INFLUENCE OF PHONON INTERACTION ON THE GROUND STATE OF EVEN–EVEN SPHERICAL NUCLEI

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Equations are obtained that make it possible to calculate the energy and structure of excited states described by a wave function containing single- and two-phonon components. Allowance is made for the phonon correlations in the nuclear ground state due to the interaction of the phonon excitation modes. The influence of the phonon correlations on the energies of the lowest excited states is estimated.

In the last few years the quasiparticle–phonon model of the nucleus [1] has been used in a series of studies (see, for example, [2]) in which allowance has been made for the interaction of simple single-phonon states with more complicated states in spherical nuclei. The excited states with angular momentum J and projection M were described by the wave function

$$\Psi_\nu(JM)=\Omega^+_{JM\nu}\Psi_0=\Big\{\sum_i R_i^\nu(J)\,Q^+_{JMi}+\sum_{\substack{\lambda_1 i_1\\ \lambda_2 i_2}} P^{\lambda_2 i_2}_{\lambda_1 i_1}(J_\nu)\,[Q^+_{\lambda_1\mu_1 i_1}Q^+_{\lambda_2\mu_2 i_2}]_{JM}\Big\}\Psi_0, \tag{1}$$

which contains single- and two-phonon components. The phonon creation operator $Q^+_{\lambda\mu i}$ is a superposition of two-quasiparticle (α^+ is the quasiparticle creation operator) configurations:

$$Q^+_{\lambda\mu i}=\frac{1}{2}\sum_{jj'}\{\psi^{\lambda i}_{jj'}\,[\alpha_{jm}{}^+\alpha^+_{j'm'}]_{\lambda\mu}-(-)^{\lambda-\mu}\varphi^{\lambda i}_{jj'}\,[\alpha_{j'm'}\alpha_{jm}]_{\lambda-\mu}\},$$

and Ψ_0 is the ground-state wave function of the even–even nucleus. In the quoted papers [2] the phonon vacuum was taken as the ground state, i.e., it was described by the same wave function as in the approximation of noninteracting phonons. However, the phonon vacuum is not an eigenfunction of the Hamiltonian, which contains terms describing phonon–phonon interaction. The interaction between the phonons leads to correlations in the ground state. This effect was taken into account in [3], in which the basic equations of the quasiparticle–phonon model were obtained using the formalism of two-time Green's functions, additional terms arising in the equations as a consequence. In the present paper, we shall show that under the condition of correct determination of the ground state the traditional methods on the quasiparticle–phonon model give the same additional terms. Equations will be obtained for spherical nuclei. We also give some numerical estimates of some effects that arise because the additional correlations in the ground state are taken into account.*

We determine the wave function of an excited state in the form of a linear combination of the operators $\Omega^+_{JM\nu}$ and $\Omega_{JM\nu}$:

$$\Psi_\tau(JM)=\Theta^+_{JM\tau}\Psi_0'\equiv\sum_\nu[\xi_\nu^{J\tau}\Omega^+_{JM\nu}-(-)^{J-M}\zeta_\nu^{J\tau}\Omega_{J-M\nu}]\,\Psi_0',$$

and as the ground state Ψ_0' of the nucleus we take the vacuum with respect to the operators $\Theta_{JM\tau}$: $\Theta_{JM\tau}\Psi_0'=\Psi_0'\Theta^+_{JM\tau}=0$.

If the wave functions of the ground state and the excited states are to be orthonormal, the operators $\Theta^+_{JM\tau}$ and $\Theta_{J'M'\tau'}$ must satisfy the commutation relations

* In what follows, we shall give the name "approximation II" to the case when the new correlations in the ground state are taken into account, in contrast to "approximation I," when only the correlations between the quasiparticles are taken into account.

