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Nuclear vorticity and the low-energy nuclear response: towards the neutron drip line ☆

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Abstract

The transition density and current provide valuable insight into the nature of nuclear vibrations. Nuclear vorticity is a quantity related to the transverse transition current. In this work, we study the evolution of the strength distribution, related to density fluctuations, and the vorticity strength distribution, as the neutron drip line is approached. Our results on the isoscalar, natural-parity multipole response of Ni isotopes, obtained by using a self-consistent Skyrme–Hartree–Fock + Continuum RPA model, indicate that, close to the drip line, the low-energy response is dominated by $L > 1$ vortical transitions.

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1. Introduction

One of the aims of the ongoing theoretical and experimental studies devoted to nuclei far from stability is to illuminate the effects of the excess protons or neutrons on the nuclear structure and response. On the neutron-rich side of the nuclear chart, in particular, a wealth of exotic phenomena is anticipated, due to the formation of neutron skins and halos, the large difference between the Fermi energies of protons and neutrons and a possible change of shell structure.

Electron scattering experiments with radioactive beams are currently being discussed, opening the possibility of detecting, besides the transition charge distributions, also transition current distributions in a broad region of the nuclear chart. The transition current density (TC), studied in conjunction with the tran-

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sition density (TD), provides valuable insight into the nature of nuclear vibrations, in particular, their "zero-sound" nature. Theoretically, the TD and TC associated with a particular type of electric excitation, approach ideal hydrodynamical behaviour in the case of very collective states such as giant resonances (GRs). It is predicted [1] that the properties of GRs in drip-line nuclei are close to the ones of β-stable nuclei in this sense. At lower energies, though, the behaviour of the TD and TC may deviate significantly from this picture, as several examples demonstrate. Microscopic studies of convection currents in stable nuclei predict that a variety of flow patterns is possible [2–4]; for instance, the isoscalar quadrupole GR of ^{208}Pb is nearly irrotational, whereas the corresponding low-lying state exhibits considerable vorticity. In (stable and unstable) neutron-rich nuclei, "pygmy" [5,6] or "soft" [7–9] dipole modes are observed, interpreted as oscillations of the excess neutrons against the isospin-saturated core; also, toroidal dipole modes are predicted to exist at a somewhat higher energy [6,10]. A special feature of the response of very neutron-rich nuclei is the so-called threshold strength, i.e., the considerable amount of strength predicted to occur just above the neutron threshold; it is of isoscalar and non-collective nature and characterized by spatially extended TDs and TCs, owing to the loosely bound neutrons [1,11–14].

The structure of the particle continuum and the nature of low-lying strength in unstable nuclei are of particular interest, as they provide information on decay properties, polarizabilities, shell structure, etc. Close to the drip line, collective "low-lying" states such as 2^+ may be located in the continuum. The energy of 2^+ states as well as other $0\hbar\omega$ modes, may be very low even for nuclei described traditionally as closed-subshell configurations, if the shell structure tends to "melt" at the extremes of isospin [11]. Such transitions can hardly be expected to be irrotational.

A standard procedure in studies of nuclear convection currents is to define a velocity field, as dictated by hydrodynamical considerations. In Ref. [4], it was shown that the degree to which a transition is irrotational, can be quantified in terms of the nuclear vorticity density. The latter is directly related to the transverse component of the TC and therefore, together with the TD, it completely specifies a transition and unambiguously reflects the zero-sound character of nuclear vibrations.

In this work, we examine the response of the isotopes ^{56,78,110}Ni, ranging from $N = Z$ and close-to-stable ($N = 28$) to extremely neutron-rich ($N = 82$). We focus on the flow properties of the various transitions. The isoscalar strength distributions $S_\rho(E)$, related to density fluctuations, and respective vorticity strength distributions $S_\omega(E)$ of multipolarity up to $L = 4$ are calculated, using a self-consistent Skyrme–Hartree–Fock plus continuum-RPA method. We consider the vorticity related to the convection current. The evolution of the distributions as the neutron number increases is discussed.

