THE ROLE OF "QUASIPARTICLE ⊗ PHONON" COMPONENTS IN GAMMA-DECAY OF HIGH-LYING STATES

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The γ decay rates of highly excited nuclear states of odd-mass nuclei into the low-lying one-quasiparticle states are calculated in a microscopic way. We conclude that not only the one-quasiparticle component but also "quasiparticle ⊗ phonon" components of the wave function of highly excited states can play an important role in their γ decay.

Recently, the first experiments on γ decay of deep-hole states in odd-mass nuclei have been performed [1,2]. In the (^{3}He, $\alpha\gamma$) reaction on even Pd and Sn isotopes, γ quanta and α particles have been detected in coincidence. By this method it is possible to examine the γ spectrum of decaying states in an energy interval fixed by the energy of outgoing α particles. The comparison of the energy distribution of direct γ_0 transitions to the ground state with the one going through low-lying states has allowed one to conclude that in this reaction not only the one-particle component of the wave function of deep-hole states could be investigated but collective ones as well [2]. From this point of view the contributions of one-particle and collective transitions to the matrix element of γ decay have to be examined. We will consider this problem here for the decay of the neutron deep-hole state $(1g_{9/2})^{-1}$ in ^{111}Sn.

The wave function of an odd-mass nucleus can be written in terms of the quasiparticle α^+_{jm} (with the shell quantum numbers $j = nlj$ and magnetic quantum number m) and phonon $Q^+_{\lambda\mu i}$ (λ, μ are the momentum of the phonon and its projection, i is the RPA-root number) operators

$$\Psi_\nu(JM) = C_{J\nu}\Big(\alpha^+_{jm} + \sum_{\lambda i j} D_j^{\lambda i}[\alpha^+_{jm}Q^+_{\lambda\mu i}]_{JM} + \sum F_{jI}^{\lambda_1 i_1 \lambda_2 i_2}(J\nu)\,[\alpha^+_{jm}[Q^+_{\lambda_1\mu_1 i_1}Q^+_{\lambda_2\mu_2 i_2}]_{I\mu}]_{JM}\Big)\Psi_0\,, \tag{1}$$

where Ψ_0 is the ground-state wave function of the neighbouring even–even nucleus. The phonon operator is defined as follows:

$$Q^+_{\lambda\mu i} = \frac{1}{2}\sum_{jj'} \psi^{\lambda i}_{jj'}[\alpha^+_{jm}\alpha^+_{j'm'}]_{\lambda\mu} - (-)^{\lambda-\mu}\varphi^{\lambda i}_{jj'}[\alpha_{j'm'}\alpha_{jm}]_{\lambda-\mu} \tag{2}$$

and describes both collective and noncollective excitations in an even–even nucleus. The amplitudes ψ and φ and one-phonon energies $\omega_{\lambda i}$ are calculated from the well-known RPA-equations for quasiparticles.

To calculate the coefficients C, D, F of the wave function (1), we apply the quasiparticle–phonon nuclear model (QPM) [3,4]. The model hamiltonian includes the phenomenological average potentials for protons and neutrons, pairing forces with constant matrix elements and separate interaction in the particle–hole channel written as multipole and spin-multipole expansions.

The coupling of the odd quasiparticle with the phonon excitation of the core is determined by the quasiparticle–phonon interaction term [5]

$$H_{\mathrm{qph}} = -\frac{1}{2\sqrt{2}} \sum_{\lambda\mu i} \sum_{j_1 j_2} \left(\frac{2j_1+1}{2\lambda+1}\right)^{1/2} \Gamma(j_1 j_2 \lambda i) \, [Q^+_{\lambda\mu i} + (-)^{\lambda-\mu} Q_{\lambda-\mu i}] B(j_1 j_2 \lambda - \mu) + \mathrm{h.c.} ,$$

$$B(j_1 j_2 \lambda - \mu) = \sum_{m_1 m_2} (-)^{j_2+m_2} \langle j_1 m_1 j_2 m_2 | \lambda - \mu \rangle \, \alpha^+_{j_1 m_1} \alpha_{j_2 - m_2} . \tag{3}$$

The value of $\Gamma(j_1 j_2 \lambda i)$ depends on the single-particle matrix elements of the residual forces, superfluid Bogolubov's coefficients and phonon characteristics and has no free parameters.