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$$[\Theta^{+}_{JM\tau}, \Theta_{J'M'\tau'}] = \delta_{JJ'}\delta_{MM'}\delta_{\tau\tau'}, \quad [\Theta^{+}_{JM\tau}, \Theta^{+}_{J'M'\tau'}] = [\Theta_{JM\tau}, \Theta_{J'M'\tau'}] = 0. \tag{2}$$

Substituting in (2) the explicit expressions for the operators $\Theta^{+}_{JM\tau}$ and $\Theta_{JM\tau}$, we obtain an equation relating the coefficients R, P, ξ, and ζ:

$$\sum_{\nu} \{[\xi_{\nu}^{J\tau}]^2 - [\zeta_{\nu}^{J\tau}]^2\} \left\{ \sum_{i} [R_i^{\nu}(J)]^2 + \sum_{\substack{\lambda_1\lambda_2 \\ i_1 i_2}} [P_{\lambda_1 i_1}^{\lambda_2 i_2}(J\nu)]^2 \right\} = 1. \tag{3}$$

To determine the energy $\eta_{J\tau}$ of the excited states described by the wave function $\Psi_{\tau}(JM)$, we use the method of linearization of the equations of motion:

$$[H, \Theta^{+}_{JM\tau}] = \eta_{J\tau}\Theta^{+}_{JM\tau}, \quad [H, \Theta_{JM\tau}] = \eta_{J\tau}\Theta_{JM\tau}. \tag{4}$$

The part of the Hamiltonian of the quasiparticle–phonon model describing the interaction of the phonons has the form

$$H = \sum_{\lambda\mu i} \omega_{\lambda i} Q^{+}_{\lambda\mu i} Q_{\lambda\mu i} + \frac{1}{2} \sum_{\substack{\lambda_1\lambda_2\lambda \\ \mu_1\mu_2\mu \\ i_1 i_2 i}} \langle \lambda_1\mu_1\lambda_2\mu_2 | \lambda -\mu \rangle [U_{\lambda_1 i_1}^{\lambda_2 i_2}(\lambda_i) Q^{+}_{\lambda_1\mu_1 i_1} Q^{+}_{\lambda_2\mu_2 i_2} Q_{\lambda-\mu i} + (-)^{\lambda-\mu} V_{\lambda_1 i_1}^{\lambda_2 i_2}(\lambda i) Q^{+}_{\lambda_1\mu_1 i_1} Q^{+}_{\lambda_2\mu_2 i_2} Q^{+}_{\lambda\mu i} + \text{h. c.}]; \tag{5}$$

its first term corresponds to the approximation of noninteracting phonons, when the excited states are treated as purely single-, two-, ... phonon states, and the second term describes the interaction of the configurations with different numbers of phonons. In principle, the Hamiltonian of the model has a more complicated form; the expression (5) is exact in the sense that the omitted terms will not contribute to any of the expressions obtained with the chosen wave function. The coefficients $U_{\lambda_1 i_1}^{\lambda_1 i_2}(\lambda i)$ and $V_{\lambda_1 i_1}^{\lambda_2 i_2}(\lambda i)$ have the form

$$U_{\lambda_1 i_1}^{\lambda_2 i_2}(\lambda i) = (-)^{\lambda_1+\lambda_2-\lambda} \sqrt{(2\lambda_1+1)(2\lambda_2+1)} \sum_{j_1 j_2 j_3} \Bigg[\frac{\langle j_1 \| f_\lambda(r) Y_\lambda \| j_2 \rangle v_{j_1 j_2}^{(-)}}{\sqrt{2\mathcal{Y}_{\lambda i}}} \begin{Bmatrix} \lambda_1 & \lambda_2 & \lambda \\ j_2 & j_1 & j_3 \end{Bmatrix} (\psi_{j_3 j_1}^{\lambda_1 i_1} \varphi_{j_2 j_3}^{\lambda_2 i_2} + \psi_{j_2 j_3}^{\lambda_2 i_2} \varphi_{j_3 j_1}^{\lambda_1 i_1}) +$$