2. Definitions and method of calculation

Let us consider the response of spherical, closed-(sub)shell nuclei to an isoscalar (IS) external field $V_{LM}(\vec{r}) = r^L Y_{LM}(\theta, \phi)$ $(L > 0)$. The IS TD $\delta\rho(\vec{r})$ and TC $\vec{j}(\vec{r})$ characterizing the excited natural-parity state $|LM\rangle$ of energy E, are determined by their radial components, $\delta\rho_L(r)$ and $j_{LL\pm1}(r)$, respectively [2,4]. From $\vec{j}(\vec{r})$ and the ground-state density $\rho(r)$, a velocity field can be defined, $\vec{u}(\vec{r}) = \vec{j}(\vec{r})/\rho(r)$. Under the assumption that a single transition exhausts the available strength of an external potential $V(\vec{r})$, it can be shown that $\vec{u}(\vec{r}) \propto \vec{\nabla} V(\vec{r})$ [15], which implies irrotational flow, $\vec{\nabla} \times \vec{u} = 0$. In classical hydrodynamics, the flow is always irrotational in ideal, non-viscous fluids.

The IS TD and TC are related through the continuity equation (CE) [2], which we write here for the radial components:

$$\frac{E}{\hbar c}\delta\rho_L(r) = \sqrt{\frac{L}{2L+1}}\left(\frac{\mathrm{d}}{\mathrm{d}r} - \frac{L-1}{r}\right) j_{LL-1}(r) - \sqrt{\frac{L+1}{2L+1}}\left(\frac{\mathrm{d}}{\mathrm{d}r} + \frac{L+2}{r}\right) j_{LL+1}(r). \tag{1}$$

The transverse part of the current, $\vec{j}^{\mathrm{tr}}$, with radial components $j^{\mathrm{tr}}_{LL\pm1}(r)$, does not contribute to the CE since, by definition, $\vec{\nabla} \cdot \vec{j}^{\mathrm{tr}} = 0$. This equation does not define $\vec{j}^{\mathrm{tr}}$ uniquely; following Ref. [16], one may set $j^{\mathrm{tr}}_{LL+1}(r) = j_{LL+1}(r)$. Then $j^{\mathrm{tr}}_{LL-1}(r)$ is determined from $\vec{\nabla} \cdot \vec{j}^{\mathrm{tr}} = 0$. The remaining part of the current is related to $\delta\rho_L$ through the CE. Therefore, measurements of $\delta\rho_L$ and $j_{LL+1}(r)$, as demonstrated, for example, in the analyses of inelastic electron scattering data in Ref. [16], determine the transition completely.

The vorticity density $\vec{\omega}(\vec{r})$ associated with a transition from the spherically symmetric ground state to the excited state $|LM\rangle$, as defined in Ref. [4], equals the curl of the transverse current introduced above [17],

$$\vec{\omega}(\vec{r}) = \vec{\nabla} \times \vec{j}^{\mathrm{tr}}(\vec{r}) \equiv (2L+1)^{-1/2}\omega_{LL}(r)\vec{Y}^{M}_{LL}(\hat{r}). \tag{2}$$

The radial part $\omega_{LL}(r)$ is given by

$$\omega_{LL}(r) = \sqrt{\frac{2L+1}{L}}\left(\frac{\mathrm{d}}{\mathrm{d}r} + \frac{L+2}{r}\right) j_{LL+1}(r). \tag{3}$$

The r^L-moment of ω_{LL} is identically zero; its r^{L+2}-moment may be used as a measure of its strength [4]. For irrotational and incompressible flow, e.g., in the Tassie model, ω_{LL} vanishes.

The strength distribution

$$S_\rho(E) = \sum_f \langle f|V_{LM}|0\rangle^2 \delta(E - E_f),$$

where $|f\rangle$ are the final states excited by the field V_{LM} and E_f their excitation energies, is related to the r^L-moment of $\delta\rho_L(r)$. Since we are dealing with continuous distributions, we write the strength in a small energy interval of width ΔE at energy E as

$$S_\rho(E) = \frac{1}{\Delta E}\left[\int_0^\infty \mathrm{d}r\, \delta\rho_L(r) r^{L+2}\right]^2.$$

We define the *vorticity* strength distribution $S_\omega(E)$ in a similar way, namely,

$$S_\omega(E) = \frac{1}{\Delta E}\left[\int_0^\infty \mathrm{d}r\, \omega_{LL}(r) r^{L+4}\right]^2.$$

Comparison of $S_\rho(E)$ and $S_\omega(E)$ allows a discrimination between collective states of longitudinal zero-sound character and excitation modes which are highly vortical and possibly of transverse zero-sound character. The latter case applies, for example, to the toroidal dipole mode [6,10]. Note that, in order to quantify how "collective" (in terms of S_ω) a seemingly strong transition is, and to what extent it is a transverse-zero-sound candidate, one would need to establish appropriate sum rules for S_ω.

