High-Energy-Resolution Inelastic Electron and Proton Scattering and the Multiphonon Nature of Mixed-Symmetry 2^+ States in ^{94}Mo

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High-energy-resolution inelastic electron scattering (at the S-DALINAC) and proton scattering (at iThemba LABS) experiments permit a thorough test of the nature of proposed one- and two-phonon symmetric and mixed-symmetric 2^+ states of the nucleus ^{94}Mo. The combined analysis reveals the one-phonon content of the mixed-symmetry state and its isovector character suggested by microscopic nuclear model calculations. The purity of two-phonon 2^+ states is extracted.

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Collective valence-shell excitations are a generic feature of strongly-coupled mesoscopic quantum systems. A prime example of a two-component system is the atomic nucleus formed by protons and neutrons. The microscopic structure of collective nuclear excitations with respect to their proton-neutron content is a central issue of nuclear structure physics with general implications for the physics of composite strongly-coupled quantum systems.

Low-energy nuclear valence-shell excitations usually possess the lowest possible isospin quantum number $T_< = |N - Z|/2$. Nevertheless, the symmetry character of their proton-neutron coupling can vary. This fact is evident in the framework [1] of the proton-neutron version of the nuclear interacting boson model (IBM-2). The *sd*-IBM-2 considers monopole (s) bosons and quadrupole (d) bosons while the number of proton and neutron bosons $N_{\pi,\nu}$ are taken as the number of respective valence nucleon pairs. The model describes quadrupole collective valence-shell excitations. The $J^\pi = 2^+$ one-phonon states and their coupling to multiphonon multiplets can be classified according to their proton-neutron symmetry by the F-spin quantum number. Fully symmetric states (FSSs) have maximum F-spin $F_{\max} = (N_\pi + N_\nu)/2$. Those with quantum numbers $F < F_{\max}$ are called mixed-symmetry states (MSSs). A characteristic feature of MSSs are enhanced magnetic dipole ($M1$) transitions to FSSs. The IBM-2 successfully accounted [2] for the strength of the nuclear $M1$ scissors mode, which is a prime example for the class of MSSs. The scissors mode was discovered [3] in deformed nuclei and subsequently investigated in other quantum systems [4].

Recently, one- and two-phonon MSSs were investigated in vibrational nuclei with proton and neutron numbers near closed shells, e.g., in the nuclide ^{94}Mo [5]. Comprehensive spectroscopic information on low-spin states has been achieved up to an excitation energy of about 4 MeV [6] allowing to identify one- and two-phonon FSSs and MSSs based on strong $M1$ and $E2$ γ decays. Properties of the one-phonon states and the mechanism of their formation have successfully been explained microscopically using one of a class of low-momentum nucleon-nucleon interactions $V_{\text{low}-k}$ [7]. Form factors or differential cross sections offer additional sensitivity to the structure of MSSs utilized, e.g., in a combination of (p, p') and (d, d') scattering for the analysis of one-phonon MSSs in a variety of nuclei [8]. However, two-phonon MSSs have never been investigated in scattering experiments before.

It is the purpose of this Letter to report a combined study of electron and proton scattering differential cross sections for $J^\pi = 2^+$ one- and two-phonon FSSs and MSSs in ^{94}Mo. The selectivity of both reactions to one-phonon components in the excited state wave functions allows to extract for the first time the small one-phonon contributions to the two-phonon candidates. The proton-neutron symmetry character can be derived since electron scattering couples to the proton distributions, while proton scattering is dominated by the isoscalar central part of the effective proton-nucleus interaction. We thereby introduce a new approach establishing a multiphonon character of nuclear MSSs based on scattering data complementary to γ-ray spectroscopy.

The (e, e') experiments were carried out at the high-energy-resolution magnetic spectrometer [9] of the Darmstadt superconducting electron linear accelerator S-DALINAC. Data were taken for kinematics broadly covering the maximum of $E2$ form factors (incident electron beam energy $E_e = 70$ MeV and scattering angles $\Theta_e = 93°–165°$) with typical beam currents of 2 μA. An enriched (91.6%) self-supporting ^{94}Mo target of 9.7 mg/cm^2 areal density was used. In the dispersion-matching mode an energy resolution $\Delta E \simeq 30$ keV (full width at half maximum, FWHM) was achieved.

