



## One- and two-phonon mixed-symmetry states in $^{94}\text{Mo}$ in high-resolution electron and proton scattering\*

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High-resolution inelastic electron scattering experiments at the S-DALINAC and proton scattering experiments at iThemba LABS permit a thorough test of the nature of proposed one- and two-phonon symmetric and mixed-symmetric  $2^+$  states of the nucleus  $^{94}\text{Mo}$ . The combined analysis reveals the one-phonon content of the mixed-symmetry state and its isovector character suggested by microscopic calculations. The purity of two-phonon  $2^+$  states is extracted.

### 1. Introduction

Low-energy nuclear valence shell excitations usually possess the lowest possible isospin quantum number  $T_z = |N - Z|/2$ . Nevertheless, the symmetry character of their proton-neutron coupling can vary. This fact is clearly formulated in the framework [1] of the proton-neutron version of the nuclear interacting boson model (IBM-2). The *sd*-IBM-2 considers monopole (*s*) bosons and quadrupole (*d*) bosons while the number of proton bosons  $N_\pi$  and of neutron bosons  $N_\nu$  are taken as the number of respective valence nucleon pairs. The model describes quadrupole-collective valence shell excitations. Basis states can be classified according to the symmetry of their proton-neutron coupling quantified by the *F*-spin quantum number. Fully symmetric states (FSSs) have maximum *F*-spin  $F_{\text{max}} = (N_\pi + N_\nu)/2$ . Those with quantum numbers  $F < F_{\text{max}}$  are called mixed-symmetry

\*Supported by the NRF and by the DFG under contracts SFB 634, NE 679/2-2 and 445 SUA-111/3/04.

states (MSSs). A characteristic feature of MSSs are enhanced magnetic dipole ( $M1$ ) transitions to fully symmetric states.

Recently, one-phonon and two-phonon MSSs were investigated in less collective vibrational nuclei, e.g. in the nuclide  $^{94}\text{Mo}$  [2–4]. Properties of these structures and the mechanism of their formation have been discussed microscopically in the framework of the shell model (SM) [5] and the quasiparticle phonon model (QPM) [6]. Experimental evidence is largely based on  $M1$  transition rates between excited nuclear states [7]. Since a  $B(M1)$  value involves an integral of the  $M1$  operator over nuclear wave functions, evidence for multi-phonon vibrational structures of MSSs is based so far on integral data only. Differential information, such as scattering form factors or differential cross sections, offers additional sensitivity for testing the structure of nuclear multi-phonon MSSs. Here, a combined study of  $^{94}\text{Mo}$  with high-resolution electron and proton scattering is reported.

## 2. Experiment and results

The ( $e,e'$ ) experiments were carried out at the high-resolution magnetic spectrometer [8] of the Darmstadt superconducting electron linear accelerator S-DALINAC. Data were taken at an incident electron beam energy  $E_e = 70$  MeV and scattering angles  $\Theta_e = 93^\circ - 165^\circ$  with typical beam currents of  $2 \mu\text{A}$ . A highly enriched (91.6 %) self-supporting  $^{94}\text{Mo}$  target was used. In dispersion-matching mode an energy resolution  $\Delta E \simeq 30$  keV (full width at half maximum, FWHM) was achieved.

High-resolution ( $p,p'$ ) measurements were performed at the cyclotron of iThemba LABS, South Africa, using a K600 magnetic spectrometer. The techniques were similar to those described in [9] except for the additional implementation of the faint beam technique [10] accelerating the optimization of dispersion-matched conditions. The target consisted of a self-supporting molybdenum foil of  $1.2 \text{ mg/cm}^2$  areal density enriched to 93.9 %  $^{94}\text{Mo}$ . Data were taken at a proton energy  $E_p = 200$  MeV and scattering angles  $\Theta_p = 6^\circ - 27^\circ$  with currents varying from 1 to 30 nA, depending on the scattering angle. Typical energy resolutions were  $\Delta E \simeq 35$  keV (FWHM).