$$\frac{\langle j_1 \| f_{\lambda_1}(r) Y_{\lambda_1} \| j_2 \rangle v_{j_1 j_2}^{(-)}}{\sqrt{2\mathcal{Y}_{\lambda_1 i_1}}} \begin{Bmatrix} \lambda_1 & \lambda_2 & \lambda \\ j_3 & j_2 & j_1 \end{Bmatrix} (\psi_{j_3 j_1}^{\lambda_2 i_2} \psi_{j_2 j_3}^{\lambda i} + \varphi_{j_2 j_3}^{\lambda i} \varphi_{j_3 j_1}^{\lambda_2 i_2}) + \frac{\langle j_1 \| f_{\lambda_2}(r) Y_{\lambda_2} \| j_2 \rangle v_{j_1 j_2}^{(-)}}{\sqrt{2\mathcal{Y}_{\lambda_2 i_2}}} \begin{Bmatrix} \lambda_1 & \lambda_2 & \lambda \\ j_1 & j_3 & j_2 \end{Bmatrix} (\psi_{j_3 j_1}^{\lambda i} \psi_{j_2 j_3}^{\lambda_1 i_1} + \varphi_{j_2 j_3}^{\lambda_1 i_1} \varphi_{j_3 j_1}^{\lambda i}) \Bigg],$$

$$V_{\lambda_1 i_1}^{\lambda_2 i_2}(\lambda i) = (-)^{\lambda_1+\lambda_2-\lambda} \sqrt{(2\lambda_1+1)(2\lambda_2+1)} \sum_{j_1 j_2 j_3} \frac{\langle j_1 \| f_\lambda(r) Y_\lambda \| j_2 \rangle v_{j_1 j_2}^{(-)}}{\sqrt{2\mathcal{Y}_{\lambda i}}} \begin{Bmatrix} \lambda_1 & \lambda_2 & \lambda \\ j_2 & j_1 & j_3 \end{Bmatrix} (\psi_{j_3 j_1}^{\lambda_1 i_1} \varphi_{j_2 j_3}^{\lambda_2 i_2} + \psi_{j_2 j_3}^{\lambda_2 i_2} \varphi_{j_3 j_1}^{\lambda_1 i_1}),$$

where $\langle j_1 \| f_\lambda(r) Y_\lambda \| j_2 \rangle$ are the reduced matrix elements of the single-particle operator generating the phonon excitations*; $v_{j_1 j_2}^{(-)} = u_{j_1} u_{j_2} - v_{j_1} v_{j_2}$ is a combination of coefficients of a Bogolyubov transformation; $\mathcal{Y}_{\lambda i}$ are normalization coefficients. The above expressions for the coefficients $U_{\lambda_1 i_1}^{\lambda_2 i_2}(\lambda i)$ and $V_{\lambda_1 i_1}^{\lambda_2 i_2}(\lambda i)$ correspond to the case when only natural-parity phonons are taken into account in the wave function (1). The generalization to the case with both natural and anomalous parity of the phonons leads merely to a change in the signs in front of certain terms (see, for example, [4]). For the overwhelming majority of the two-quasiparticle amplitudes $\psi_{j_1 j_2}^{\lambda i} \gg \varphi_{j_1 j_2}^{\lambda i}$, and therefore the main term in the expression for $U_{\lambda_1 i_1}^{\lambda_2 i_2}(\lambda i)$ is the term $\sim\psi\psi$. The coefficients $V_{\lambda_1 i_1}^{\lambda_2 i_2}(\lambda i)$ contain only weak terms $\sim\psi\varphi$.