High-energy-resolution (p, p') measurements were performed at the cyclotron of iThemba LABS, South Africa, using a K600 magnetic spectrometer. The experimental techniques were similar to those described in [10]. The target consisted of a self-supporting molybdenum foil enriched to 93.9% ^{94}Mo of 1.2 mg/cm^2 areal density. Data were taken at the highest possible incident proton energy $E_p = 200$ MeV in order to enhance one-step contributions to the cross sections. Scattering angles $\Theta_p = 6°–27°$ were covered with currents varying from 1 to 30 nA, depending on scattering angle. The average energy resolution was $\Delta E \simeq 35$ keV (FWHM).

Examples of electron and proton scattering spectra are shown in Fig. 1. The prominent peaks correspond to the elastic line, the collective 2_1^+ ($E_x = 0.871$ MeV) and 3_1^- states, and the one-phonon 2^+ MSS (the 2_3^+ level, $E_x = 2.067$ MeV). The two-phonon FSS (2_2^+, $E_x = 1.864$ MeV) and MSS (2_5^+, $E_x = 2.870$ MeV) are only weakly excited, but the insets of Fig. 1 demonstrate their observation as well as that of all other 2^+ states up to 4 MeV [6].

To extract quantitative information on the phonon character of the observed states we analyze the momentum-transfer dependence of the cross sections (for (e, e') normalized to the Mott cross section) using microscopic quasiparticle-phonon model (QPM), shell model (SM), and IBM-2 transition form factors for comparison.

Wave functions of 2^+ states in ^{94}Mo were obtained by diagonalizing a QPM Hamiltonian in the space of interacting one-, two-, and three-phonon configurations (see, e.g., [11,12] and references therein). Multiphonon configurations have been built from phonons with $J^\pi = 1^\pm - 6^\pm$. The approach is similar to the one in Ref. [13] except for the treatment of the particle-particle channel of the residual interaction. The calculation reproduces the number of experimentally known 2^+ states in the energy interval studied and also predicts the excitation energies of the FSSs and MSSs with an accuracy of better than 300 keV allowing for a one-to-one correspondence with the data. Additionally, we present results ("pure" QPM) in which excited states are described as pure one- or two-phonon states with the same phonons as in the full calculation but with the interaction between them being artificially switched off.

The shell-model calculations employed a valence space of 4 protons and 2 neutrons with ^{88}Sr as inert core using the microscopic low-momentum interaction $V_{\text{low}-k}$ [7]. The IBM-2 description of (e, e') form factors followed the approach suggested in [14]. The radial dependence of the transition densities was obtained in a generalized-seniority shell-model calculation [15] while the vibrational $U(5)$ limit was used to describe the dominant transitions within the IBM. For the qualitative discussion below this approximation should show little difference to a calculation with realistic IBM parameters.

Theoretical cross sections for electron and proton scattering were calculated in the distorted wave Born approximation using the code of Ref. [16] and DWBA05 [17], respectively. The t-matrix parametrization of Franey and Love [18] at 200 MeV was used as effective projectile-target interaction for the latter.

Figure 2 presents the results for the transitions populating the one-phonon FSS and MSS in ^{94}Mo. The dominance of the transitions to the 2_1^+ and 2_3^+ states observed in Fig. 1

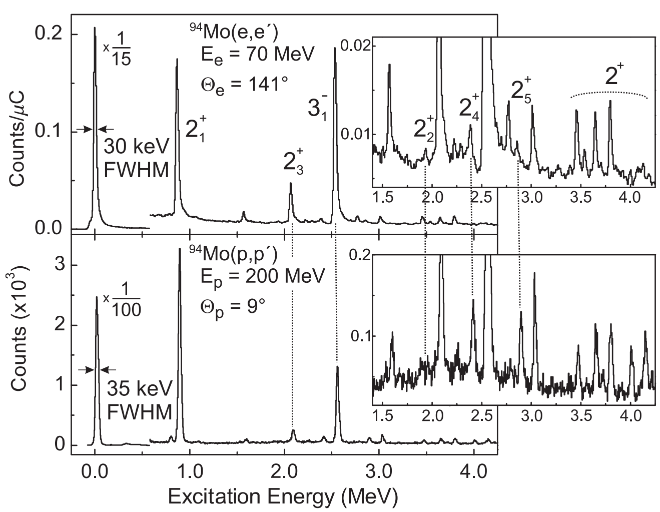

FIG. 1. Top: Spectrum of the ^{94}Mo(e, e') reaction at $E_e = 70$ MeV and $\Theta_e = 141°$. Bottom: Spectrum of the ^{94}Mo(p, p') reaction at $E_p = 200$ MeV and $\Theta_p = 9°$. Insets: zoom on the $E_x = 1.5$–4 MeV region of the respective spectra.

