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On double resonances in spherical nuclei

V.Yu. Ponomarev [a,b] and V.V. Voronov [b]

[a] *Faculteit Natuurkunde en Sterrenkunde, Vrije Universiteit, De Boelelaan 1081, NL-1081 HV Amsterdam, The Netherlands*
[b] *Laboratory of Theoretical Physics, Joint Institute for Nuclear Research, Dubna, Head Post Office, P.O. Box 79, Moscow, Russia*

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Double giant resonances in spherical nuclei formed by the low-lying collective one-phonon state and giant resonance as well as of two giant resonances are analyzed within the quasiparticle phonon model. The position, photoexcitation probability and the width of these resonances of low multipolarity are calculated in ^{208}Pb.

Up to the now one of the most interesting problems concerning giant resonances is the existence of the multiphonon giant resonance states. The idea of the multiphonon giant resonance states is simple and has been proposed many years ago (see for example refs. [1,2]). The prospects for the direct observation of two-phonon states in which at least one of the phonons is the giant dipole resonance (GDR) phonon are discussed in ref. [3].

We present here the results of our calculations of direct photoexcitation of the two-phonon giant resonance states from the ground state in ^{208}Pb. In heavy ion experiments at high bombarding energy, which are the most suitable tools now to investigate properties of double resonances, the Coulomb excitation plays the major role [4,5] and the photo cross sections can be recalculated into heavy ion reaction cross sections. The same approach can be applied to other spherical nuclei with weak interaction between one- and two-phonon configurations. Calculations have been performed within the quasiparticle phonon model (QPM). Excited states in even–even nuclei are considered in the QPM as a combination of one-, two-, ... phonon states built on the wave function of the ground state $\Psi_{\text{g.s.}}$ treated as phonon vacuum. In the present paper we truncate out the basis up to two-phonon terms and thus the wave function of the νth state with momentum λ and projection μ has the form

$$\Psi_\nu(\lambda\mu)=\Big\{\sum_i R_i(\lambda\nu)Q^+_{\lambda\mu i} + \sum_{\lambda_1 i_1 \lambda_2 i_2} P^{\lambda_2 i_2}_{\lambda_1 i_1}(\lambda\nu)[Q^+_{\lambda_1\mu_1 i_1}Q^+_{\lambda_2\mu_2 i_2}]_{\lambda\mu}\Big\}\Psi_{\text{g.s.}}\,, \qquad (1)$$

where $Q^+_{\lambda\mu i}$ is the phonon creation operator with the momentum λ, projection μ and the RPA root number i. Phonons are constructed as a linear combination of pairs of quasiparticle creation α^+_{jm} and annihilation α_{jm} operators with the shell quantum numbers $jm\equiv|n,l,j,m\rangle$ as follows:

$$Q^+_{\lambda\mu i}=\tfrac{1}{2}\sum_{jj'}^{N,Z}\{\psi^{\lambda i}_{jj'}[\alpha^+_{jm}\alpha^+_{j'm'}]_{\lambda\mu} + (-1)^{\lambda-\mu}\phi^{\lambda i}_{jj'}[\alpha_{jm}\alpha_{j'm'}]_{\lambda-\mu}\}\,. \qquad (2)$$

To obtain the phonon basis [i.e., the energy of the one-phonon excitation and the structure coefficients $\psi^{\lambda i}_{jj'}$ and $\phi^{\lambda i}_{jj'}$ in eq. (2)], we solve the RPA equations with the effective hamiltonian, which includes the average field for neutrons and protons, pairing interaction and residual interaction in separable form with the Bohr–Mottelson radial dependence, for each λ^π. The coefficients $R_i(\lambda\nu)$ and $P^{\lambda_2 i_2}_{\lambda_1 i_1}(\lambda\nu)$ of the wave function of excited states (1) and the excitation energies of these states are obtained after diagonalization of our effective hamiltonian on the basis of these wave functions. The detailed description of the QPM formalism can be found in refs. [6–8]. As soon as the one-phonon basis is obtained, no free parameters are involved in calculations.

