beta_3, \gamma_3} + 2\delta_{\alpha_3, \beta_3, \gamma_3}}} \right\} | \rangle_{ph}, \quad (1)$$

where  $Q_\alpha^+$  is a phonon creation operator ( $\alpha, \beta, \gamma \equiv (\lambda^\pi, i)$  and  $i$  is the RPA root number for given momentum  $\lambda^\pi$ );  $| \rangle_{ph}$  is the phonon vacuum. The index  $\nu$  ( $= 1, 2, 3 \dots$ ) is the order number of the states (1) with the given  $J$  value. It is assumed that any combination  $\alpha, \beta, \gamma$  of phonons appears only once. The second and the third terms in Eq. (1) include phonons of different multiplicities.

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The nuclear Hamiltonian arranges the coupling between different configuration in the wave function (1). An interaction between multiphonon configurations with the same number of phonons produces an energy shift  $\Delta$  of each configuration from its unperturbed value. This effect is known as an anharmonicity effect. A coupling of the configuration with the  $n$  number of phonons to  $(n + 1)$ -phonon configurations results in fragmentation of the strength of simple configurations over more complex ones, the density of which is much higher. We say that simple configuration fragments or gets a spreading width. It is known that in heavy nuclei the spreading width is the main part for the total width of giant resonance.

The diagonalization of the model Hamiltonian within the basis of states defined in Eq. (1) leads to secular equation. We have solved it in the space of two-phonon states:

$$\det \left\| \left( \omega_{\alpha_2} + \omega_{\beta_2} - E_x \right) [\delta_{\alpha_2\beta_2, \alpha'_2\beta'_2} + \tilde{K}_J(\beta_2\alpha_2 | \alpha'_2\beta'_2)] + \Delta(\alpha_2\beta_2, \alpha'_2\beta'_2) - \sum_{\alpha_1} \frac{U_{\alpha_2\beta_2}^{\alpha_1}(J) U_{\alpha'_2\beta'_2}^{\alpha_1}(J)}{\omega_{\alpha_1} - E_x} - \sum_{\alpha_3\beta_3\gamma_3} \frac{U_{\alpha_2\beta_2}^{\alpha_3\beta_3\gamma_3}(J) U_{\alpha'_2\beta'_2}^{\alpha_3\beta_3\gamma_3}(J)}{\omega_{\alpha_3} + \omega_{\beta_3} + \omega_{\gamma_3} - E_x} \right\| = 0, \quad (2)$$

where  $\omega_\alpha$  is the RPA energy for the  $i_\alpha^{\text{th}}$  state of multipolarity  $\lambda$ . The quantities  $U_{\alpha_2\beta_2}^{\alpha_1}(J)$  and  $U_{\alpha_3\beta_3\gamma_3}^{\alpha_2\beta_2}(J)$  are matrix elements of the interaction connecting two-phonon configurations  $\{\alpha_2\beta_2\}$  with one-  $\{\alpha_1\}$  and three-phonon  $\{\alpha_3\beta_3\gamma_3\}$  configurations, respectively, while  $\Delta(\alpha_2\beta_2, \alpha'_2\beta'_2)$  is the energy shift of the configuration  $\{\alpha_2\beta_2\}$  due to its interaction with the configuration  $\{\alpha'_2\beta'_2\}$ , both belonging to the two-phonon response. These quantities are calculated making use of the model Hamiltonian and the microscopic fermion structure of phonons. The solution of Eq. (2) provides the eigenvalues  $E_x^\nu$  associated with the states introduced in Eq. (1) and the coefficients  $S_{\alpha_1}^\nu(J)$ ,  $D_{\alpha_2\beta_2}^\nu(J)$ , and  $T_{\alpha_3\beta_3\gamma_3}^\nu(J)$ .