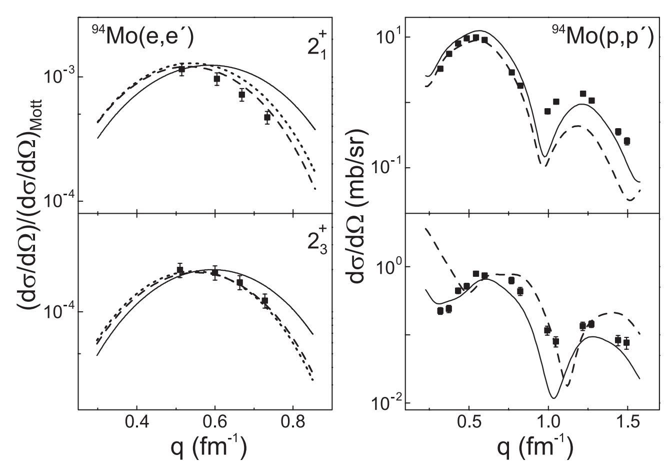

FIG. 2. Momentum-transfer dependence of the one-phonon FSS (top) and MSS (bottom) excitation cross sections in ^{94}Mo. Left: electron scattering. Right: proton scattering. The data (full squares) are compared to QPM (solid lines), SM (dashed lines), and IBA-2 (dotted lines) predictions described in the text.

already indicates the concentration of one-phonon strength in their wave functions. We first note the similarity of the data for both states. The SM results provide a good description of the (e, e') form factors and the (p, p') cross sections except at higher momentum transfers, where correlations outside the valence space become important. The IBM-2 form factor predictions are very similar to the SM results. Considering an overall uncertainty of about 25% due to the choice of the effective interaction [19], the QPM accounts well for the proton scattering results but shows a systematic shift of the form factor maximum compared to the electron scattering data. This shift results from an underestimate of the experimental charge radius by the global Woods-Saxon potential used for the calculations. An artificial increase of the potential radius would allow for a reproduction of the data comparable to SM and IBM-2. However, the structure of the ^{94}Mo ground state with two valence protons in the $1g_{9/2}$ shell does not leave room for a modification of the proton mean-field parameters.

Overall, all models agree on the one-phonon character of the 2_1^+ FSS and 2_3^+ MSS. Because of their different sensitivity to proton and neutron degrees of freedom, the combined information from electron and proton scattering results permits an extraction of their F-spin character. Based on the successful SM description one can analyze the structure of the one-phonon states in terms of their main configurations. The sign difference between the dominant terms in the wave functions (defining $p = \pi 2p_{1/2}, g = \pi 1g_{9/2}, d = \nu 2d_{5/2}$) $\psi_{2_1^+} = 0.66(p^2g^2)_{0^+}(d^2)_{2^+} + 0.42(p^2g^2)_{2^+}(d^2)_{0^+} \ldots,$ $\psi_{2_3^+} = 0.45(p^2g^2)_{0^+}(d^2)_{2^+} - 0.64(p^2g^2)_{2^+}(d^2)_{0^+} \ldots$ confirms the isovector nature of the excitation to the MSS within the valence shell.

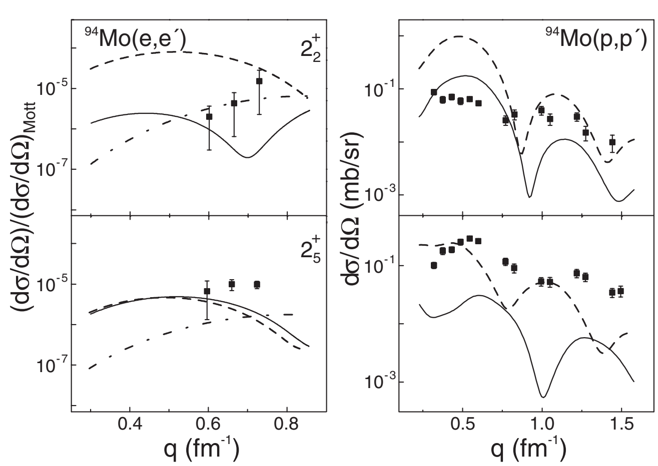

FIG. 3. Momentum-transfer dependence of the two-phonon FSS (top) and MSS (bottom) excitation strengths in ^{94}Mo. Left: electron scattering. Right: proton scattering. The data (full squares) are compared to full QPM (solid lines), SM (dashed lines), and simplified (pure) QPM (dashed-dotted line) calculations described in the text.