The quantities introduced above are calculated using a Skyrme–Hartree–Fock (HF) plus Continuum-RPA model. For the HF ground-state, the numerical code of Reinhard [18] is used. The calculation of the response function is formulated in coordinate space, as described in [19–22]. First, for given energy E, multipolarity L and isospin character τ_z, the radial part $G^0_{\mu_1\mu_2}(r, r'; E)$ of the unperturbed *ph* Green function $G^0(E)$ is calculated. The indices $\mu_i = 1, 2, \ldots$ enumerate the operators whose propagation is described by the Green function, namely,

$$Y_{LM}, \quad \left[Y_L \otimes \left(\nabla^2 + \nabla'^2\right)\right]_{LM}, \quad \left[Y_{L\pm1} \otimes \left(\vec{\nabla} \pm \vec{\nabla}'\right)\right]_{LM}.$$

Spin-dependent terms have been omitted in the present calculation. The continuum is fully taken into account, as described in [19,21]. A small but finite $\mathrm{Im}\,E$ ensures that bound transitions acquire a finite width [19]. The RPA *ph* Green function is obtained by solving the equation

$$G^{\mathrm{RPA}}(E) = \left[1 + G^0(E)V_{\mathrm{res}}\right]^{-1} G^0(E) \tag{4}$$

as a matrix equation in coordinate space (represented by a radial mesh), isospin character and operators μ_i. The *ph* residual interaction V_{res} is derived self-consistently from the Skyrme–HF energy functional [21–23]. The radial functions $\delta\rho_L$, j_{LL-1} and j_{LL+1}, at given energy E, can be calculated from the RPA Green function; then, ω_{LL} is obtained using Eq. (3) [24,25].

3. Results and discussion

We have performed calculations for the IS natural-parity response of Ni isotopes for $L \leqslant 4$. Since pairing is not included in our calculation scheme, only the doubly closed isotopes ^{56}Ni ($N = 28$), ^{78}Ni ($N = 50$) and ^{110}Ni ($N = 82$) have been studied. The nucleus ^{56}Ni lies close to the valley of stability, while ^{110}Ni serves as a numerical example of a nucleus with extreme neutron excess lying close to the drip line. In all cases we examined, we have checked that the CE, Eq. (1), is fulfilled to a satisfactory degree. We present results for the IS 2^+, 3^- and 4^+ strength distribution $S_\rho(E)$ and the corresponding vorticity strength distribution $S_\omega(E)$. The distributions are normalized to 1,

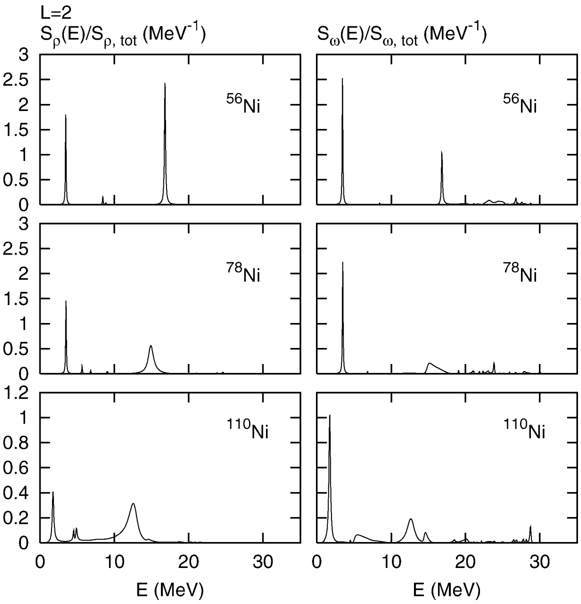

Fig. 1. Quadrupole strength distribution (left) and vorticity-strength distribution (right), normalized to 1, for the isotopes ^{56,78,110}Ni. The Skyrme parametrization SkM* has been used.