A direct calculation of the DGDR states with the wave function (1) are not possible because of a very high density of complex configurations in the DGDR energy region. Thus, one needs to truncate the basis states. To calculate the width of the DGDR we have selected  $\{1^-i, 1^-i'\} = \{\alpha_2, \beta_2\}$  two-phonon configurations displaying the largest  $B(E1) \times B(E1)$  values. They are built up out of the most collective RPA roots from the GDR energy region which carry the largest  $B(E1)$  values. Two-phonon states of collective character and made of phonons with quantum numbers different from  $1^-$  were not included in the calculations. The three-phonon states  $\{\alpha_3\beta_3\gamma_3\}$  were built out of phonons with angular momentum and parity  $1^-, 2^+, 3^-$  and  $4^+$ . Only those configurations where either  $\alpha_3, \beta_3$  or  $\gamma_3$  were equal to  $\alpha_2$  or  $\beta_2$  were chosen. This is because other configurations lead to matrix elements  $U_{\alpha_3\beta_3\gamma_3}^{\alpha_2\beta_2}(J)$  of the interaction between two- and three-phonon states, which are orders of magnitude smaller than those associated with the above mentioned three-phonon configurations.

The cross section of the DGDR states excitation in RHIC have been calculated in the second order perturbation theory [10] with the GDR states as intermediate ones in the two-step process. Only, transitions between one-phonon GDR and two-phonon DGDR components were taken into consideration since it was proved that transitions between complex GDR and DGDR configurations play a marginal role in this reaction [6].

The final results of our calculation for the width of the DGDR in  $^{136}\text{Xe}$  [4] and  $^{208}\text{Pb}$  [5] is that its value is very close to  $\sqrt{2}$  times the width of the single GDR. This is not surprising for this calculation because the selection of three-phonon configurations included in wave function (1) was done on a 'boson level'. But we hope that omitted three-phonon states with weak coupling matrix elements to two-phonon  $[1^-i \times 1^-i']$  DGDR states will not change sufficiently the results.

Anharmonicity effects are rather weak in the present calculation. In  $^{136}\text{Xe}$  we obtain the energy shift of the centroid of the DGDR with respect to that expected in the harmonic picture equal to  $-90$  keV and  $-120$  keV for  $J^\pi = 2^+$  and  $J^\pi = 0^+$ , respectively. It may be argued that the calculated shift is somewhat underestimated, because of the limitations used in selecting two-phonon basis states used in the calculation. This can be checked in calculation with relatively large two-phonon (or 2p2h) basis. Of course, in this case one has to exclude three-phonon (or 3p3h) configurations from the wave function (1) to make them possible. Such calculations have been performed and they confirm that anharmonic effects for the DGDR do not exceed 100–200 keV in heavy nuclei [11,12].

### 3. Double giant dipole resonance in deformed nuclei

The possibility to observe the DGDR in deformed nuclei with the present state of art experimental techniques is still questionable. This is mainly due to the fact that one has to expect a larger width of these resonances as compared to spherical nuclei. In a phenomenological approach the GDR is considered as a collective vibration of protons against neutrons. In spherical nuclei this state is degenerate in energy for different values of the spin  $J = 1^-$  projection  $M = 0, \pm 1$ . The same is true for the  $2^+$  component of the DGDR with projection  $M = 0, \pm 1, \pm 2$ . In deformed nuclei with an axial symmetry like  $^{238}\text{U}$ , the GDR is split into two components  $I^\pi(K) = 1^-(0)$  and  $I^\pi(K) = 1^-(\pm 1)$  corresponding to vibrations against two different axes. In this approach one expects a three-bump structure for the DGDR with the value  $K = 0$ ,  $K = \pm 1$  and  $K = 0, \pm 2$ , respectively. Actually, the GDR possesses a width and the main mechanism responsible for it in deformed nuclei is the Landau damping. Thus, the conclusion on how three bumps overlap and what is the real shape of the DGDR in these nuclei, i.e., either a three-bump or a flat broad structure, can be drawn out only from some consistent microscopic studies.

