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Microscopic studies of multi-phonon resonances

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A systematic microscopic study of the anharmonic properties of the double giant resonance (DGDR) has been carried out for some nuclei with mass number $40 \leq A \leq 208$. It is found that the corrections of the energy centroid of the DGDR from its harmonic limit are negative, have a value of the order of few hundred keV and follow an A^{-1} dependence.

1. INTRODUCTION

The discovery of the double giant dipole resonance in nuclei (DGDR) [1–5] and the observation of small deviations from the harmonic picture concerning the excitation energy and the spreading width, combined with the large deviations of the associated Coulomb excitation cross sections measured in relativistic heavy ion collisions [4], demand a better understanding of the role anharmonicities play in the spectrum of the DGDR. In fact, anharmonicities influence electromagnetic DGDR cross sections in several ways: a) the energy shifts of the DGDR states from the harmonic values can affect in an important way the electromagnetic cross section, in keeping with the exponential dependence of these quantities with the Q-value of the process [6], b) anharmonicities lead to changes in the E1 transition matrix elements to preserve the energy weighted sum rule (EWSR) [7] which eventually reinforce these effects, c) anharmonicities which are a consequence of the mixing of states with different number of phonons, give rise to many paths, other than the (harmonic) two-step one, to excite the DGDR in electromagnetic processes. While all these questions inspired much theoretical work, [8–17], no clear picture has emerged of the DGDR anharmonicity question, let alone an explanation of the "Coulomb excitation anomaly". In particular, no consensus exists concerning the mass-number dependence of the energy shifts from the harmonic values.

The first systematic calculation of the spectrum of the DGDR in a complete one- and two-phonon basis (the effect of the 3 phonon states on the anharmonicity is small [12]) for nuclei with mass number A spanning the whole mass table has been performed recently [18]. In this paper we discuss results of the microscopic studies of the DGDR anharmonicity.

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2. FORMULAE AND NUMERICAL DETAILS

The Hamiltonian used in describing the system contains, aside from a mean field term which determines the single-particle motion of protons and neutrons, a monopole pairing interaction and a separable multipole-multipole force with strengths adjusted so as to reproduce the odd-even mass differences and the spectrum of low-lying vibrations and of giant resonances respectively [19,20].

Since we focus our studies on anharmonic properties of two-phonon excited states with a total spin J and projection M, we describe them by a wave function

$$|\Psi^{\nu}_{JM}>=\left\{\sum_i R_i(J\nu)Q^+_{Ji}+\sum_{\lambda_1 i_1\le\lambda_2 i_2} P^{\lambda_2 i_2}_{\lambda_1 i_1}(J\nu)\frac{[Q^+_{\lambda_1\mu_1 i_1}Q^+_{\lambda_2\mu_2 i_2}]_{JM}}{\sqrt{1+\delta_{\lambda_1 i_1,\lambda_2 i_2}}}\right\}|>_{ph}\ , \tag{1}$$

which is a superposition of different one- and two-phonon configuration and $|>_{ph}$ is a phonon vacuum. Coupling of two-phonon states to three-phonon ones leads to a fragmentation of a strength of two-phonon states and has been considered in Ref. [12]. In Eq. (1) the following notation is used

$$[Q^+_{\lambda_1\mu_1 i_1}Q^+_{\lambda_2\mu_2 i_2}]_{JM}=\sum_{\mu_1\mu_2} C^{JM}_{\lambda_1\mu_1\lambda_2\mu_2}Q^+_{\lambda_1\mu_1 i_1}Q^+_{\lambda_2\mu_2 i_2}\ ,$$

where C is a Clebsh-Gordon coefficient.