Under the assumption that the number of phonons in the ground state of the nucleus is small,

$$N_i^J = \frac{1}{2J+1} \left\langle \Phi_0' \middle| \sum_M Q^{+}_{JMi} Q_{JMi} \middle| \Psi_0' \right\rangle \approx 0 \tag{6}$$

the equations of motion (4) lead to a system of equations for the coefficients $\xi_{\nu}^{J\tau}$, $\zeta_{\nu}^{J\tau}$, $R_i^{\nu}(J)$ and $P_{\lambda_1 i_1}^{\lambda_2 i_2}(J\nu)$:

$$\sum_{\nu} \left\{ \xi_{\nu}^{J\tau} \left[(\omega_{Ji} - \eta) R_i^{\nu}(J) + \sum_{\substack{\lambda_1\lambda_2 \\ i_1 i_2}} U_{\lambda_1 i_1}^{\lambda_2 i_2}(Ji) P_{\lambda_1 i_1}^{\lambda_2 i_2}(J\nu) \right] + 3\zeta_{\nu}^{J\tau} \sum_{\substack{\lambda_1\lambda_2 \\ i_1 i_2}} V_{\lambda_1 i_1}^{\lambda_2 i_2}(Ji) P_{\lambda_1 i_1}^{\lambda_2 i_2}(J\nu) \right\} = 0,$$

*In the quasiparticle–phonon model, the phonon excitations are generated by separable multipole (phonons of natural parity) $V_\lambda(r_1, r_2) \sim f_\lambda(r_1) f_\lambda(r_2) Y_{\lambda\mu}(\Omega_1) Y^{*}_{\lambda\mu}(\Omega_2)$ and spin-multipole (phonons of anomalous parity) $V_{\sigma\lambda}(r_1, r_2) \sim f_\lambda(r_1) f_\lambda(r_2) [\sigma_1 Y_L(\Omega_1)]_{\lambda\mu} [\sigma_2 Y_L(\Omega_2)]^{*}_{\lambda\mu}$ forces.

$$\sum_{\nu}\left\{\xi_{\nu}^{J\tau}\left[(\omega_{\lambda_1 i_1}+\omega_{\lambda_2 i_2}-\eta)P_{\lambda_1 i_1}^{\lambda_2 i_2}(J\nu)+\frac{1}{2}\sum_{i}U_{\lambda_1 i_1}^{\lambda_2 i_2}(Ji)R_i^{\nu}(J)\right]+\frac{3}{2}\zeta_{\nu}^{J\tau}\sum_{i}V_{\lambda_1 i_1}^{\lambda_2 i_2}(Ji)R_i^{\nu}(J)\right\}=0,$$

$$\sum_{\nu}\left\{\zeta_{\nu}^{J\tau}\left[(\omega_{Ji}+\eta)R_i^{\nu}(J)+\sum_{\substack{\lambda_1\lambda_2\\ i_1 i_2}}U_{\lambda_1 i_1}^{\lambda_2 i_2}(Ji)P_{\lambda_1 i_1}^{\lambda_2 i_2}(J\nu)\right]+3\xi_{\nu}^{J\tau}\sum_{\substack{\lambda_1\lambda_2\\ i_1 i_2}}V_{\lambda_1 i_1}^{\lambda_2 i_2}(Ji)P_{\lambda_1 i_1}^{\lambda_2 i_2}(J\nu)\right\}=0, \tag{7}$$

$$\sum_{\nu}\left\{\zeta_{\nu}^{J\tau}\left[(\omega_{\lambda_1 i_1}+\omega_{\lambda_2 i_2}\mp\eta)P_{\lambda_1 i_1}^{\lambda_2 i_2}(J\nu)+\frac{1}{2}\sum_{i}U_{\lambda_1 i_1}^{\lambda_2 i_2}(Ji)R_i^{\nu}(J)\right]+\frac{3}{2}\xi_{\nu}^{J\tau}\sum_{i}V_{\lambda_1 i_1}^{\lambda_2 i_2}(Ji)R_i^{\nu}(J)\right\}=0,$$

$$i=1,2,\ldots,n_1;\quad \tau,\nu=1,2,\ldots,(n_1+n_2),$$

where n_1 and n_2 are, respectively, the total numbers of single- and two-phonon components in the wave function (1). The system (7) can also be obtained by using the variational procedure

$$\delta\{\langle\Psi_0'|\Theta_{JM\tau}H\Theta_{JM\tau}^{+}|\Psi_0'\rangle-\langle\Psi_0'|H|\Psi_0'\rangle-\eta\langle\Psi_0'|\Theta_{JM\tau}\Theta_{JM\tau}^{+}|\Psi_0'\rangle\}=0,$$

the variables R, P, ξ, and ζ being varied independently.