Examples of the electron and proton scattering spectra are shown in Fig. 1. The prominent peaks correspond to the elastic line, the collective  $2_1^+$  and  $3_1^-$  states, and the suggested one-phonon  $2^+$  MSS (the  $2_3^+$  level). The candidates for the two-phonon FSS ( $2_2^+$ ) and MSS ( $2_5^+$ ) are weakly excited only, but the extended view (r.h.s. of Fig. 1) demonstrate that they could be identified in the data as well as all other  $2^+$  states known [7] in the excitation region up to 4 MeV. The dominance of the excitations to the  $2_1^+$  and  $2_3^+$  states indicates already the concentration of one-phonon proton and neutron strength in their wave functions.

## 3. Combined analysis of electron and proton scattering

To extract further information on the phonon character of the observed states we analyze the momentum-transfer dependence of the cross sections (for electron scattering normalized to the Mott cross section) of the observed transitions. The experimental findings for the one- and two-phonon candidates shown in Fig. 2 are compared to microscopic quasiparticle phonon model (QPM), shell model (SM), and IBM-2 calculations.

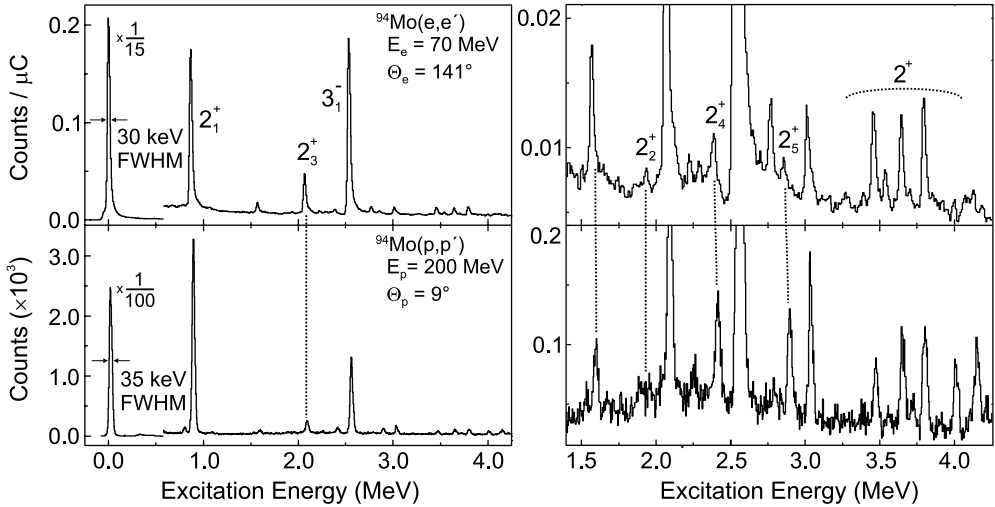


Figure 1. L.h.s.: Spectrum of the  $^{94}\text{Mo}(e,e')$  reaction at  $E_e = 70$  MeV and  $\Theta_e = 141^\circ$  (top), and spectrum of the  $^{94}\text{Mo}(p,p')$  reaction at  $E_p = 200$  MeV and  $\Theta_p = 9^\circ$  (bottom). R.h.s.: Zoom on the  $E_x = 1.5 - 4$  MeV region of the respective spectra.

### 3.1. Model calculations

QPM wave functions of  $2^+$  states in  $^{94}\text{Mo}$  have been obtained by diagonalizing a model Hamiltonian in the space of interacting one-, two-, and three-phonon configurations. Multi-phonon configurations have been built from phonons with  $J^\pi = 1^\pm - 6^\pm$ . The approach is similar to Ref. [6] except for the treatment of the particle-particle channel of the residual interaction. The calculation exactly reproduces the number of experimentally known  $2^+$  states in the studied energy interval and also predicts their excitation energies to a level allowing for a one-to-one correspondence between calculated states and the data.

The shell-model calculations considered a valence space of 4 protons and 2 neutrons with  $^{88}\text{Sr}$  as inert core using the parameters of [5]. The IBM-2 description of  $(e,e')$  form factors followed the approach suggested in [11]. The radial dependence of the transition densities was obtained in a generalized-seniority shell-model calculation [12]. The vibrational  $U(5)$  limit was used to describe the dominant transitions within the IBM. For the qualitative discussion below, this approximation should show little difference to a calculation with realistic IBM parameters.