In our approach, phonons posses an internal fermion structure. A phonon creation operator $Q^+_{\lambda\mu i}$ with a multipolarity λ, projection μ and order number i is a linear combination of two quasiparticle creation α^+_{jm} and annihilation α_{jm} operators with shell quantum numbers $j\equiv(n,l,j)$ and m:

$$Q^+_{\lambda\mu i}=\frac{1}{2}\sum_{\tau}^{n,p}\sum_{jj'}\left\{\psi^{\lambda i}_{jj'}[\alpha^+_j\alpha^+_{j'}]_{\lambda\mu}-(-1)^{\lambda-\mu}\varphi^{\lambda i}_{jj'}[\alpha_{j'}\alpha_j]_{\lambda-\mu}\right\}\ . \tag{2}$$

An energy spectrum of one-phonon states $\omega_{\lambda i}$ is obtained by solving quasiparticle-RPA equations. These equations also yield the values of the forward $\psi^{\lambda i}_{jj'}$ and backward $\varphi^{\lambda i}_{jj'}$ amplitudes in Eq. (2). In the case of a separable form of a residual interaction with a radial formfactor $f_\lambda(r)$, they are

$$\begin{pmatrix}\psi\\ \varphi\end{pmatrix}^{\lambda i}_{jj'}(\tau)=\frac{1}{\sqrt{2\mathcal{Y}^{\lambda i}_{\tau}}}\cdot\frac{f^{\lambda}_{jj'}(\tau)(u_jv_{j'}+v_ju_{j'})}{\varepsilon_j+\varepsilon_{j'}\mp\omega_{\lambda i}}$$

where $f^{\lambda}_{jj'}=<j'||f_\lambda(r)||j>$; u_j and v_j are coefficients of Bogoliubov transformation from particle a_{jm} to quasiparticle operators: $a^+_{jm}=u_j\alpha^+_{jm}+(-1)^{j-m}v_j\alpha_{j-m}$; ε_j is a quasiparticle energy and $\mathcal{Y}_\tau$ are normalization coefficients (see, e.g. Ref. [20] for an explicit form). The quasiparticle spectrum and u_j, v_j coefficients are obtained from solving BSC equations.

To obtain a spectrum $E_\nu(J)$ of states (1) and their structure, i.e. the coefficients R and P, we diagonalize the following matrices:

$$\left|\left|<\Psi^{\nu'}_J|H|\Psi^{\nu}_J|>-E<\Psi^{\nu'}_J|\Psi^{\nu}_J|>\right|\right|_{[\nu\times\nu']}$$

$$= \left\| \begin{array}{cc} < [Q_{\lambda_4 i_4} Q_{\lambda_3 i_3}]_J |H| [Q^+_{\lambda_1 i_1} Q^+_{\lambda_2 i_2}]_J > & U^{\lambda_1 i_1}_{\lambda_2 i_2}(Ji) \\ U^{\lambda_4 i_4}_{\lambda_3 i_3}(Ji) & \omega_{Ji} - E \end{array} \right\| = 0 \tag{3}$$

for a definite value of J. Our model Hamiltonian may be written in terms of quasiparticle and phonon operators as following:

$$H = \sum_{jm} \varepsilon_j \alpha^+_{jm} \alpha_{jm} - \frac{1}{4} \sum_{\lambda \mu i i'} \sum_{\tau}^{n,z} \frac{X^{\lambda i}_\tau + X^{\lambda i'}_\tau}{\sqrt{\mathcal{Y}^{\lambda i}_\tau \mathcal{Y}^{\lambda i'}_\tau}} Q^+_{\lambda \mu i} Q_{\lambda \mu i'} + H_{int} \tag{4}$$

where

$$X^{\lambda i}_\tau = \frac{1}{2\lambda + 1} \sum_{jj'}^{\tau} \frac{[f^{\lambda}_{jj'}(\tau)(u_j v_{j'} + v_j u_{j'})]^2 (\varepsilon_j + \varepsilon_{j'})}{(\varepsilon_j + \varepsilon_{j'})^2 - \omega^2_{\lambda i}}$$

and H_{int} is a term responsible for an interaction between quasiparticles and phonons. Coefficients in the second term of (4) are such that the model Hamiltonian in the form (4) is diagonal in the space of one-phonon configurations. Thus, no double counting of quasiparticle operators occur due to the fact that they appear twice in the first and second terms because the phonons themselves posses an internal fermion structure.