To get the spectrum of the eigenvalues $\eta_{J\nu}$ of the QPM hamiltonian for the basic vectors (1), we have to solve the following system of equations:

$$\epsilon_J - \eta_{J\nu} - \sum_{\lambda i j} \Gamma(J j \lambda i) D_j^{\lambda i}(J\nu) = 0 ,$$

$$D_j^{\lambda i}(J\nu) \left(\epsilon_j + \omega_{\lambda i} - \eta_{J\nu} - \sum_{j'\lambda' i'} \frac{\Gamma^2(jj'\lambda' i')}{\epsilon_{j'} + \omega_{\lambda i} + \omega_{\lambda' i'} - \eta_{J\nu}} \right)$$

$$- \frac{1}{2} \sum_{i'} D_j^{\lambda i'}(J\nu) \sum_{\lambda_1 i_1 \lambda_2 i_2} \frac{U^{\lambda_2 i_2}_{\lambda_1 i_1}(\lambda i) \, U^{\lambda_2 i_2}_{\lambda_1 i_1}(\lambda i')}{\epsilon_j + \omega_{\lambda_1 i_1} + \omega_{\lambda_2 i_2} - \eta_{J\nu}}$$

$$+ \sum_{j_1 \lambda_1 i_1} D_{j_1}^{\lambda_1 i_1}(J\nu) \Bigg[\sum_{\lambda_2 i_2} \Gamma(j j_1 \lambda_2 i_2) \hat{j} \begin{Bmatrix} \lambda_1 & \lambda_2 & \lambda \\ j & J & j_1 \end{Bmatrix} (-)^{\lambda+\lambda_2+j_1+J}$$

$$\times \left(\frac{\hat{\lambda}_1 U^{\lambda i}_{\lambda_2 i_2}(\lambda_1 i_1)}{\epsilon_{j_1} + \omega_{\lambda i} + \omega_{\lambda_2 i_2} - \eta_{J\nu}} + \frac{\hat{\lambda} U^{\lambda_2 i_2}_{\lambda_1 i_1}(\lambda i)}{\epsilon_j + \omega_{\lambda_1 i_1} + \omega_{\lambda_2 i_2} - \eta_{J\nu}} \right)$$

$$- \sum_{j_2} (-)^{\lambda+\lambda_1+j+j_1} \begin{Bmatrix} \lambda_1 & j_2 & j \\ \lambda & J & j_1 \end{Bmatrix} \frac{\Gamma(j j_2 \lambda i) \Gamma(j j_2 \lambda_1 i_1) \hat{j}_1 \hat{j}}{\epsilon_{j_2} + \omega_{\lambda i} + \omega_{\lambda_1 i_1} - \eta_{J\nu}} \Bigg] = \Gamma(J j \lambda i) ,$$

$$\Gamma(j_1 j_2 \lambda i) \equiv \langle \alpha_{j_1 m_1} \| H_{\mathrm{qph}} \| [\alpha^+_{j_2 m_2} Q^+_{\lambda\mu i}]_{j_1 m_1} \rangle ,$$

$$U^{\lambda_1 i_1}_{\lambda_2 i_2}(\lambda i) \equiv \langle Q_{\lambda\mu i} \| H_{\mathrm{qph}} \| [Q^+_{\lambda_1 \mu_1 i_1} Q^+_{\lambda_2 \mu_2 i_2}]_{\lambda\mu} \rangle , \quad \hat{\lambda} = (2\lambda+1)^{1/2} . \tag{4}$$

Here ϵ_j is the energy of the one-quasiparticle state. As one can see from (4) the $\Gamma(j_1 j_2 \lambda i)$ are the coupling strengths of the quasiparticle $\alpha^+_{j_1 m_1}$ and "quasiparticle ⊗ phonon" $[\alpha^+_{j_2 m_2} Q^+_{\lambda\mu i}]_{j_1 m_1}$ states. The interaction matrix elements $U^{\lambda_1 i_1}_{\lambda_2 i_2}(\lambda i)$ describe the so-called anharmonic corrections, i.e., they describe the interaction between the phonons of the core.