In ref. [9] we considered in detail different properties of one-phonon giant resonances and low lying states in ^{208}Pb within the same approach. It has been shown that the agreement with the available experimental information [3] and other theoretical calculations [10,11], even for γ decay of GQR to the low lying states which is rather sensitive to the details of calculations, is very good. Having proved this way the quality of our phonon basis, we can now proceed with two-phonon states and consider the properties of double-phonon resonances.

The reduced probability of photoexcitation of states (1) from the ground state has the form

$$
\begin{aligned}
B(\mathrm{E}\lambda; 0^+_{\mathrm{g.s.}} \to \lambda^\pi_\nu) = \Bigg| & \sum_{j_1 j_2 \tau} e^{(\lambda)}_\tau f^\lambda_{j_1 j_2}(\tau) \\
& \times \Bigg(\sum_i R_i(\lambda\nu) \cdot \tfrac{1}{2} u^{(+)}_{j_1 j_2} (\psi^{\lambda i}_{j_1 j_2} + \phi^{\lambda i}_{j_1 j_2}) \\
& + \sum_{\lambda_1 i_1 \lambda_2 i_2 j_3} P^{\lambda_2 i_2}_{\lambda_1 i_1}(\lambda\nu) \sqrt{(2\lambda_1+1)(2\lambda_2+1)}\, v^{(-)}_{j_1 j_2} \\
& \times \begin{Bmatrix} \lambda_2 & \lambda_1 & \lambda \\ j_1 & j_2 & j_3 \end{Bmatrix} (\psi^{\lambda_2 i_2}_{j_2 j_3} \phi^{\lambda_1 i_1}_{j_3 j_1} + \psi^{\lambda_1 i_1}_{j_3 j_1} \phi^{\lambda_2 i_2}_{j_2 j_3}) \Bigg) \Bigg|^2 ,
\end{aligned} \qquad (3)
$$

where $e^{(\lambda)}_\tau$ is the effective charge; $f^\lambda_{j_1 j_2}(\tau)$ is the two-quasiparticle matrix element of the $B(\mathrm{E}\lambda)$ transitions; $u^{(+)}_{j_1 j_2}$ and $v^{(-)}_{j_1 j_2}$ are combinations of Bogolubov transformation coefficients. The second term is responsible for the photoexcitation of double-phonon states from the ground state. One of the most simple ways to calculate this matrix element is to use the Marumori expansion [12] of one-body fermion electromagnetic operator $a^+_{j_1 m_1} a_{j_2 m_2}$ in the infinite sum of boson phonon operators.

According to our calculations in ^{208}Pb the mixing between one- and two-phonon giant resonances is very weak and such resonances have very different excitation energies. Though single giant resonances and giant resonances built on the low-lying states are separated only by the energy of the low-lying state the coupling between them is also very weak in ^{208}Pb. Practically one can do calculations for the two-phonon resonances without taking into account this coupling for this double closed nucleus.

In this paper we take into account only terms which are proportional to the $\psi\phi$. The complete set of diagrams which have to be included in the calculations for the excitation of two-phonon states is presented in ref. [13]. As one can see from the analytical expressions (see ref. [13]) the main part of the contributions from different terms disappears due to the cancellation between particles and holes. In future we are going to estimate the contributions of all possible diagrams.

Another subject to be investigated in detail is the influence of the polarization contributions to the effective charge (especially for E1-transitions [14]) since in the present calculation we have used the "free" values as $e_Z = N/A$, $e_N = -Z/A$ for $\lambda^\pi = 1^-$ and $e_Z = 1$, $e_N = 0$ for other λ^π. [For the $B(\mathrm{M}\lambda)$ transitions we use effective gyromagnetic factors $g^{\mathrm{eff}}_s = 0.8 g^{\mathrm{free}}_s$ and $g^{\mathrm{eff}}_l = g^{\mathrm{free}}_l$ for both protons and neutrons.] Some changes of the effective charges may lead to the renormalization of the excitation absolute values within a range of less than a factor of two.