### 3.2. One-phonon excitations

Theoretical proton scattering cross sections with the microscopic wave functions of the QPM and SM were determined in distorted wave Born approximation (DWBA) with the code DWBA05 [13] using the  $t$ -matrix parametrization [14] of Franey and Love at 200 MeV as effective projectile-target interaction. DWBA corrections were also applied for the electron scattering results.

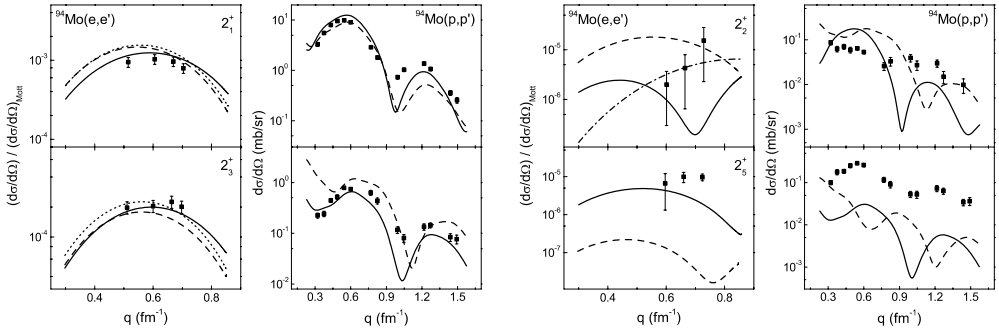


Figure 2. Momentum-transfer dependence of the excitation of the one-phonon (l.h.s.) and two-phonon (r.h.s.) FSS (top) and MSS (bottom) in  $^{94}\text{Mo}$  observed in electron and proton scattering, respectively. The data (full circles) are compared to QPM (solid lines), SM (dashed lines), and IBA-2 (dotted lines) calculations described in the text.

The l.h.s. of Fig. 2 presents the results for the transitions populating the one-phonon FSS and MSS in  $^{94}\text{Mo}$ . We first note the similarity of data for both states. All models provide a good description of the form factors. The underlying symmetric and antisymmetric coupling of the main proton and neutron components of the wave functions found in all model calculations confirms the picture of a FSS and MSS. Considering an overall uncertainty of about 25% due to the choice of the effective interaction [15], the QPM accounts well for the proton scattering results. The SM results encounter some difficulties at higher momentum transfers tested in the proton scattering data, where correlations outside the valence space become important.

### 3.3. Two-phonon excitations

The transitions to the two-phonon state candidates are much more sensitive to details of the wave functions since the transition strengths are strongly influenced by the complex interplay of the weakly excited, but large two-phonon components with strongly excited, but small one-phonon admixtures. The r.h.s. of Fig. 2 shows the comparison of the SM and QPM results to the corresponding (e,e') and (p,p') data. Here, the SM fails for the electron scattering results; the predicted cross sections are significantly too large for the FSS and too small for the MSS. This might be traced back again to the limited SM space which does not allow for sufficient 4-particle 4-hole components in the two-phonon state wave functions. The QPM with larger model space indeed provides cross sections of the correct magnitude for the symmetric two-phonon state although it predicts a pronounced minimum at a momentum transfer  $q \simeq 0.72 \text{ fm}^{-1}$  due to an interference of the one- and two-phonon components not observed in the data. However, a simplified QPM calculations considering the basic one- and two-phonon states allows a good description is achieved. This indicates the symmetric two-phonon state to be very pure. For the mixed-symmetry candidate, the full QPM results are somewhat small but account roughly for the momentum transfer dependence. Still, a dominant two-phonon character prevails. We

note, however, that the full QPM calculation also predicts non-negligible three-phonon components (about 17%).