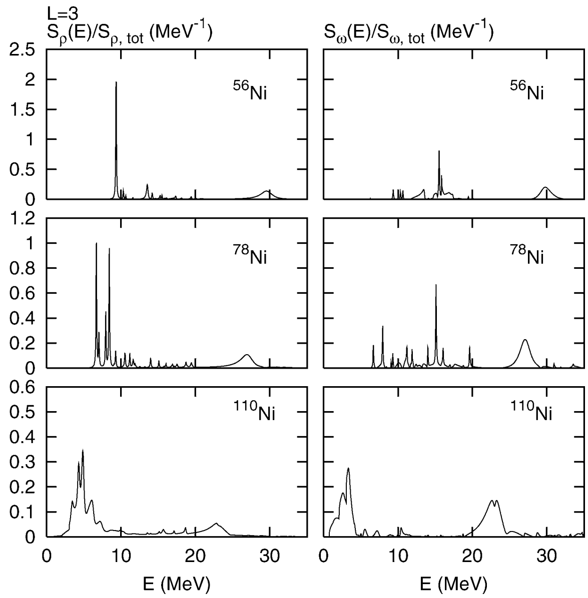

Fig. 2. As in Fig. 1, octupole strength distributions.

i.e., $S_{\rho,\omega}(E)/S_{\rho,\omega,\text{ tot}}$ is shown, where $S_{\rho,\omega,\text{ tot}}$ is the integrated strength in the region 0–50 MeV. For the results shown we have used the Skyrme-force parametrization SkM* [26], tailored to describe GRs of stable nuclei and employed in previous studies of the response of exotic nuclei as well (e.g., in [1,11–13]).

As we observe in Fig. 1, most of the IS quadrupole strength is concentrated in two peaks, a low-energy collective state and the GR. The width of the GR increases and its energy decreases as we approach the neutron drip line. The GR carries a large fraction of $S_{\rho,\text{ tot}}$ and little vorticity. It has been verified that, in the three Ni isotopes studied, its velocity field resembles that of ideal hydrodynamical flow. On the contrary, the lowest 2^+ state is strongly vortical. The octupole response of ^{56}Ni (^{78}Ni), Fig. 2, is dominated by one (two) collective state(s) carrying almost no vorticity strength; a large part of the S_ω lies in the $3\hbar\omega$ region. The situation is quite different in the case of ^{110}Ni, where highly vortical threshold strength appears at low energy. In order to illustrate the difference, in Fig. 3 we plot the velocity field corresponding to the 9.3 MeV state in ^{56}Ni and the 3.4 MeV transition in ^{110}Ni, in an arbitrary scale. For ^{56,78}Ni, it is interesting to notice a very weak octupole state with considerable vorticity strength, at around 15 MeV. The hexadecapole strength distribution of ^{56,78}Ni, Fig. 4, appears fragmented. Vorticity strength is carried by all peaks. In ^{110}Ni a striking amount of vorticity develops at very low energy.

The neutron-rich nucleus ^{78}Ni is possibly doubly magic [27]. We notice that its low-energy response does not differ dramatically from the response of ^{56}Ni. The transition strength of ^{110}Ni is located in the continuum in all examined cases. We have performed calculations also for the 0^+ and 1^- response of the three isotopes. In ^{110}Ni, the threshold energy for these multipoles is higher than for the $L = 2, 3, 4$ multipoles. The threshold strength of ^{110}Ni for $L > 0$ carries increased vortical strength. This result should be anticipated given the single-particle character of the threshold strength. The present results indicate that, close to the neutron drip line, the low-energy response is dominated by $L > 1$ transitions of transverse character. 1^- as well as 3^- excitations may gain importance with respect to 2^+ and 4^+ in the special case of nuclei described as $n\ell$-closed configurations.

We have also used the Skyrme force MSk7 [28], whose parameters were determined by fitting the values of nuclear masses, calculated using the HF + BCS method, to the measured ones, for 1888 nuclei with various values of $|N - Z|/A$. The two forces have sim-