A matrix element of interaction between two-phonon configurations has the form:

$$\begin{aligned} &\sum_{all\ \mu} < [Q_{\lambda_4 \mu_4 i_4} Q_{\lambda_3 \mu_3 i_3}]_{JM} |H| [Q^+_{\lambda_1 \mu_1 i_1} Q^+_{\lambda_2 \mu_2 i_2}]_{JM} > \\ &= (\omega_{\lambda_1 i_1} + \omega_{\lambda_2 i_2}) \cdot \left\{ \delta_{\lambda_1 i_1, \lambda_3 i_3} \delta_{\lambda_2 i_2, \lambda_4 i_4} + K^{\lambda_4 i_4 \lambda_3 i_3}_{\lambda_1 i_1 \lambda_2 i_2}(J) \right\} \\ &- \frac{1}{4} \sum_{i\tau} \left[\frac{X^{\lambda_3 i_3}_\tau + X^{\lambda_3 i}_\tau}{\sqrt{\mathcal{Y}^{\lambda_3 i_3}_\tau \mathcal{Y}^{\lambda_3 i}_\tau}} K^{\lambda_4 i_4 \lambda_3 i}_{\lambda_1 i_1 \lambda_2 i_2}(J) + \frac{X^{\lambda_4 i_4}_\tau + X^{\lambda_4 i}_\tau}{\sqrt{\mathcal{Y}^{\lambda_4 i_4}_\tau \mathcal{Y}^{\lambda_4 i}_\tau}} \right. \\ &\times K^{\lambda_4 i \lambda_3 i_3}_{\lambda_1 i_1 \lambda_2 i_2}(J) + \sum_{\substack{\lambda_5 i_5 \\ \lambda_6 i_6}} \frac{X^{\lambda_5 i_5}_\tau + X^{\lambda_5 i}_\tau}{\sqrt{\mathcal{Y}^{\lambda_5 i_5}_\tau \mathcal{Y}^{\lambda_5 i}_\tau}} K^{\lambda_4 i_4 \lambda_3 i_3}_{\lambda_5 i_5 \lambda_6 i_6}(J) \\ &\left. \times K^{\lambda_6 i_6 \lambda_5 i}_{\lambda_1 i_1 \lambda_2 i_2}(J) \right] \cdot \frac{1}{\sqrt{(1 + \delta_{\lambda_1 i_1, \lambda_2 i_2})(1 + \delta_{\lambda_3 i_3, \lambda_4 i_4})}} \end{aligned} \tag{5}$$

where coefficients K^J are complex functions of phonon's amplitudes ψ and φ in Eq. (2) and have the form:

$$\begin{aligned} &K^{\lambda_4 i_4 \lambda_3 i_3}_{\lambda_1 i_1 \lambda_2 i_2}(J) = \sum_{\substack{j_1 j_2 \\ j_3 j_4}} (-1)^{j_2 + j_4} \hat{\lambda_1} \hat{\lambda_2} \hat{\lambda_3} \hat{\lambda_4} \\ &\times \left[(-1)^{\lambda_2 + \lambda_4} \left\{ \begin{array}{ccc} j_1 & j_2 & \lambda_4 \\ j_4 & j_3 & \lambda_3 \\ \lambda_1 & \lambda_2 & J \end{array} \right\} \left(\psi^{\lambda_3 i_3}_{j_3 j_4} \psi^{\lambda_1 i_1}_{j_1 j_4} \psi^{\lambda_2 i_2}_{j_3 j_2} \psi^{\lambda_4 i_4}_{j_1 j_2} \right. \right. \\ &\left. - \varphi^{\lambda_3 i_3}_{j_3 j_4} \varphi^{\lambda_1 i_1}_{j_1 j_4} \varphi^{\lambda_2 i_2}_{j_3 j_2} \varphi^{\lambda_4 i_4}_{j_1 j_2} \right) \\ &- (-1)^{\lambda_1 + \lambda_2} \left\{ \begin{array}{ccc} \lambda_1 & \lambda_2 & J \\ j_2 & j_4 & j_3 \end{array} \right\} \left\{ \begin{array}{ccc} \lambda_3 & \lambda_4 & J \\ j_2 & j_4 & j_1 \end{array} \right\} \\ &\left. \times \left(\varphi^{\lambda_3 i_3}_{j_1 j_4} \varphi^{\lambda_1 i_1}_{j_4 j_3} \psi^{\lambda_2 i_2}_{j_2 j_3} \psi^{\lambda_4 i_4}_{j_1 j_2} - \psi^{\lambda_3 i_3}_{j_1 j_4} \psi^{\lambda_1 i_1}_{j_4 j_3} \varphi^{\lambda_2 i_2}_{j_2 j_3} \varphi^{\lambda_4 i_4}_{j_1 j_2} \right) \right] \end{aligned} \tag{6}$$