One can hope that for the nuclei with weak coupling like ^{208}Pb the strength distribution for the double resonances will not be changed essentially due to the effects mentioned above. That is why, even with these shortcomings, which will be removed in future, we think that the results of this calculation would be important for the estimation of experimental cross sections for future possible experiments in this subject.

We present in fig. 1 only two figures of direct photoexcitation of double-phonon states in ^{208}Pb: the E2 excitation of two-phonon states $[1^- \times 1^-]_{2^+}$ (top) and the E1 excitations of $[1^- \times 2^+]_{1^-}$ states (bottom). The main feature of the top figure is that just all two-phonon states which form this double-phonon resonance are constructed from the one-phonon 1^- states belonging to the GDR in the RPA picture. The structure of the $[1^- \times 2^+]_{1^-}$ strength distribution is much more complicated. The substructure in the energy range from 15 to 20 MeV is formed mainly by 1^- phonons from the GDR region coupled to the 2^+_1 state. The small substructure above 32 MeV is formed from the GDR 1^- phonons coupled to the 2^+ phonons which form the isovector GQR. And in the broad structure from 20 to 30 MeV not only two-phonon states made of the GDR 1^- and isoscalar GQR 2^+ phonons but many other collective two-phonon configurations play an essential role. Thus, if

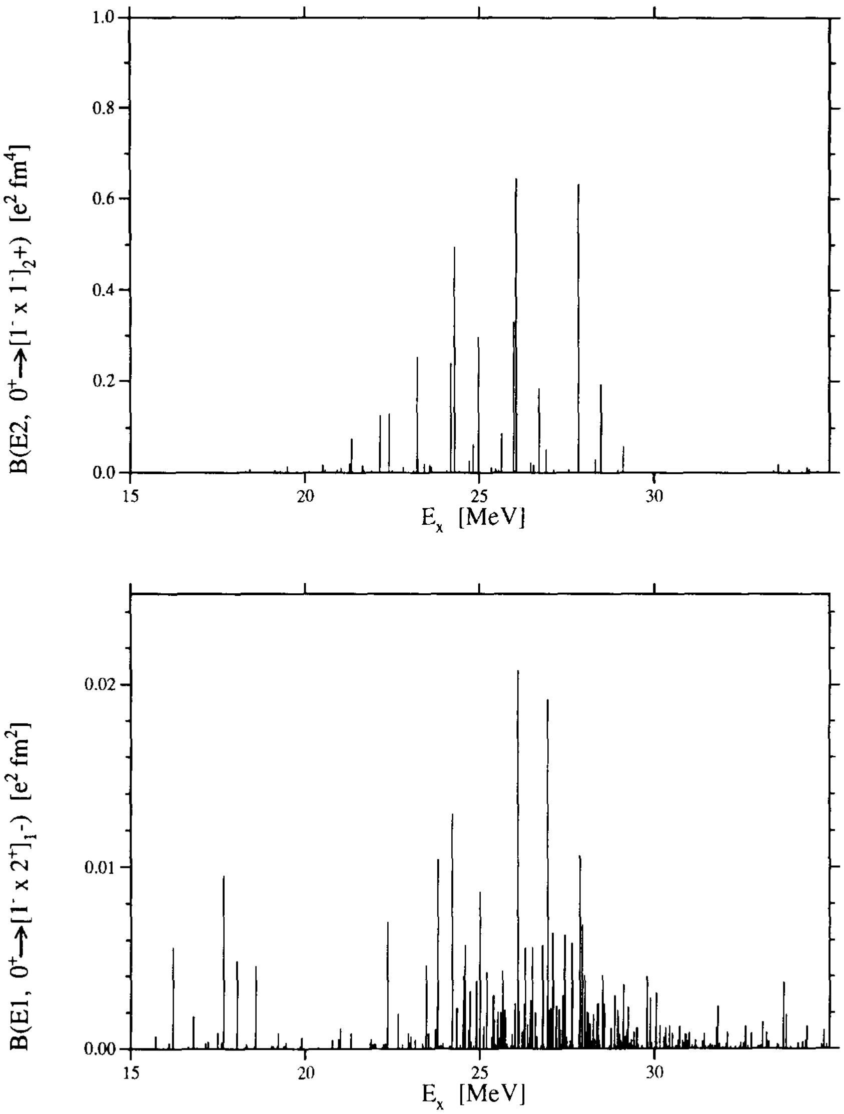

Fig. 1. E2 strength distribution over $[1^- \times 1^-]_{2^+}$ configurations (top) and E1 strength distribution over $[1^- \times 2^+]_{1^-}$ configurations (bottom) in ^{208}Pb.