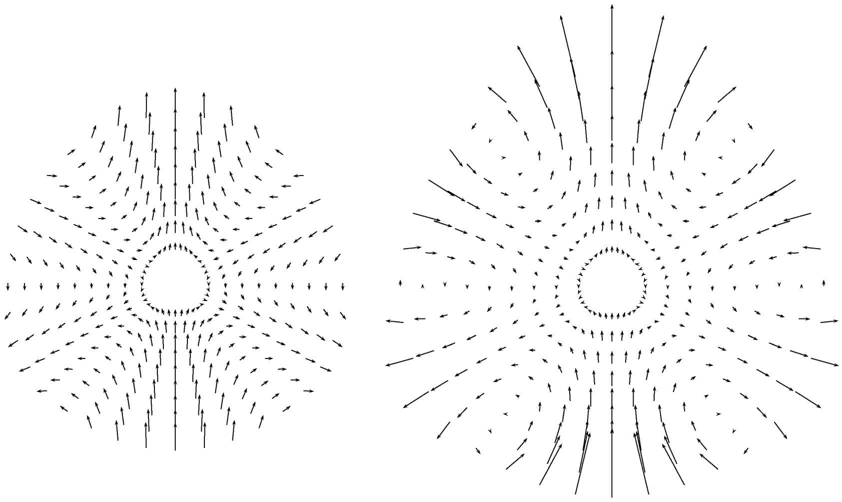

Fig. 3. Left (right): velocity field for the octupole excitation of ^{56}Ni at $E = 9.3$ MeV (^{110}Ni at $E = 3.4$ MeV) plotted up to a radial distance of 6 fm (7.6 fm). The scale of the velocity amplitude is arbitrary. The z-axis is along the page.

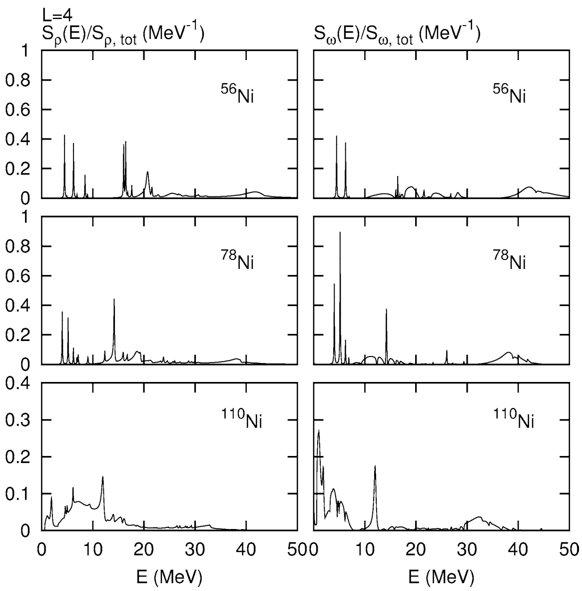

Fig. 4. As in Fig. 1, hexadecapole strength distributions.

ilar nuclear-matter properties, except for the effective mass m^*. The SkM* force ($m^*/m = 0.786$) predicts that the last occupied neutron state of ^{110}Ni is bound. The nucleus appears soft against excitations of vortical nature. In the case of the MSk7 force ($m^*/m = 1.05$), the last neutron state of ^{110}Ni appears as a resonant state at positive energy and therefore the nucleus lies beyond the neutron drip line. In general, the results obtained with the two Skyrme parametrizations agree qualitatively. The strength distributions are systematically shifted towards lower energies when MSk7 is used.

We should stress that the nucleus ^{110}Ni was studied as an academic example of a closed-shell Ni isotope close to the neutron drip line, as has been done before in Ref. [11]. This particular choice was dictated by the limitations of our model, which does not account for pairing correlations. According to SHF results, the $N = 82$ closure may still be valid in the Ni region [11], although not conclusively. Ideally, one should perform a (preferably self-consistent) quasiparticle-RPA (QRPA) calculation including the full continuum, as proposed in Ref. [29]. The correct treatment of the continuum when pairing is present, in the ground state as well as in the excited states, is particularly important in weakly bound neutron systems, where the pairing field couples the neutrons in the bound states with those in the low-energy continuum [30–32]. In general, pairing correlations are not expected to modify dramatically the electric transitions of even–even nuclei (the same may not hold for magnetic transitions, however) [33]. They can introduce new transitions and change the energy and strength of the ones that we have already examined [29,34]. Therefore, we expect that our conclusions would still hold qualitatively, after pairing is taken into account. Of course, they are restricted to spherical nuclei.

In conclusion, our results for the IS, natural-parity response of Ni isotopes, obtained using a self-consistent Skyrme–Hartree–Fock + Continuum RPA model, indicate that, close to the neutron drip line, instabilities develop in the form of $L > 1$ transitions of transverse character. We have used the vorticity strength distribution to quantify the irrotational character of the various transitions. Future work should account for the spin current, besides the convection current. It would be useful to examine the magnetic response as well, for a more complete picture. Ideally, a chain of even-N isotopes should be studied using a QRPA method including the continuum. The proton drip line is also interesting to explore; modifications of the strength distributions, in the direction of sharper low-lying transitions, are expected due to the Coulomb barrier.