Next, we discuss the structure of two-phonon state candidates. Figure 3 shows the comparison of the SM and QPM results to the data. Again, the momentum-transfer dependence of the transitions to the 2_2^+ and 2_5^+ states is quite similar while it differs qualitatively from that of the one-phonon states in Fig. 2. The (e, e') form factors are sensitive to the interference between large two-phonon components, which are weakly excited only, and small one-phonon admixtures with a large excitation probability, leading to contributions of comparable magnitude. Here, the SM significantly overshoots the (e, e') data on the FSS indicating too large one-phonon components in the wave function. This is in line with large seniority-2 contributions of about 45% in the SM wave function (although they do not provide a direct measure of the one-phonon component). The QPM provides cross sections of the correct magnitude although it predicts a pronounced minimum at a momentum transfer $q \simeq 0.72$ fm^{-1} due to an interference of the main two-phonon component (81%) with an admixture (19%) of the 2_4^+ state. Because of a dominant neutron $(3s_{1/2}2d_{5/2}^{-1})$ nature, its $(e, e)'$ cross section is small. In the pure QPM calculations—considering the basic one- and two-phonon states only—a good description is achieved, indicating a high purity of the symmetric two-phonon state. On the other hand, results for the MSS are about an order of magnitude too small, pointing to significant one-phonon components. The SM and the full QPM results are very similar but still somewhat below the data. An increase of the predicted one-phonon admixtures of about 3% to 8%–15% (depending on the assumed configuration) would lead to a quantitative agreement with experiment. In any case, a dominant two-phonon character prevails.

As is visible in the right-hand side (rhs) of Fig. 3, both SM and QPM results fail to describe the (p, p') results for both two-phonon 2^+ states. A possible explanation is the neglect of two-step processes in the (p, p') reaction mechanism. Such contributions are small for collective transitions at a beam energy of 200 MeV but are important [20] for the extreme case of very weak one-step excitations and strong two-step excitations through collective levels encountered here. To estimate the two-step processes at least qualitatively, a coupled-channel analysis was performed with the code CHUCK3 [21]. It is based on the collective model describing nuclear excitations as surface vibrations of multipolarity L, whose amplitudes are proportional to a coupling strength c_L. This approximation is insensitive to the isospin nature of the transitions to the one-phonon states; the only requirement is that of collectivity demonstrated above for the case of ^{94}Mo. The transition potential is taken to be the derivative of the optical potential. Starting from the global set of Ref. [22], optical model parameters for the present reaction were determined by a fit to the elastic scattering cross sections.

The left-hand side of Fig. 4 indicates the coupling schemes taken into account for the two-phonon FSS and

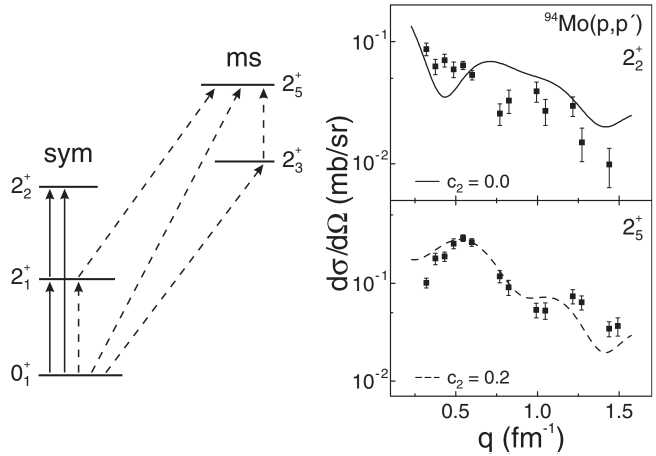

FIG. 4. Coupled-channel analysis for the excitation of the two-phonon states in the ^{94}Mo(p, p') experiment. Left: Coupling scheme. Right: Best fits to the data using the indicated coupling strengths (uncertainty of about 5%) for the transitions to the FSS (top) and to the MSS (bottom), respectively.