The coefficients $D_j^{\lambda i}$ of the wave function (1) are obtained from the system (4); the coefficients F are expressed as linear combinations of D's; the one-quasiparticle amplitude is determined by the norm of the wave function (1). The application of the QPM-formalism to the odd nuclei could be found in more detail in refs. [6–8].

In spite of the simplifications due to the separable form of the effective forces and phonon representation of the core excitations, to solve the system (4) is a hard problem since in realistic calculations we have to diagonalize a high dimension matrix. To overcome this difficulty we use the strength function method [9]. The extension of the method to the calculation of the coefficients $D_j^{\lambda i}$ and electromagnetic transition rates has been worked out in refs. [7,8]. It should be noted that in calculations of ref. [8] the terms $\sim\Gamma U$ of eqs. (4) were omitted.

The standard one-body operator of the γ transition in nuclei after simple transformations can be written in terms of the quasiparticle and phonon operators in the following way:

$$\mathcal{M}(E,M\lambda) = \hat{\lambda}^{-1} \sum_{j_1 j_2} \langle j_1 \| \mathcal{M}(E,M\lambda) \| j_2 \rangle \Big((u_{j_1} u_{j_2} \mp v_{j_1} v_{j_2}) B(j_1 j_2 \lambda\mu)$$

$$+ \tfrac{1}{2} (u_{j_1} v_{j_2} \pm u_{j_2} v_{j_1}) \sum_i (\psi_{j_1 j_2}^{\lambda i} \pm \varphi_{j_1 j_2}^{\lambda i}) [Q_{\lambda\mu i}^{+} - (-)^{\lambda-\mu} Q_{\lambda-\mu i}] \Big) . \qquad (5)$$

The first term in (5) causes γ transitions between the components of initial (i) and final (f) states of the same complexity, i.e., $|\alpha^+\rangle_i \Rightarrow |\alpha^+\rangle_f$, $|\alpha^+ Q^+\rangle_i \Rightarrow |\alpha^+ Q^+\rangle_f$, etc. In principle, due to this term the transitions with the exchange of two phonons (e.g., $|\alpha^+ Q^+ Q^+\rangle_i \Rightarrow |\alpha^+\rangle_f$) are allowed, as follows from the boson expansion technique [10], but they will be strongly suppressed by a factor $\sim\psi\varphi$ as compared to the transitions $|\alpha^+\rangle_i \Rightarrow |\alpha^+\rangle_f$. In the Tamm–Dancoff approximation the transitions $|\alpha^+ Q^+ Q^+\rangle_i \Rightarrow |\alpha^+\rangle_f$ are forbidden ($\varphi \equiv 0$). The second part of the operator (5) causes transitions with the exchange of one phonon, i.e., $|\alpha^+ Q^+\rangle_i \Rightarrow |\alpha^+\rangle_f$. In the transitions of this type the quasiparticle quantum numbers do not change. Which of the two terms in (5) is dominant depends on the matrix element of each partial transition and on the corresponding coefficients C, D, F of the i and f states.

We consider here the γ decay of $9/2^+$ states that are excited in the one-neutron pick-up reaction ^{112}Sn(^{3}He,α)^{111}Sn into the ground state $7/2_1^+$ and the low-lying states $11/2_1^-$ and $5/2_1^+$ of the ^{111}Sn nucleus. These final states have practically the one-quasiparticle structure. We get for them $C_f^2 \geqslant 0.9$ in agreement with experimental data. So, we suggest that D_f and F_f coefficients vanish. The structure of the high-lying $9/2^+$ states is much more complicated and we describe it by the wave function (1). We are especially interested in excitations within the energy range $E^* = 3$–6 MeV where the main part of $1g_{9/2}$-neutron hole strength is concentrated. The $9/2^+$ states in this range have rather discernible values of $C^2(1g_{9/2})$, and, moreover, they have large admixtures of more complex components. The crucial role is played by the competition between partial transitions $|\alpha^+\rangle_i \Rightarrow |\alpha^+\rangle_f$ and $|\alpha^+ Q^+\rangle_i \Rightarrow |\alpha^+\rangle_f$. In that case the components $|\alpha^+ Q^+ Q^+\rangle_i$ are important as they are coupled with $|\alpha^+ Q^+\rangle_i$ configurations and influence their fragmentation. So, without "quasiparticle $\otimes$ two phonons" components it is impossible to obtain a correct energy distribution of the $|\alpha^+ Q^+\rangle_i \Rightarrow |\alpha^+\rangle_f$ part of the transition strength.

