Two-phonon 1^- state in ^{112}Sn observed in resonant photon scattering

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(Received 25 October 2005; published 24 January 2006)

Results of a photon scattering experiment on ^{112}Sn using bremsstrahlung with an endpoint energy of $E_0 = 3.8$ MeV are reported. A $J = 1$ state at $E_x = 3434(1)$ keV has been excited. Its decay width into the ground state amounts to $\Gamma_0 = 151(17)$ meV, making it a candidate for a $[2^+ \otimes 3^-]1^-$ two-phonon state. The results for ^{112}Sn are compared with quasiparticle-phonon model calculations as well as the systematics of the lowest lying 1^- states established in other even-mass tin isotopes. In contrast to findings in the heavier stable even-mass Sn isotopes, no 2^+ states between 2 and 3.5 MeV excitation energy have been detected in the present experiment.

PACS number(s): 21.10.−k, 23.20.−g, 25.20.Dc, 27.60.+j

Collective quadrupole and octupole vibrations are well-established features of nuclear structure [1]. From the coupling of these elementary excitations one expects multiplets of states whose energies and transition strengths may provide insight into, for example, the anharmonicities of vibrations, the underlying microscopic structure of elementary excitations, or the purity of multiphonon states. For the specific case of the coupling of a quadrupole and an octupole vibration, a quintuplet of states with $J^\pi = 1^-$ to 5^- [2,3] arises whose 1^- member is accessible by real photon scattering [4]. Experimentally, the two-phonon structure of such 1^- states has been established from the decay pattern in the case of some $N = 82$ isotones [5,6].

A systematic study of two-phonon 1^- states using photon scattering has been carried out for ^{116,118,120,122,124}Sn by Bryssinck and co-workers [7]. Here, a surprisingly uniform behavior concerning excitation energies and strengths has been identified, in spite of a (slow) variation of the octupole collectivity with the mass number. In addition, a number of $E2$ excitations between 2 and 4 MeV excitation energy could be detected in the even-even $^{116-124}$Sn nuclei in the photon scattering experiments [8]. Fair agreement of a microscopic analysis with the experimental data has been found.

The present study aims at an extension of this survey to the most neutron-deficient stable tin isotope, ^{112}Sn, whose negative-parity states, identified from inelastic neutron scattering [9] recently, are proposed to represent the members of the quadrupole-octupole-coupled quintuplet. Similar to the works of Bryssinck *et al.* [7,8], a photon scattering experiment has been performed at the nuclear resonance fluorescence (NRF) setup at the Stuttgart Dynamitron accelerator [4]. Unpolarized bremsstrahlung was produced from a 3.8-MeV dc electron beam with average beam current of 200 μA. Photons scattered from a 1990-mg ^{112}Sn target (enrichment $\geqslant$99.5%) have been measured in three high-purity germanium detectors placed at 90°, 127°, and 150° with respect to the photon beam. The detector at 127° was surrounded by BGO detectors for Compton background suppression. Aluminum platelets [10] and ^{13}C powder [11] were placed around the Sn target for photon flux calibration. Data were taken for 69 h.

Figure 1 displays the measured spectrum for the example of the detector placed at 127° with respect to the incident beam. Besides transitions from the reference materials, ^{13}C and ^{27}Al, one recognizes only one strong transition at 3434(1) keV to which dipole character can be unambiguously assigned on the basis of the measured angular distribution. Aside from this transition and from the decay of the 2_1^+ state to the ground state (g.s.), no further transitions has been observed that could be attributed to ^{112}Sn. The sensitivity of the present experiment was comparable to the work of Bryssinck *et al.* [7,8]. A decay of the new 3434-keV state into the 2_1^+ state has also not been detected. An upper limit of 1.5% for the branching ratio of this decay can be extracted from the data.

To extract the excitation strength, the detector efficiency and the bremsstrahlung photon spectrum need to be determined. The relative shape of the former was obtained from a measurement using a radioactive ^{56}Co source and that of the latter from the transitions in ^{13}C and ^{27}Al. For a smooth interpolation between the measured transitions, several assumptions about the shape of the photon distribution are possible: (i) The shape of the photon spectrum may be simulated using Monte Carlo methods, (ii) it may be approximated by the Schiff function [12] using the endpoint energy of the spectrum as a free parameter, and (iii) it may be fit freely to the measured transitions including the endpoint at which the photon intensity drops to zero. One finds that none of these approaches can appropriately describe the 3088-keV transition from ^{13}C if