MSS, respectively. The coupling strengths of the one-phonon transitions to the 2_1^+ and 2_3^+ states were determined by a fit to the data. Unknown (like $2_1^+ \rightarrow 2_5^+$) or poorly known (like $2_1^+ \rightarrow 2_2^+$) transition strengths [6] were fixed assuming harmonic vibrations. The CHUCK3 results for the two-phonon states are displayed on the rhs of Fig. 4. The best description of the 2_2^+ state is achieved for a vanishing coupling strength $c_2 = 0$; i.e., the cross sections are explained by two-step processes entirely. This in turn confirms the conclusion of a nearly pure two-phonon nature drawn from the electron scattering results. A value of $c_2 = 0.2$ is obtained for the transition to the 2_5^+ state. The corresponding one-step cross section implies a one-phonon component roughly (depending on anharmonicities and optical model parameters) in accord with the estimate from the (e, e') results. Thus, after consideration of two-step contributions to the (p, p') cross sections a consistent picture is obtained with both experimental probes: The one-phonon components of the predominant two-phonon states are <10% for the FSS and 8%–15% for the MSS. In both cases they are small indeed.

To summarize, we have tested the nature of one- and two-phonon symmetric and mixed-symmetry 2^+ states in ^{94}Mo through high-energy-resolution inelastic electron and proton scattering experiments in a combined analysis for the first time. Results from QPM, SM, and IBM-2 calculations confirm the dominant one-phonon structure of the transitions to the first and third 2^+ state. The combined data reveal the isovector character of the transition to the one-phonon MSS within the valence shell. Excitation of the two-phonon states is sensitive to admixtures of one-phonon components, which are found to be small. Consistent conclusions can be drawn from both experimental probes when two-step contributions to the proton scattering cross sections are taken into account.

Clearly, the combination of electromagnetic and hadronic scattering is a versatile tool for detailed studies of nuclear wave functions. This work opens up a new experimental avenue for future investigations of MSSs. One obvious application would be the study of ^{92}Zr, where a description in terms of symmetric and mixed-symmetric multiphonon structures seems to fail [23,24].

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[1] F. Iachello and A. Arima, *The Interacting Boson Model* (Cambridge University Press, Cambridge, England, 1987).
[2] F. Iachello, Nucl. Phys. A **358**, 89c (1981).
[3] D. Bohle *et al.*, Phys. Lett. B **137**, 27 (1984).
[4] V. O. Nesterenko *et al.*, Phys. Rev. Lett. **83**, 57 (1999); L. Serra, A. Puente, and E. Lipparini, Phys. Rev. B **60**, R13 966 (1999); O. M. Maragó *et al.*, Phys. Rev. Lett. **84**, 2056 (2000).
[5] N. Pietralla *et al.*, Phys. Rev. Lett. **83**, 1303 (1999); **84**, 3775 (2000); C. Fransen *et al.*, Phys. Lett. B **508**, 219 (2001).
[6] C. Fransen *et al.*, Phys. Rev. C **67**, 024307 (2003).
[7] J. D. Holt *et al.*, arXiv:nucl-th/0612070 [Phys. Rev. C (to be published)].
[8] M. Pignanelli *et al.*, Phys. Lett. B **202**, 470 (1988); R. de Leo *et al.*, Phys. Lett. B **226**, 202 (1989).
[9] A. W. Lenhardt *et al.*, Nucl. Instrum. Methods Phys. Res., Sect. A **562**, 320 (2006).
[10] A. Shevchenko *et al.*, Phys. Rev. Lett. **93**, 122501 (2004).
[11] V. Yu. Ponomarev and P. von Neumann-Cosel, Phys. Rev. Lett. **82**, 501 (1999).
[12] N. Ryezayeva *et al.*, Phys. Rev. Lett. **89**, 272502 (2002).
[13] N. Lo Iudice and Ch. Stoyanov, Phys. Rev. C **62**, 047302 (2000); **65**, 064304 (2002).
[14] A. E. L. Dieperink *et al.*, Phys. Lett. B **76**, 135 (1978).
[15] O. Scholten and H. Kruse, Phys. Lett. B **125**, 113 (1983); H. Sagawa *et al.*, Nucl. Phys. A **462**, 1 (1987).
[16] J. Heisenberg and H. P. Blok, Annu. Rev. Nucl. Part. Sci. **33**, 569 (1983).
[17] J. Raynal, code DWBA05, NEA data bank NEA-1209.
[18] M. A. Franey and W. G. Love, Phys. Rev. C **31**, 488 (1985).
[19] F. Hofmann *et al.*, Phys. Lett. B **612**, 165 (2005).
[20] R. De Leo *et al.*, Phys. Rev. C **53**, 2718 (1996).
[21] P. D. Kunz, computer code CHUCK3 (unpublished).
[22] P. Schwandt *et al.*, Phys. Rev. C **26**, 55 (1982).
[23] C. Fransen *et al.*, Phys. Rev. C **71**, 054304 (2005).
[24] N. Lo Iudice and Ch. Stoyanov, Phys. Rev. C **73**, 037305 (2006).