By means of the second and fourth equations of the system (7) we eliminate $P_{\lambda_1 i_1}^{\lambda_2 i_2}(J\nu)$ from the other two equations and go over to the new variables

$$\mathcal{P}_i^{J\tau}=\sum_{\nu}\xi_{\nu}^{J\tau}R_i^{\nu}(J);\quad \mathcal{R}_i^{J\tau}=\sum_{\nu}\zeta_{\nu}^{J\tau}R_i^{\nu}(J).$$

Then the system (7) goes over into a system of linear homogeneous equations in the new variables:

$$M\cdot\mathbf{L}=\begin{pmatrix} \mathrm{I} & & \mathrm{II} \\ & M_{kk'} & \\ \mathrm{III} & & \mathrm{IV}\end{pmatrix}\begin{pmatrix}\mathcal{P}_1\\ \mathcal{P}_2\\ \vdots\\ \mathcal{P}_{n_1}\\ \mathcal{R}_1\\ \mathcal{R}_2\\ \vdots\\ \mathcal{R}_{n_1}\end{pmatrix}=0. \tag{8}$$

The rank of the matrix M is 2_{n_1}, and its elements (depending on the values of the indices k and k′) can be calculated in accordance with the following formulas.

I. $1\leqslant k,\quad k'\leqslant n_1$:

$$M_{kk'}=(\omega_{Jk}-\eta)\delta_{kk'}-\frac{1}{2}\sum_{\substack{\lambda_1\lambda_2\\ i_1 i_2}}\left[\frac{U_{\lambda_1 i_1}^{\lambda_2 i_2}(Jk)U_{\lambda_1 i_1}^{\lambda_2 i_2}(Jk')}{(\omega_{\lambda_1 i_1}+\omega_{\lambda_2 i_2}-\eta)}+9\frac{V_{\lambda_1 i_1}^{\lambda_2 i_2}(Jk)V_{\lambda_1 i_1}^{\lambda_2 i_2}(Jk')}{(\omega_{\lambda_1 i_1}+\omega_{\lambda_2 i_2}+\eta)}\right].$$

II. $1\leqslant k\leqslant n_1,\quad n_1+1\leqslant k'\leqslant 2n_1$:

$$M_{kk'}=-\frac{3}{2}\sum_{\substack{\lambda_1\lambda_2\\ i_1 i_2}}\left[\frac{U_{\lambda_1 i_1}^{\lambda_2 i_2}(Jk)V_{\lambda_1 i_1}^{\lambda_2 i_2}(Jk')}{(\omega_{\lambda_1 i_1}+\omega_{\lambda_2 i_2}-\eta)}+\frac{U_{\lambda_1 i_1}^{\lambda_2 i_2}(Jk')V_{\lambda_1 i_1}^{\lambda_2 i_2}(Jk)}{(\omega_{\lambda_1 i_1}+\omega_{\lambda_2 i_2}+\eta)}\right]$$