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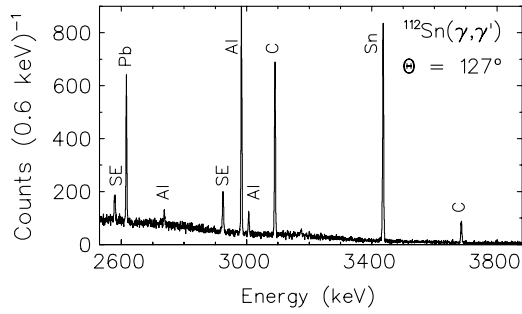
FIG. 1. BGO-suppressed nuclear resonance fluorescence spectrum measured at an angle of 127° with respect to the incoming beam direction. A strong dipole excitation at 3434 keV can be attributed to ^{112}Sn; the other transitions stem from ^{13}C and ^{27}Al used for the determination of the incoming photon spectrum. Single-escape peaks as well as a ^{208}Pb background line are indicated.

one takes the latest literature value for the integrated cross section determined from a self-absorption experiment [11]. This behavior has been noticed before and is discussed in Ref. [13]. We have chosen to omit the 3088-keV line from the data points for the photon spectrum determination and to use approach (ii) for the quantitative analysis as this resulted in a very good description of the measured reference points. Including the lower ^{13}C point will decrease the photon flux and hence increase the extracted $B(E1)$ value by about 10%. The shape of the photon spectrum as generated by a Monte Carlo simulation using the code GEANT 3.21 does not describe the data very well, in contrast to previous analyses (see, e.g., the work by Belic *et al.* [14], where the simulation successfully described the shape of the photon distribution).

Table I lists the results for the observed dipole excitation in ^{112}Sn from the present work in comparison with the results of the other even-mass stable Sn isotopes from Ref. [7]. Although the multipole character of the 3434-keV transition and thus the parity of the $J = 1$ state was not established in the experiment, we will henceforth assume that the transition stems from the depopulation of the two-phonon 1^- state. This is also suggested by the recent work of Kumar and colleagues [9]. If a magnetic dipole character was assumed instead, the measured transition width into the g.s. of $\Gamma_0 = 151(17)$ meV would correspond to $B(M1)\uparrow = 0.97(11)\,\mu_N^2$. This would be larger than in neighboring Cd [15] and Te [16,17] nuclei, where magnetic dipole strength was identified that might be attributed to the orbital $M1$ scissors mode [18,19]. No
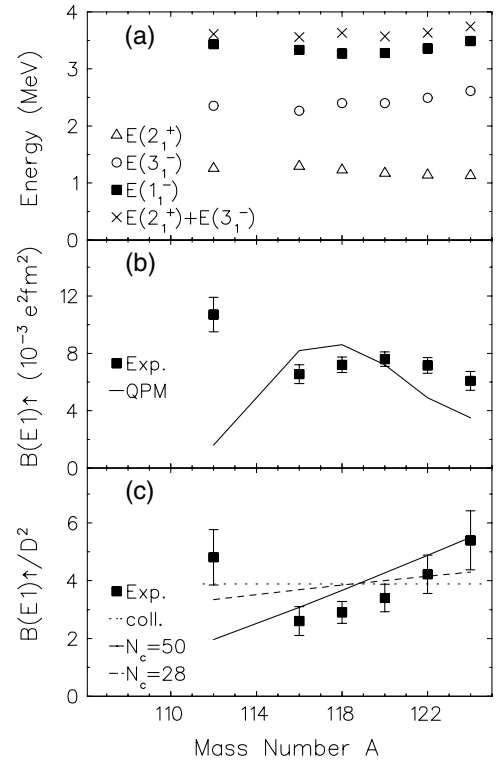

FIG. 2. Systematics of the $[2_1^+ \otimes 3_1^-]1^-$ two-phonon states in stable even-mass tin isotopes, from this work and Ref. [7]. (a) Excitation energy of 1^- states compared with the excitation energies of the 2_1^+ and 3_1^- states and their sums. (b) Measured $E1$ excitation strengths (solid squares) compared with the results of a quasiparticle-phonon model calculation (solid line). (c) Ratio of the measured $E1$ strength and the square of the dynamical dipole moment (solid squares) according to Eq. (1). The lines show the scaling behavior for a purely collective picture (dotted), for the assumption of an $N_c = Z_c = 50$ core (solid), and for an $N_c = 28$ core (dashed). The lines are scaled to the mean of the experimental $B(E1)/D^2$ values.

low-lying $M1$ strength was reported for the other even-mass tin isotopes [7,8].