Let us now turn to the results of the calculations. The excitation of the deep-hole state in the (^{3}He, α) reaction goes mainly through the one-quasiparticle component of the wave function. So, firstly, we need to calculate the energy distribution of the $1g_{9/2}$-neutron hole strength. These calculations had long been done in ref. [11]. Our present results slightly differ from those of ref. [11] due to minor changes in the truncation of the basis of complex configurations and to the influence of the anharmonic terms in eqs. (4) [i.e., the terms with $U_{\lambda_2 i_2}^{\lambda_1 i_1}(\lambda i)$ coefficients] that had not been taken into account in ref. [11]. The calculated energy distribution $C_{9/2}^2(E^*)$ bears a qualitative resemblance with the energy distribution of the intensity of direct γ_0 transitions from the resonance-like deep-hole bump to the ground state [1,2]. But our goal is to show that no conclusions on the decay mechanism can be drawn from this resemblance because although the excitation of deep-hole states goes through the one-quasiparticle component of the wave function, the decay of the excited state may go through the "quasiparticle $\otimes$ phonon" component.

To analyse the decay mechanism of $9/2^+$ states in ^{111}Sn, we calculate the absolute intensities $T(E,M\lambda)$ of γ transitions. The results are presented in fig. 1. The contribution of the one-quasiparticle part of the transition rate ($|\alpha^+\rangle_i \Rightarrow |\alpha^+\rangle_f$) for the M1 transition $9/2^+ \to 7/2_{g.s.}^+$ and the E1 transition $9/2^+ \to 11/2_1^-$ is shown by the dashed curve. The shape of the curve coincides with the function $C_{9/2}^2(E^*) \times (E^*)^{2\lambda+1}$ (λ is the transition multipolarity). The E^* dependence comes from the expression for $T(E,M\lambda)$:

$$T(E,M\lambda)=\frac{8\pi(\lambda+1)}{\lambda[(2\lambda+1)!!]^2}\frac{1}{\hbar}\left(\frac{E^*}{\hbar c}\right)^{2\lambda+1}B(E,M\lambda)\,.$$

The solid curve shows the calculations where both partial transitions ($|\alpha^+\rangle_i \Rightarrow |\alpha^+\rangle_f$ and $|\alpha^+Q^+\rangle_i \Rightarrow |\alpha^+\rangle_f$) are taken into account. As in ref. [8], the results are presented in the form of the strength functions with the averaging parameter $\Delta = 0.5$ MeV. We also display in fig. 1 the intensity of the E2 transitions $9/2^+ \to 7/2^+_{g.s.}$ that go exclusively via the $|\alpha^+Q^+\rangle_i \Rightarrow |\alpha^+\rangle_f$ way. For any reasonable value of the neutron effective charge the transition $|\alpha^+\rangle_i \Rightarrow |\alpha^+\rangle_f$ is more than two orders of magnitude less than the $|\alpha^+Q^+\rangle_i \Rightarrow |\alpha^+\rangle_f$ transition. We get the same picture for E2 transitions $9/2^+ \to 5/2^+_1$.

The direct γ transitions to the low-lying states from the energy range $E^* > 6$ MeV go predominantly through the "quasiparticle $\otimes$ phonon" components. This is quite natural because the contribution of the one-quasiparticle component to the wave functions of the $9/2^+$ states at these energies is small, as follows from our calculation.

The most interesting range to discuss is $E^* < 6$ MeV. The intensity of the M1 transitions from this range is more than one-half of the E1 transition intensity (note, we use effective spin-gyromagnetic factors $g_s^{\text{eff}} = 0.8 g_s^{\text{free}}$), and the intensity of the E2 transitions is much lower. The interesting feature is that the character of E1 and M1 transitions is different. An admixture of collective components $|1h_{11/2} \otimes Q^+(1^-)\rangle_i$ in the wave functions of the $9/2^+$ states at $E^* < 6$ MeV is extremely small; that is why the E1 transitions $9/2^+ \to 11/2^-_1$ are determined by the one-quasiparticle part $|\alpha^+\rangle_i \Rightarrow |\alpha^+\rangle_f$.