As is visible in the r.h.s. of Fig. 2, both SM and QPM results are off the (p,p') data. A possible explanation is the neglect of two-step processes in the reaction mechanism. Such contributions are indeed small for collective transitions at an incident energy of 200 MeV/nucleon but are enhanced [16] for the extreme case of very weak one-step excitations and strong two-step excitations through collective levels encountered here. To estimate the two-step processes at least qualitatively, a coupled-channel analysis was performed with the code CHUCK3 [17]. It is based on the collective model, where nuclear excitations are described as small surface vibrations of multipolarity  $L$ . The amplitude of these vibrations is proportional to the coupling strength  $\beta$ . The transition potential is taken to be the derivative of the optical potential. Starting from the global parameter set of Ref. [18], optical model parameters for the present reaction were determined by a fit to the elastic scattering cross sections. The l.h.s. of Fig. 3 indicates the coupling schemes taken into

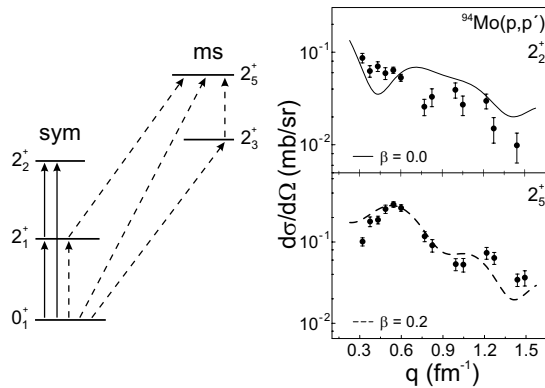


Figure 3. Coupled-channel analysis for the excitation of the two-phonon FSS and MSS in the  $^{94}\text{Mo}(p, p')$  experiment. Left: Coupling scheme. Right: Best fits to the data (solid lines) using  $\beta = 0.0$  for the FSS (top) and  $\beta = 0.2$  for the MSS (bottom).

account for the two-phonon FSS and MSS, respectively. The coupling strengths of the one-phonon transitions to the  $2_1^+$  and  $2_3^+$  states were determined by a fit to the data. Unknown (like  $2_1^+ \rightarrow 2_5^+$ ) or poorly [7] known (like  $2_1^+ \rightarrow 2_2^+$ ) transition strengths were fixed assuming harmonic vibrations. The CHUCK3 results for the two-phonon states are displayed on the r.h.s. of Fig. 3. The best description of the  $2_2^+$  state is achieved for a vanishing one-step amplitude ( $\beta = 0.0$ ), i.e., the conclusion of a nearly pure two-phonon nature drawn from the electron scattering results is confirmed. For the mixed-symmetry state, the best description is achieved with  $\beta = 0.2$ . The corresponding one-step cross section implies a one-phonon component in the excited-state wave function which agrees well with the estimate obtained from the (e,e') results. Thus, after inclusion of two-step

contributions to the  $(p,p')$  cross sections a fully consistent picture is obtained from both experimental probes.

#### 4. Conclusions

To summarize, we have tested the nature of proposed one- and two-phonon symmetric and mixed-symmetry  $2^+$  states in  $^{94}\text{Mo}$  through high-resolution inelastic electron and proton scattering experiments in consistent measurements for the first time. A microscopic analysis with QPM, SM and IBM-2 calculations confirms the dominant one-phonon structure of the transitions to the first and third  $2^+$  state. Because of the different isospin sensitivity of both experimental probes, the model description of the combined data reveals the isovector character of the one-phonon MSS within the valence shell. The excitation of the two-phonon candidate states is found to be very sensitive to admixtures of small one-phonon components in the wave functions. Consistent conclusions can be drawn from both experimental probes when two-step contributions to the proton scattering cross sections are taken into account. The two-phonon FSS is found to be quite pure. The MSS has about 10% one-phonon admixture, but the dominant two-phonon character prevails.

Clearly, the combination of electromagnetic and hadronic scattering is a versatile tool for detailed studies of nuclear wave functions. This work opens up a new experimental avenue for future investigations of MSSs. One obvious application would be the study of  $^{92}\text{Zr}$ , where a description in terms of symmetric and mixed-symmetric states seems to fail [19,20].

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