To do that [7], first, the phonon basis for the  $K = 0$  and  $K = \pm 1$  components of the GDR in  $^{238}\text{U}$  is constructed by solving the quasiparticle-RPA equations for each projection. All one-phonon states with the energy lower than 20 MeV and with the  $B(E1)$  value larger than  $10^{-4} e^2 fm^2$  are accounted for. Their total number equals to 447 and 835 for the  $K = 0$  and  $K = \pm 1$  components, respectively. The  $B(E1, 0_{g.s.}^+ \rightarrow 1^-)$  strength distribution known from experiment for this nucleus is reproduced very well in this calculation. The wave function of the  $0^+$  and  $2^+$  states belonging to the DGDR are obtained by the folding of two  $1^-$  phonons from RPA calculations.

Second, we have calculated the excitation of the DGDR in  $^{238}\text{U}$  projectiles (0.5 GeV·A) incident on  $^{120}\text{Sn}$  and  $^{208}\text{Pb}$  targets. These calculations have been performed in the second order perturbation theory [10], in which the DGDR states are excited within a two-step process:  $g.s. \rightarrow \text{GDR} \rightarrow \text{DGDR}$ . As intermediate states, the full set of one-phonon  $1_{\bar{K}=0}(i)$  and  $1_{\bar{K}=\pm 1}(i')$  states was used. We have also calculated the GDR excitation to first order

Table 1

The properties of the different components of the GDR and the DGDR in  $^{238}\text{U}$ . The energy centroid  $E_c$ , the second moment of the strength distribution  $m_2$  in RHIC, and the cross sections  $\sigma$  for the excitation of the projectile are presented for: a)  $^{238}\text{U}$  (0.5 GeV·A) +  $^{120}\text{Sn}$ , and b)  $^{238}\text{U}$  (0.5 GeV·A) +  $^{208}\text{Pb}$ .

	$E_c$ [MeV]	$m_2$ [MeV]	$\sigma$ [mb]	
			a)	b)
GDR( $K = 0$ )	11.0	2.1	431.2	1035.4
GDR( $K = \pm 1$ )	12.3	2.6	1560.2	3579.1
GDR(total)	12.0	2.6	1991.4	4614.5
DGDR $_{0^+}$ ( $K = 0$ )	25.0	3.4	18.3	88.9
DGDR $_{2^+}$ ( $K = 0$ )	24.4	3.5	11.8	58.7
DGDR $_{2^+}$ ( $K = \pm 1$ )	23.9	3.2	22.7	115.4
DGDR $_{2^+}$ ( $K = \pm 2$ )	25.3	3.4	49.7	238.3
DGDR(total)	24.8	3.4	102.5	501.3

for the same systems. The results of our calculations are summarized in Table 1 and Fig. 1. We plot only the smeared strength functions of the energy distributions because the number of two-phonon states involved is numerous.

The two-bump structure can still be seen in the curve representing the cross section of the GDR excitation in  $^{238}\text{U}$  in RHIC as a function of the excitation energy. But its shape differs appreciably from the B(E1) strength distribution. The reason for that is the role of the virtual photon spectra. First, for the given value of the excitation energy and impact parameter it is larger for the  $K = \pm 1$  component than that for the  $K = 0$  one (see also the first two lines in Table 1). Second, for both components it has a decreasing tendency with an increase of the excitation energy [10].