where $\hat{\lambda}_i = \sqrt{2\lambda_i + 1}$. The terms in $K^J \sim \psi^2\varphi^2$ and $\sim \varphi^4$ do not exceed 1-2% of the main term ψ^4 for low-lying collective phonons and they are negligibly small for phonons belonging to giant resonances.

A matrix element of interaction between one- and two-phonon configurations equals:

$$
\begin{aligned}
&U^{\lambda_1 i_1}_{\lambda_2 i_2}(\lambda i) = \sum_{all\ \mu} < [Q_{\lambda\mu i}|H|[Q^+_{\lambda_1\mu_1 i_1} Q^+_{\lambda_2\mu_2 i_2}]_{JM} > \\
&= (-1)^{\lambda_1+\lambda_2-\lambda} \hat{\lambda}_1 \hat{\lambda}_2 \sum_{\tau}^{n,p} \sum_{j_1 j_2 j_3} \\
&\left[\frac{f^{\lambda}_{j_1 j_2} v^{(-)}_{j_1 j_2}}{\sqrt{\mathcal{Y}^{\lambda i}_{\tau}}} \begin{Bmatrix} \lambda_1 & \lambda_2 & \lambda \\ j_2 & j_1 & j_3 \end{Bmatrix} \left(\psi^{\lambda_1 i_1}_{j_3 j_1}\, \varphi^{\lambda_2 i_2}_{j_2 j_3} + \psi^{\lambda_2 i_2}_{j_2 j_3}\, \varphi^{\lambda_1 i_1}_{j_3 j_1} \right) \right. \\
&+ \frac{f^{\lambda_1}_{j_1 j_2} v^{(-)}_{j_1 j_2}}{\sqrt{\mathcal{Y}^{\lambda_1 i_1}_{\tau}}} \begin{Bmatrix} \lambda_1 & \lambda_2 & \lambda \\ j_3 & j_2 & j_1 \end{Bmatrix} \left(\varphi^{\lambda i}_{j_2 j_3}\, \varphi^{\lambda_2 i_2}_{j_3 j_1} + \psi^{\lambda i}_{j_2 j_3}\, \psi^{\lambda_2 i_2}_{j_3 j_1} \right) \\
&\left. + \frac{f^{\lambda_2}_{j_1 j_2} v^{(-)}_{j_1 j_2}}{\sqrt{\mathcal{Y}^{\lambda_2 i_2}_{\tau}}} \begin{Bmatrix} \lambda_1 & \lambda_2 & \lambda \\ j_1 & j_3 & j_2 \end{Bmatrix} \left(\psi^{\lambda i}_{j_3 j_1}\, \psi^{\lambda_1 i_1}_{j_2 j_3} + \varphi^{\lambda i}_{j_3 j_1}\, \varphi^{\lambda_1 i_1}_{j_2 j_3} \right) \right]
\end{aligned}
\tag{7}
$$

for phonons of natural parity where $v^{(-)}_{j_1 j_2} = u_{j_1} u_{j2} - v_{j_1} v_{j_2}$.