From Table I as well as from Fig. 2(a), one recognizes that the excitation energies of the low-lying 2_1^+ and 3_1^- states vary only slowly with mass number, as does the energy of the 1_1^- state. The latter can be found, in all cases including ^{112}Sn, about 5%–8% below the sum energy $E_x(2_1^+) + E_x(3_1^-)$.

In contrast to the very uniform behavior of the excitation energies, the $E1$ excitation strength $B(E1)\uparrow = 10.7(12) \times$

TABLE I. Measured and calculated excitation energies and strengths for the lowest 1^- states in ^{112,116,118,120,122,124}Sn.

	^{112}Sn[a]	^{116}Sn[b]	^{118}Sn[b]	^{120}Sn[b]	^{122}Sn[b]	^{124}Sn[b]
$E_x(1^-)_{\text{exp}}$ (keV)	3434	3334	3271	3279	3359	3490
$E_x(1^-)_{\text{QPM}}$ (keV)	3240	3350	3290	3320	3420	3570
$E_x(2_1^+) + E_x(3_1^-)$ (keV)	3612	3560	3631	3572	3634	3646
$B(E1)\uparrow_{\text{exp}}$ ($10^{-3}\,e^2\,\text{fm}^2$)	10.7(12)	6.6(7)	7.2(5)	7.6(5)	7.2(5)	6.1(7)
$B(E1)\uparrow_{\text{QPM}}$ ($10^{-3}\,e^2\,\text{fm}^2$)	1.6	8.2	8.6	7.2	4.9	3.5

[a]This work.
[b]Ref. [7].

$10^{-3} e^2$ fm^2 of the 1_1^- state in ^{112}Sn is about 50% higher than in the heavier even-mass Sn isotopes. This result is depicted by the full squares in Fig. 2(b). Our value from NRF is smaller than the one reported by the $(n, n'\gamma)$ work [9], which finds $B(E1)\uparrow = (18_{-5}^{+18}) \times 10^{-3} e^2$ fm^2. Kumar and co-workers were also able to determine a rough estimate for the $B(E1; 3_1^- \to 2_1^+)$ value, which is listed as $0.9(22) \times 10^{-3} e^2$ fm^2. The ratio $B(E1; 1^- \to 0_1^+)/B(E1; 3_1^- \to 2_1^+)$ would be much larger than expected from phenomenological [20] and microscopic [21] analyses, but has a significant experimental uncertainty dominated by the measurement of the lifetime and the multipole mixing ratio in the decay of the 3_1^- state.

To analyze the large $E1$ strength we used a phenomenological approach based on a collective dynamical dipole moment as introduced by Bohr and Mottelson [22] and Strutinsky [23]

$$D_{\text{BMS}} = (5.38 \times 10^{-4})(Z + N)Z\beta_2\beta_3 \, e \, \text{fm}, \quad (1)$$

where the dipole moment arises from the dynamical quadrupole and octupole deformations β_2, β_3 as determined from the $B(E2; 0_1^+ \to 2_1^+)$ and $B(E3; 0_1^+ \to 3_1^-)$ values, respectively, Z denotes the proton and N the neutron number. The ratio of the measured $B(E1)$ values and D_{BMS}^2 is shown in Fig. 2(c) and should be constant within a collective approach (dotted line) in which the underlying shell structure does not play a role. Whereas the heavier isotopes show a systematic increase with mass number, the value for ^{112}Sn is large and comparable to the result found for ^{122,124}Sn. Within the Bohr-Mottelson approach therefore, the uniformity of the $B(E1)$ values in the heavier tin isotopes is surprising. We note that in open-shell nuclei the ratio $B(E1)/D^2$ is smaller by about an order of magnitude, as discussed by Refs. [15,24].

Recently, Kohstall *et al.* [15] have applied a simple collective model description to the two-phonon $E1$ excitations in the $Z \approx 50$ region. Here, an effective dipole moment is extracted:

$$D_{\text{eff}} = \Delta e \frac{NZ}{A} \left(\frac{Z_v}{Z} - \frac{N_v}{N} \right), \quad (2)$$

where Z_v and N_v denote the number of valence protons and neutrons, respectively, and Δ represents the distance between the centers of gravity of the proton and neutron bodies. If one neglects the shell structure, the dipole moment scales with $e\Delta NZ/A$ so that a correction factor K for an isotopic chain can be introduced to account for shell effects:

$$[K(Z, N)]^2 = \left(\frac{N_c}{N} - \frac{Z_c}{Z} \right)^2. \quad (3)$$

This factor depends on the choice of the proton and neutron core, $Z_c = Z - Z_v$ and $N_c = N - N_v$, respectively. Scaled to the average, the correction factor K^2 can be compared to the experimental ratios $B(E1)/D_{\text{BMS}}^2$ as shown in Fig. 2(c). The full line indicates the assumption of an $N_c = Z_c = 50$ core; the dashed line indicates a core with $N_c = 28$ and $Z_c = 50$. Whereas the heavier Sn isotopes nicely scale with the correction factor given by the $N_c = Z_c = 50$ core, the value for ^{112}Sn is found close to the assumption of an $N_c = 28$ core with the proton $Z_c = 50$ shell left intact, or it could be described by a collective picture where the underlying shell structure no longer plays a role. However, although the experimental

data are compatible with such an analysis, it is not clear why the $N_c = 50$ shell closure should disappear when removing neutrons from a half-filled sdg neutron shell. The unexpectedly large $E1$ strength in ^{112}Sn thus remains unexplained.

To analyze the structure of the dipole excitation in ^{112}Sn, we have also performed calculations within the quasiparticle-phonon nuclear model (QPM [25]) along the lines of Refs. [7,26,27]. The results are displayed in Table I with the experimental data for the 1_1^- states. The calculated excitation strengths are displayed as a solid line in Fig. 2(b) and compared to the experimental data. The QPM fails to describe both the uniformity of the $E1$ strength found in the heavier tin isotopes and the large $B(E1)$ value reported here for ^{112}Sn. The predicted $E1$ strength in this lightest naturally occurring tin isotope is very small. Closer inspection reveals that the large contributions from protons and neutrons to the transition matrix element connecting the ground with the two-phonon state nearly cancel each other. The total transition matrix element is almost two orders of magnitude smaller than its proton and neutron parts. The calculation is thus very sensitive to the choice of parameters and does not exhibit too much predictive power in this special case.

A few comments on the electric quadrupole strength distribution between 2 and 4 MeV are in order. Several 2^+ states are known in ^{116,118,120,122,124}Sn. A large fraction of the states below 4 MeV exhibit a strong decay branch into the g.s. so that some of these states could be excited in the photon scattering study by Bryssinck *et al.* [8]. In the literature [28] for ^{112}Sn, a number of 2^+ states are listed below 3.5 MeV. The recent $(n, n'\gamma)$ work by the Kentucky group [9] has found new 2^+ states and corroborated previous $J^\pi = 2^+$ assignments, but different branching ratios were reported in several cases.

The $E2$ excitation strengths and upper limits reported by Refs. [9,28] are consistent with the upper limits derived from the detection threshold of the present experiment using the branching ratios listed in the Nuclear Data Sheets [28]. Summing the upper limits of all possible $E2$ excitations in the energy interval between 2 and 3.5 MeV, one finds the maximum possible $E2$ strength to be $<260 \, e^2$ fm^4. This is of the same order of magnitude as detected in the heavier Sn isotopes [8]. As no $E2$ transition could be detected in our experiment, it is likely that the $E2$ strength will continue the trend of the heavier isotopes to an even smaller summed $B(E2)$ value than in ^{116}Sn, which amounted to $143(19) \, e^2$ fm^4. Calculations within the QPM expect nearly a dozen 2^+ states between the 2_1^+ state and 3.8 MeV, none of which is predicted to be strong enough to be detectable in the present experiment.

In conclusion, we have performed a nuclear resonance fluorescence experiment using bremsstrahlung with an end-point energy of 3.8 MeV on the semi-magic ^{112}Sn nucleus. The strength of a dipole excitation at 3434 keV has been determined, and this excitation is assumed to arise from the coupling of the low-lying quadrupole and octupole vibrations. Compared with the heavier even-mass stable tin isotopes, the $E1$ strength of the 3434-keV excitation in ^{112}Sn is about 50% larger. We have tried a phenomenological analysis, but this does not lead to a qualitative understanding of the large $B(E1)$ value in ^{112}Sn. The detected $E1$ strength cannot be reproduced

by a microscopic quasiparticle-phonon model calculation, which turns out to be very sensitive to the interplay of proton and neutron amplitudes in this transition. No experimental evidence for $E2$ strength above the 2_1^+ state was found. Measuring the form factors of the low-lying 1^- and 2^+ states in electron scattering could provide a quantitative estimate of the composition of the wave function and admixtures from collective excitations. To elucidate the change of the $E1$ strengths of the 1^- two-phonon states in ^{112}Sn and ^{116}Sn, it would be desirable to obtain information about the only missing stable even-mass tin isotope, the rare ^{114}Sn. Also,

the multipole character of the 3434-keV excitation should be investigated.

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