The discrepancy between the solid and dashed curves in fig. 1c at $E^* < 6$ MeV is of artificial origin: since we use the strength function averaging, it is due to an admixture of the slowly sloping Lorentz's tail of $|1h_{1/2} \otimes Q^+(1^-)\rangle_i$ states with large $T(\text{E1})$ values from the range $E^* > 6$ MeV.

As for the M1 transitions from the range $E^* < 6$ MeV, both components, i.e., $|\alpha^+Q^+\rangle_i \Rightarrow |\alpha^+\rangle_f$ and $|\alpha^+\rangle_i \Rightarrow |\alpha^+\rangle_f$, give approximately the same contribution to it. This is vividly seen from fig. 2, in which the strength function of the *reduced* transition probability $B(\text{M1}, 9/2^+ \to 7/2^+_{g.s.})$ is shown. This calculation is performed with the averaging parameter $\Delta = 0.1$ MeV. The strong peak at $E^* = 3.75$ MeV will be suppressed in the cross section of

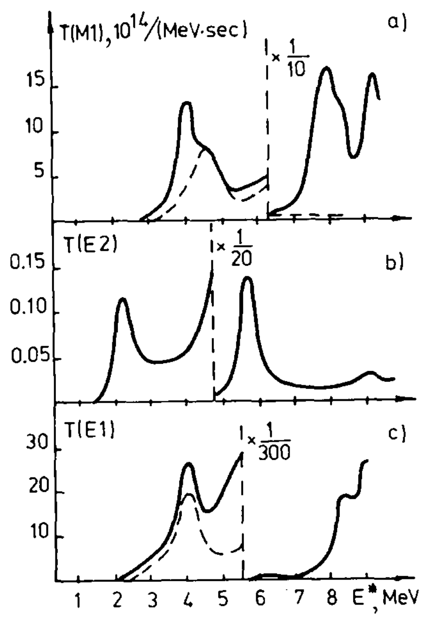

Fig. 1. Strength functions of the γ transition rates $T(E,M\lambda)$. The one-quasiparticle parts of the transition rate are displayed by the dashed curves, the solid curves are the results of the complete calculations. (a) M1 transitions $9/2^+ \to 7/2^+_{g.s.}$; (b) E2 transitions $9/2^+ \to 7/2^+_{g.s.}$; (c) E1 transitions $9/2^+ \to 11/2^-_1$.

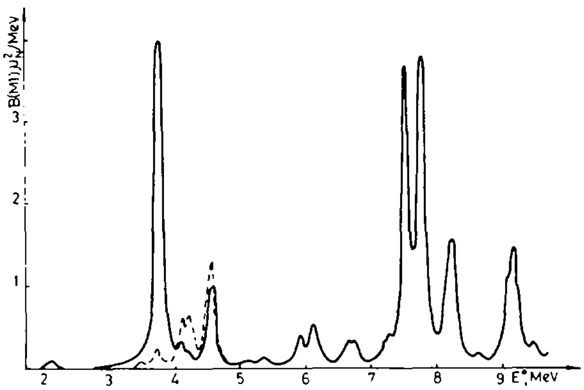

Fig. 2. Strength function of the reduced M1-transition probability $B(\mathrm{M1}, 9/2^+ \to 7/2^+_{\mathrm{g.s.}})$. For notation see fig. 1.

the (^{3}He, $\alpha\gamma$) reaction because of the small contribution of the one-quasiparticle component in the wave functions of $9/2^+$ states. This peak is due to the collective component of their wave functions, that is of the type $|1g_{7/2} \otimes Q^+(1^+)\rangle$. The strong interference of both comppnents $|\alpha^+\rangle_i \Rightarrow |\alpha^+\rangle_f$ and $|\alpha^+Q^+\rangle_i \Rightarrow |\alpha^+\rangle_f$ of the M1 transitions takes place in the energy interval $4 < E^* < 5$ MeV. The interference has a destructive character and changes essentially the result obtained only with the $|\alpha^+\rangle_i \Rightarrow |\alpha^+\rangle_f$ component. That is why we want to point out that the partial transitions of the $|\alpha^+Q^+\rangle_i \Rightarrow |\alpha^+\rangle_f$-type cannot be neglected in considering the γ decay of high-lying single-particle (or hole) states especially if in the range of the one-quasiparticle strength concentration the "quasiparticle $\otimes$ phonon" states with the same J^π are placed.