Diagrams corresponding to different terms of Eqs. (5) and (7) can be found in Ref. [20].

An orthogonality relation between pure two-phonon configurations has the form:

$$
\begin{aligned}
&\sum_{all\ \mu} < [Q_{\lambda_4\mu_4 i_4} Q_{\lambda_3\mu_3 i_3}]_{JM} \mid [Q^+_{\lambda_1\mu_1 i_1} Q^+_{\lambda_2\mu_2 i_2}]_{JM} > \\
&= \delta_{\lambda_1 i_1, \lambda_3 i_3} \delta_{\lambda_2 i_2, \lambda_4 i_4} + \frac{K^{\lambda_4 i_4 \lambda_3 i_3}_{\lambda_1 i_1 \lambda_2 i_2}(J)}{\sqrt{(1+\delta_{\lambda_1 i_1, \lambda_2 i_2})(1+\delta_{\lambda_3 i_3, \lambda_4 i_4})}} .
\end{aligned}
\tag{8}
$$

Equations (5,7,8) have been obtained by applying exact commutation relation between phonon operators, i.e. taking into account their internal fermion structure.

To exclude an energy dependence in a two-step process of the DGDR excitation in relativistic heavy ion collision which also modifies the energy centroid of the DGDR in respect to a twice energy of the single GDR, we have calculated an energy independent quantity

$$
\begin{aligned}
&B_{\nu}([E1 \times E1]_J) = \left| \sum_i \langle \Psi^{\nu}_J | E1 | \Psi^i_{1^-} \rangle \cdot \langle \Psi^i_{1^-} | E1 | \Psi_{g.s.} \rangle \right|^2 \\
&= \frac{2J+1}{3} \left| 2 \sum_{i_1 \le i_2} P^{1^- i_2}_{1^- i_1}(J\nu) \frac{M_{i_1}(E1) M_{i_2}(E1)}{\sqrt{1+\delta_{i_1, i_2}}} \right|^2
\end{aligned}
\tag{9}
$$

for the DGDR excitation, where $M_i(E1)$ is a reduced transition probability of an $E1$-excitation of an i^{th} one-phonon 1^- configuration from the ground state. Intermediate states $\Psi^i_{1^-}$ belonging to the GDR are described in an one-phonon approximation. The results will be compared to harmonic picture of nuclear excitation in which coefficients

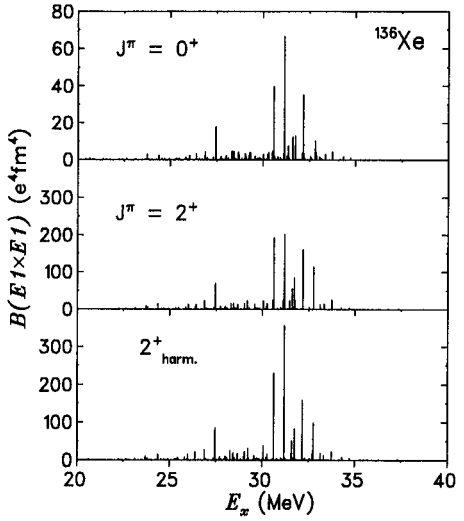

Figure 1. Energy distributions of the $B(E1 \times E1)$ values associated with the excitation of the 0^+ and 2^+ components of the DGDR in ^{136}Xe, in comparison with the same quantity for the 2^+ component in the harmonic limit. Scales are chosen proportionally to $(2J+1)$.

$P^{\lambda_2 i_2}_{\lambda_1 i_1}(J\nu) = \delta_{\lambda_1 i_1 \lambda_2 i_2, 1^- i_1 1^- i_2}$ and an excitation energy for a two-phonon configuration $[1^- i_1 \times 1^- i_2]_J$ equals exactly $(\omega_{1^- i_1} + \omega_{1^- i_2})$.