III. $n_1+1\leqslant k\leqslant 2n_1,\quad 1\leqslant k'\leqslant n_1$:

$$M_{kk'}=-\frac{3}{2}\sum_{\substack{\lambda_1\lambda_2\\ i_1 i_2}}\left[\frac{U_{\lambda_1 i_1}^{\lambda_2 i_2}(Jk)V_{\lambda_1 i_1}^{\lambda_2 i_2}(Jk')}{(\omega_{\lambda_1 i_1}+\omega_{\lambda_2 i_2}+\eta)}+\frac{U_{\lambda_1 i_1}^{\lambda_2 i_2}(Jk')V_{\lambda_1 i_1}^{\lambda_2 i_2}(Jk)}{(\omega_{\lambda_1 i_1}+\omega_{\lambda_2 i_2}-\eta)}\right].$$

IV. $n_1+1\leqslant k,\quad k'\leqslant 2n_1$:

$$M_{kk'}=(\omega_{Jk}+\eta)\delta_{kk'}-\frac{1}{2}\sum_{\substack{\lambda_1\lambda_2\\ i_1 i_2}}\left[\frac{U_{\lambda_1 i_1}^{\lambda_2 i_2}(Jk)U_{\lambda_1 i_1}^{\lambda_2 i_2}(Jk')}{(\omega_{\lambda_1 i_1}+\omega_{\lambda_2 i_2}+\eta)}+9\frac{V_{\lambda_1 i_1}^{\lambda_2 i_2}(Jk)V_{\lambda_1 i_1}^{\lambda_2 i_2}(Jk')}{(\omega_{\lambda_1 i_1}+\omega_{\lambda_2 i_2}-\eta)}\right].$$

The condition of existence of a nontrivial solution of the system (8) leads to the equation $\det\|M(\eta)\|=0$, and solving this we obtain the energy spectrum $\eta_{J\tau}$ of the excited states described by the wave function $\Psi_{\tau}(JM)$. Substituting then in the system (7) the value $\eta=\eta_{J\tau}$ and using the normalization condition (3), we find the coefficients $\xi_{\nu}^{J\tau}$, $\zeta_{\nu}^{J\tau}$, $R_i^{\nu}(J)$, and $P_{\lambda_1 i_1}^{\lambda_2 i_2}(J\nu)$, i.e., we determine the wave function itself. Note that in the limiting case $\xi_{\nu}^{J\tau}\to\delta_{\nu,\tau}$, $\zeta_{\nu}^{J\tau}\to 0$ Eqs. (3), (7), and (8) go over into the equations corresponding to approximation I

TABLE 1

	^{116}Sn	^{120}Te	^{144}Sm	^{146}Sm		^{116}Sn	^{120}Te	^{144}Sm	^{146}Sm
U	0,158	0,615	0,690	0,941	$\eta_1^{(II)}$	1,21	0,19	1,50	0,59
V	0,099	0,255	0,165	0,304	2ω	2,46	1,44	3,32	2,26
ω	1,23	0,72	1,66	1,13	$\eta_2^{(I)}$	2,47	1,64	3,45	2,57
$\eta_1^{(I)}$	1,22	0,52	1,53	0,82	$\eta_2^{(II)}$	2,46	1,53	3,43	2,47

(in this case, the rank of the matrix $M(\eta)$ is halved, and the matrix itself has the form of I if we set $V_{\lambda_1 i_1}^{\lambda_2 i_2}(\lambda k)=0$). Allowance for the phonon correlations in the ground state leads to the pole terms $\sim(\omega_{\lambda_1 i_1}+\omega_{\lambda_2 i_2}-\eta)^{-1}$ in the equations being augmented by non-pole terms $\sim(\omega_{\lambda_1 i_1}+\omega_{\lambda_2 i_2}+\eta)^{-1}$. Similar non-pole terms are also present in the equations obtained in the framework of the theory of finite Fermi systems when allowance is made for the correlations in the ground state due to the interaction of the 1p1h and 2p2h configurations [5].