The curves representing the cross sections of the excitation of the  $K = \pm 1$  and  $K = \pm 2$  components of the DGDR in  $^{238}\text{U}$  in RHIC have typically a one-bump structure. It is because they are made of two-phonon  $2^+$  states of one type,  $[1_{K=0}^-(i) \otimes 1_{K=\pm 1}^-(i')]_{2_{K=\pm 1}^+}$  and  $[1_{K=\pm 1}^-(i'_1) \otimes 1_{K=\pm 1}^-(i'_2)]_{2_{K=\pm 2}^+}$ , respectively. The  $K = 0$  components of the DGDR include two group of states:  $[1_{K=0}^-(i_1) \otimes 1_{K=0}^-(i_2)]_{0(2)_{K=0}^+}$  and  $[1_{K=\pm 1}^-(i'_1) \otimes 1_{K=\mp 1}^-(i'_2)]_{0(2)_{K=0}^+}$ . Its strength distribution has two-bumps. The excitation of the states with  $K = 0$  in RHIC is enhanced due to their lower energies, while the enhancement of the excitation of the states with  $K = \pm 2$  is related to the strongest response of the  $K = \pm 1$  components to the external E1 Coulomb field in both stages of the two-step process. Summing together all components of the DGDR yields a broad one-bump distribution for the cross section for the excitation of the DGDR in  $^{238}\text{U}$ , as a function of excitation energy. It is presented by the solid curve in Fig. 1b. Another interesting result of our calculations is related to the position of the DGDR energy centroid and to the second moment of the DGDR cross section. The centroid of the DGDR in RHIC is shifted to the higher energies by about 0.8 MeV from the expected value of two times the energy of the GDR centroid. The origin for this shift is in the energy dependence of the virtual photon spectra.

Our studies also indicate that the DGDR width in deformed nuclei is smaller than  $\sqrt{2}$  times the width of the GDR, as observed with spherical nuclei. Our calculation yields the value 1.33 for the ratio  $\Gamma_{\text{DGDR}}/\Gamma_{\text{GDR}}$  in RHIC. The origin for this effect is in the different

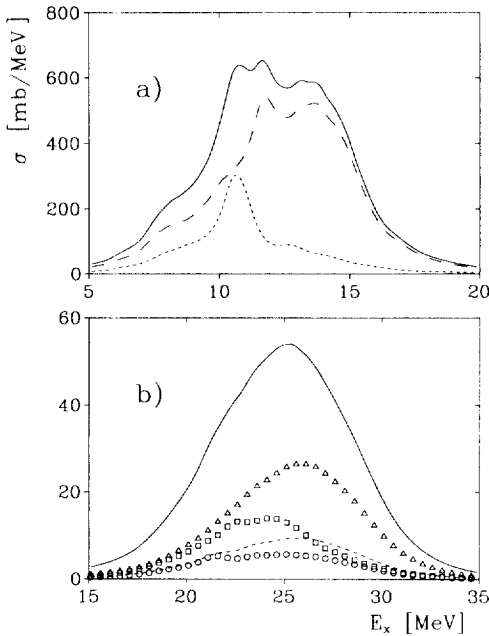


Figure 1. The strength functions for the excitation: a) of the GDR, and b) of the DGDR in  $^{238}\text{U}$  in the  $^{238}\text{U} (0.5 \text{ GeV}\cdot\text{A}) + ^{208}\text{Pb}$  reaction. In a), the short-dashed curve corresponds to the GDR ( $K = 0$ ) and the long-dashed curve to the GDR ( $K = \pm 1$ ). In b) the dashed curve corresponds to the  $\text{DGDR}_{0-}$  ( $K = 0$ ), the curve with circles to the  $\text{DGDR}_{2-}$  ( $K = 0$ ), the curve with squares to the  $\text{DGDR}_{2+}$  ( $K = \pm 1$ ), and the curve with triangles to the  $\text{DGDR}_{2+}$  ( $K = \pm 2$ ). The solid curve is the sum of all components. The strength functions are calculated with the smearing parameter equal to 1 MeV. The results for  $^{238}\text{U} (0.5 \text{ GeV}\cdot\text{A}) + ^{120}\text{Sn}$  reaction look very similar and differ only by the absolute value of cross sections.

contributions of the GDR  $K = 0$  and  $K = \pm 1$  components to the total cross section, due to the reaction mechanism. It should be remembered that only the Landau damping is accounted for the width of both the GDR and the DGDR. But we think that the effect of narrowing of the DGDR width still holds if the coupling to complex configurations is included in the calculation.

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