In calculations, we have used natural parity phonons with the multipolarity λ from 0 to 4. All one-phonon configuration up to 50 MeV excitation energy have been included for the first term of wave function (1). The basis of two-phonon configurations has been slightly truncated to make calculation possible according to: All collective and weakly collective phonons which contribute to an energy weighted sum rule (EWSR) with more than 0.2% for dipole phonons and 1.0% for other multipolarities have been accounted for.

3. RESULTS

Calculations have been performed for nuclei from different A-mass regions. In Fig. 1 we show the quantity $B_\nu([E1 \times E1]_J)$ for the different (two-phonon) states ν, eigenstates of the total Hamiltonian with angular momentum and parity 0^+ and 2^+ of the nucleus ^{136}Xe.

Results of our calculations are listed in the first column of Table 1.

In the second column of this table an exhaust of TRK EWSR by electric dipole phonons included in a model space is provided. In the third column an exhaust of a DGDR sum rule $S_2^{(1)}$ introduced in Ref. [7] is given for a sum of 0^+ and 2^+ components of the DGDR in our calculations. This quantity

$$S_2^{(1)} = 4 S_1^{(0)} S_1^{(1)}$$

Table 1
Percentage of the EWSR exhausted by the GDR and DGDR of the atomic nuclei indicated in the first column. In column 4 and 5 is displayed the anharmonicity shift $\Delta E_c(J^\pi)$ of the energy centroid of the $J^\pi = 0^+$ and 2^+ components of the DGDR from its harmonic limit.

A	EWSR, %		$\Delta E_c(J^\pi)$, keV	
Nucl.	GDR	DGDR	$J^\pi = 0^+$	$J^\pi = 2^+$
^{40}Ca	104.	103.	-643	-740
^{58}Ni	104.	103.	-476	-495
^{86}Kr	106.	105.	-309	-271
^{120}Sn	106.	105.	-199	-194
^{136}Xe	103.	102.	-203	-179
^{208}Pb	94.	94.	-108	-158

where $S_1^{(1)}$ is the TRK sum rule for the GDR and for

$$S_1^{(0)} = \sum_i \left| < 1_i^- | E1 | g.s. > \right|^2$$

the value from our calculation of the GDR excitation in one-phonon approximation has been used. A small difference between the exhaust of the EWSR for the DGDR and the GDR is due to the fact that the ground state is considered as a phonon vacuum in the present approach and the ground state correlations which arise from an interaction between multi-phonon configurations are not accounted for.

In the last two columns of Table 1 the values of an anharmonicity shift of the DGDR energy centroid

$$E_c(J) = \frac{\sum_\nu B_\nu([E1 \times E1]_J) \cdot E_\nu(J)}{\sum_\nu B_\nu([E1 \times E1]_J)}$$

in respect to the energy centroid of the DGDR in harmonic approximation, i.e. the twice of the energy centroid of the GDR, is presented for 0^+ and 2^+ components of the DGDR separately.

Pauli principle corrections are responsible for this energy shift. Excluding four-quasiparticle configurations which violate Pauli principle reduces somehow a collectivity of two-phonon configurations. Thus, from general arguments we should expect a positive sign of the shift for two-phonon states built up of two isoscalar phonons and a negative sign when we are dealing with isovector phonons as in the case of the DGDR.

An absolute value of the anharmonicity shift of the 0^+ (starts) and 2^+ (triangles) components of the DGDR are also presented in a graphic form in Fig. 2 as a function of the atomic mass A. Lines represent predictions of macroscopic approaches for the A dependence of this quantity: A^{-1} (solid line) in Ref. [19] and $A^{-5/3}$ (dashed line) in Ref. [14].