Since we do not calculate the cross section of the (^{3}He, $\alpha\gamma$) reaction but only the decay properties of the excited states, we substitute the direct comparison with the experimental data of refs. [1,2] by a qualitative one. We assume that in this one-nucleon transfer reaction the excitation goes exclusively through the one-quasiparticle component of the wave function (1), thus each $(9/2)^+$ state is excited in accordance with its C_ν^2 value. So, to get the model cross section we have to multiply the $C_{9/2}^2(E^*)$ distribution by the $T(E^*)$ distribution. To compare our results (fig. 3c) with the experimental data of refs. [1,2] (figs. 3a, 3b), we normalize all solid histograms to obtain the integral cross section equal to 100 arbitrary units. The histogram corresponding only to $|\alpha^+\rangle_i \Rightarrow |\alpha^+\rangle_f$ decay transitions (dashed line) has a lack of strength below $E^* = 4$ MeV. Our solid histogram (fig. 3c) has a less pro-

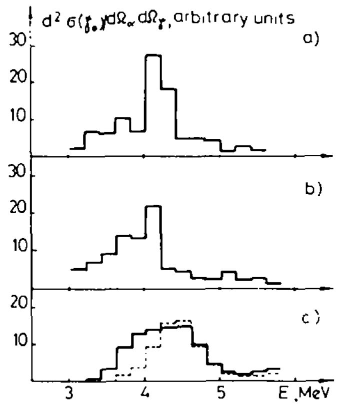

Fig. 3. The cross section of the ^{112}Sn(^{3}He, $\alpha\gamma_0$)^{111}Sn reaction for the γ_0 decay to the ground state in arbitrary units: (a) from ref. [1]; (b) from ref. [2]; (c) our calculations. For details see text.

nounced peak than the experimental ones, but this is partly caused by the artificial reason as we use the strength function averaging with $\Delta = 0.5$ MeV. In spite of the strong γ transitions predicted in fig. 1 at $E^* > 5$ MeV the cross section of the reaction rapidly decreases at those excitation energies. The reason is the decreasing of the $C^2(1g_{9/2})$ value and the vanishing of the excitation probability of initial states. It should be noted that on the whole we obtain a good qualitative agreement with experiment.

A very sensitive characteristic of the admixture of collective components is the E2/M1 mixing ratio δ. If no collective transitions occur, it will be close to zero since for the neutron $|\alpha^+\rangle_i \Rightarrow |\alpha^+\rangle_f$ transition $e^{(2)}_{\text{eff}} \simeq 0$. In ref. [1] it has been estimated to be $\delta = -0.16^{+0.28}_{-0.35}$ for the region $E^* = 4.0$–4.5 MeV. Our calculations give the result $\delta = -0.03$ for this region and $\delta = -0.08$ for a wider region $E^* = 3.0$–6.0 MeV. Surely, with such large experimental uncertainties it is hard to judge about the role of a collective mechanism.

In summary, in the framework of the microscopic approach we have presented for the first time the calculations of the γ-decay rates of the neutron $1g_{9/2}$ deep-hole resonance-like structure in ^{111}Sn to the ground state and low-lying one-quasiparticle states. We have found the almost pure one-quasiparticle character for the E1 transitions $9/2^+ \to 11/2^-_1$ and strong destructive interference between one-quasiparticle and collective "quasiparticle $\otimes$ phonon" terms for the M1 transitions $9/2^+ \to 7/2_{\text{g.s.}}$. So, in some cases the collective "quasiparticle $\otimes$ phonon" components of the wave function may play a crucial role in γ decay of high-lying single-hole and/or single-particle structures. The schematic calculation of the $^{112}\text{Sn}(^3\text{He},\alpha\gamma_0)^{111}\text{Sn}$ cross section gives a qualitative agreement with the experimental data [1,2]. Our QPM formalism can be applied to the study of γ decay in a large number of medium and heavy odd-mass spherical nuclei.

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