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References

[1] I. Hamamoto, H. Sagawa, X.Z. Zhang, Nucl. Phys. A 648 (1999) 203.
[2] E.E. Serr, T.S. Dimitrescu, T. Suzuki, C.H. Dasso, Nucl. Phys. A 404 (1983) 359.
[3] E. Wüst, U. Mosel, J. Kunz, A. Schuh, Nucl. Phys. A 406 (1983) 285.
[4] D.G. Ravenhall, J. Wambach, Nucl. Phys. A 475 (1987) 468.
[5] T. Hartmann, J. Enders, P. Mohr, K. Vogt, S. Voltz, A. Zilges, Phys. Rev. Lett. 85 (2001) 274.
[6] N. Ryezayeva, et al., Phys. Rev. Lett. 89 (2002) 272502.
[7] A. Leistenschneider, et al., Phys. Rev. Lett. 86 (2001) 5442.
[8] E. Tryggestad, et al., Phys. Lett. B 541 (2002) 52.
[9] S. Nakayama, et al., Phys. Rev. Lett. 85 (2000) 262.
[10] D. Vretenar, N. Paar, P. Ring, T. Niksic, Phys. Rev. C 65 (2002) 021301(R).
[11] I. Hamamoto, H. Sagawa, X.Z. Zhang, Phys. Rev. C 53 (1996) 765.
[12] I. Hamamoto, H. Sagawa, Phys. Rev. C 54 (1996) 2369.
[13] I. Hamamoto, H. Sagawa, X.Z. Zhang, Phys. Rev. C 56 (1997) 3121.
[14] F. Catara, E.G. Lanza, M.A. Nagarajan, A. Vitturi, Nucl. Phys. A 624 (1997) 449.
[15] T. Suzuki, Ann. Phys. (Paris) 9 (1984) 535.
[16] J. Heisenberg, J. Lichtenstadt, C.N. Papanicolas, J.S. McCarthy, Phys. Rev. C 25 (1982) 2292.
[17] E.C. Caparelli, E.J.V. de Passos, J. Phys. G 25 (1999) 537.
[18] P.-G. Reinhard, Skyrme–Hartree–Fock model, in: K. Langanke, J.E. Maruhn, S.E. Koonin (Eds.), Computational Nuclear Physics I—Nuclear Structure, Springer, New York, 1991, p. 28.
[19] G. Bertsch, in: K. Langanke, J.E. Maruhn, S.E. Koonin (Eds.), Computational Nuclear Physics I—Nuclear Structure, Springer, New York, 1991, p. 75.
[20] G.F. Bertsch, S.F. Tsai, Phys. Rep. C 18 (1975) 127.
[21] N. Van Giai, Nuclear excitations in the nuclear response theory, in: D. Bucuresku, V. Ceausescu, N.V. Zamfir (Eds.), Nuclear Collective Dynamics, World Scientific, Singapore, 1983, p. 356.
[22] J. Ryckebusch, Ph.D. thesis, University of Gent, 1988.
[23] S.F. Tsai, Phys. Rev. C 17 (1978) 1862.
[24] P. Papakonstantinou, Ph.D. thesis, University of Athens, 2004.
[25] P. Papakonstantinou, J. Wambach, E. Mavrommatis, V.Yu. Ponomarev, in: M. Buballa, et al. (Eds.), Proc. XXXII Int. Workshop on Gross Properties of Nuclei and Nuclear Excitations, Hirschegg, Kleinwalsertal, Austria, 11–17 January, 2004, p. 159.
[26] J. Bartel, P. Quentin, M. Brack, C. Guet, H.-B. Hakansson, Nucl. Phys. A 386 (1982) 79.
[27] J.M. Daugas, R. Grzywacz, M. Lewitowicz, L. Achouri, et al., Phys. Lett. B 476 (2000) 213.
[28] S. Goriely, F. Tondeur, J.M. Pearson, At. Data Nucl. Data Tables 77 (2001) 311.
[29] M. Matsuo, Nucl. Phys. A 696 (2001) 371.
[30] K. Bennaceur, J.F. Berger, B. Ducomet, Nucl. Phys. A 708 (2002) 205.
[31] I. Hamamoto, B.R. Mottelson, Phys. Rev. C 68 (2003) 034312.
[32] E. Khan, N. Sandulescu, M. Grasso, N. Van Giai, Phys. Rev. C 66 (2002) 024309.
[33] D.J. Rowe, Nuclear Collective Motion, Methuen, London, 1970.
[34] A. Bohr, B.R. Mottelson, Nuclear Structure, vol. II, Benjamin, New York, 1975.