The results of our calculations follow very well the solid line, although both double- and semimagic nuclei have been included in consideration. Weighting equally the 0^+ and 2^+ components of the DGDR we obtain from a χ^2 analysis of the results in Fig. 2

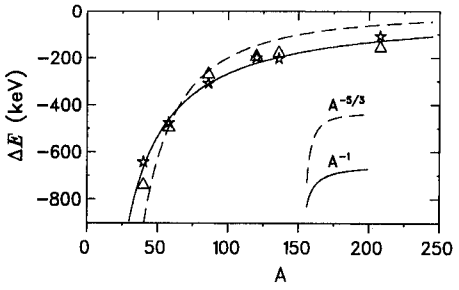

Figure 2. Shift of the DGDR energy centroid (0^+ - stars and 2^+ triangles) from the harmonic limit. The continuous and dashed curves represent fits, assuming an A^{-1} and an $A^{-5/3}$ dependence respectively, of the results of the microscopic calculations.

a $|\Delta E| \sim A^{-\alpha}$ dependence with $\alpha = 1.08 \pm 0.06$. To better appreciate the results, we should point out that the present calculations of the shift have been performed with no free parameters. The most essential parameter for this calculation, the strength of the isovector dipole residual interaction, has been fixed to reproduce the energy of the GDR centroid in each nuclei known from experiment or systematic.

Comparing the results of our present calculation of the DGDR properties with previous ones published in Ref. [12] we should comment on the following. The main idea of the calculation in Ref. [12] was to describe the width of the DGDR and for this reason a three-phonon term was added to the wave function (1), but to make numeric calculation possible we had to truncate very strongly a basis of the two-phonon configurations. Thus, only a few the most important two-phonon configurations of the type $[1_i^- \times 1_{i'}^-]$ have been included in a model space. For this reason, the shift of the DGDR centroid in Ref. [12] was underestimated but not very much as the present calculation show, and 0^+ and 2^+ components of the DGDR were practically degenerated. In the present calculation a model space of two-phonon configurations is rather complete and the $B([E1 \times E1]_J)$ strength distribution over the two component of the DGDR is different. It is presented in Fig. 1a) for 0^+ and in Fig. 1b) for 2^+ components of the DGDR in ^{136}Xe in a comparison with the strength distribution in a harmonic limit in Fig 1c).

We have also did a simplified calculation in which only the most collective two-phonon $[1_i^- \times 1_i^-]_{J^\pi}$ configuration and a complete set of one-phonon configurations with $J^\pi = 0^+$ (or 2^+) have been included in a model space. It yields a set of states, one of each carrying about 95% of the two-phonon configuration in every nucleus. There are two mechanisms contributing to a shift of such states. The first one is a Pauli principle correction, already discussed in the case of complete calculation, which is always negative This term scales very nicely as A^{-1}. The second mechanism arises from an interaction of the $[1_i^- \times 1_i^-]_{J^\pi}$ configuration with all one-phonon configurations. It has a positive sign and is cancelling strongly with the first term.

An analysis of the structure of one-phonon configurations to which the most collective two-phonon $[1_i^- \times 1_i^-]_{J^\pi}$ configuration prefers to couple reveals no specific features of

them. They are not very collective and some of them are practically pure 1p1h excitations. Indeed, the collective isoscalar one-phonon configurations which have the largest matrix element U with this two-phonon configuration are far below in energy to mix. The one-phonon configurations belonging to the isovector GQR have a very small value of U because of a different with the DGDR value of the isospin. Also, the one-phonon configurations which admix to the $[1_i^- \times 1_i^-]_{J^\pi}$ configuration are not the closest in energy because the ones in a vicinity have a small U value. They are just the one-phonon configurations which are not very far in energy from the $[1_i^- \times 1_i^-]_{J^\pi}$ configuration and have a moderate value of the coupling matrix element U due to their internal fermion structure and the structure of the two-phonon configuration. So, the second term keeps an information on a shell structure in each specific nucleus when only a few 1p1h configurations are essential and it does not scale with A at all.

It should be pointed out that the cancellation mentioned above and a discussion in the two previous paragraphs corresponds only to the shift of the state which carries the main portion of the strongest $[1_i^- \times 1_i^-]_{J^\pi}$ configuration in respect to its unperturbed energy.

4. CONCLUSION

We conclude that the deviation of the energy centroid of the double giant dipole resonance from the harmonic limit displays a behaviour with mass number A typical of that associated with the global properties characterizing the system, like e.g. the energy centroid of the giant dipole resonance.

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