some measurements in coincidence with two γ decays are performed the shape of the resonance in this region will vary strongly depending on what γ energies are measured. Also, the reduction of the high energy part of resonances due to the exponential decline of Coulomb excitation has to be taken into account in the calculation of heavy ion cross sections.

In spite of a rather strong fragmentation of the single dipole resonance strength the calculated strength distributions demonstrate a pronounced resonance-like behavior for excitation energies about 17 MeV and 25 MeV. Keeping in mind that only strong transitions from double resonance to intermediate states and from the intermediate states to the ground state can be detected in coincidence measurements, we have calculated the properties of double-phonon resonances made of one-phonon configurations of either the first low-lying state or from the one-phonon res-

Table 1
The integral characteristics of the two-phonon resonances in ^{208}Pb.

Configuration	Centroid (MeV)	Width (MeV)	$\Sigma B(\mathrm{E(M)}\lambda\uparrow)$	$\int\sigma(E)\,\mathrm{d}E$ (MeV mb)
$[1^-_{\mathrm{GDR_{iv}}}\times 2^+_1]_{1^-}$	17.4	2.2	$2.4\times10^{-2}\,e^2\,\mathrm{fm}^2$	1.7
$[1^-_{\mathrm{GDR_{iv}}}\times 2^+_1]_{2^-}$	17.2	2.1	$5.0\,\mu_0^2\,\mathrm{fm}^2$	8.7×10^{-4}
$[1^-_{\mathrm{GDR_{iv}}}\times 2^+_1]_{3^-}$	17.7	3.4	$19.8\,e^2\,\mathrm{fm}^6$	4.9×10^{-5}
$[1^-_{\mathrm{GDR_{iv}}}\times 3^-_1]_{2^+}$	15.3	3.9	$4.7\,e^2\,\mathrm{fm}^4$	5.2×10^{-2}
$[1^-_{\mathrm{GDR_{iv}}}\times 2^+_{\mathrm{GQR_{is}}}]_{1^-}$	25.1	3.8	$9.4\times10^{-2}\,e^2\,\mathrm{fm}^2$	9.6
$[1^-_{\mathrm{GDR_{iv}}}\times 1^-_{\mathrm{GDR_{iv}}}]_{2^+}$	25.5	4.4	$4.2\,e^2\,\mathrm{fm}^4$	0.22

onance region. Some of these double-phonon resonances in ^{208}Pb are presented in table 1. Calculating the resonance widths we used the standard relation between the width and dispersion for a gaussian distribution. The widths of double resonances presented in table 1 are only the lower limit of the real width because we did not take into account the coupling of two-phonon states with more complex configurations. Another problem which has to be mentioned in such calculations is related with the Pauli principle. Constructing the double resonance from the RPA phonons we violate the Pauli principle. To check this effect we calculated the Pauli principle corrections for the two-phonon giant resonance state using the exact fermion commutation relations [7,8]. For the double giant resonances under consideration such corrections contribute not more than (2–3)% and can be neglected.

The last column of table 1 presents the integrated photo absorption cross sections calculated using the well known relation between cross sections and transition probabilities [14]. As one can see from this table the double resonances with $J^\pi = 1^-, 2^+$ have the largest excitation cross sections and we expect that these resonances would be easier to discover.

From the present study one can conclude that the strength of the double resonances including the GDR in ^{208}Pb is concentrated within definite energy intervals and can be investigated in the coincidence experiments. It is possible to explore such calculations for the estimation of the branch ratios for the future γ-coincidence measurements.

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