We estimate the number of phonons in the ground state of the nucleus, which was assumed to be small in the derivation of Eqs. (7) and (8). For this, we first express the wave function Ψ_0' of the new ground state in terms of the wave function Ψ_0 of the phonon vacuum. Applying the procedure described in detail in the monograph [6] (pp. 382-384), we obtain

$$\Psi_0'=\frac{1}{\sqrt{N}}\exp\left\{\frac{1}{2}\sum_{JM\tau}\sum_{\nu\nu'}(\xi^{-1})_{\nu'}{}^{J\tau}\zeta_{\nu}{}^{J\tau}(-)^{J-M}\overset{+}{\Omega}_{JM\nu}\overset{+}{\Omega}_{J-M\nu'}\right\}\Psi_0, \tag{9}$$

where the matrix $(\xi^{-1})_\nu{}^{J\tau}$ is the inverse of $\xi_\nu{}^{J\tau}$ (i.e., $\sum_\nu(\xi^{-1})_\nu{}^{J\tau}\xi_\nu{}^{J\tau'}=\delta_{\tau\tau'}$), and N is a normalization coefficient, determined by the condition $\langle\Psi_0'|\Psi_0'\rangle=1$. Substituting (9) in the definition of N_i^J, we readily obtain an expression for the total number of phonons with angular momentum J in the ground state Ψ_0':

$$N^J=\sum_i N_i^J=\sum_\tau\left\{2\sum_\nu(\zeta_\nu{}^{J\tau})^2+\sum_i\left(\sum_\nu R_i^\nu(J)\zeta_\nu{}^{J\tau}\right)^2\right\},$$

i.e., the assumption (6) is well satisfied in the case when the coefficients $\zeta_\nu{}^{J\tau}$ are small.

The solution of the obtained equations is a complicated computational problem, since it is necessary to diagonalize a matrix of high rank $M(\eta)$ that depends nonlinearly on the variable η. In the present paper, we do not attempt such a task. However, to obtain a certain quantitative picture of the part played by the phonon correlations in the ground state, we consider the simplest case. From the complete set of phonons in the definition of the operator $\Omega_{JM\nu}^+$, we take into account only the lowest 2^+ phonon: $Q_{JMi}^+=Q_{\lambda_1\mu_1 i_1}^+=Q_{\lambda_2\mu_2 i_2}^+=Q_{2_1^+}^+\equiv Q^+$ (with energy $\omega\equiv\omega_{2_1^+}$). In this special case, all the equations can be solved analytically. In the approximation of noninteracting phonons, the system has two excited states with energies ω ($\Psi=Q^+\Psi_0$) and 2ω ($\Psi=Q^+Q^+\Psi_0$). Allowance for the interaction of the phonons in the wave function of the excited states (approximation I),

$$\Psi_\nu=\Omega_\nu^+\Psi_0=\{R^\nu Q^+ + P(\nu)[Q^+Q^+]\}\Psi_0,\quad \nu=1, 2,$$

leads to the well-known effect for the two-level problem – repulsion of the roots:

$$\eta_1^{(I)}=\frac{3\omega-\sqrt{\omega^2+2U^2}}{2},\quad \eta_2^{(I)}=\frac{3\omega+\sqrt{\omega^2+2U^2}}{2}.$$

The first solution $\eta_1^{(I)}$ is the lowest 2^+ state, the single-phonon component being predominant in its wave function; for the second solution $\eta_2^{(I)}$, the two-phonon component is predominant.

Allowance for the phonon–phonon interaction in the ground state (approximation II) gives the solutions

$$\eta_1^{(II)}=\sqrt{\frac{5\omega^2+U^2-9V^2-3\omega\sqrt{\omega^2+2(U^2-V^2)}}{2}},\quad \eta_2^{(II)}=\sqrt{\frac{5\omega^2+U^2-9V^2+3\omega\sqrt{\omega^2+2(U^2-V^2)}}{2}}.$$

We calculated $\eta_{1,2}^{(I)}$ and $\eta_{1,2}^{(II)}$ for some nuclei, using the program GIRES [4] and taking the parameters of the model such as to satisfy approximately the condition $\eta_1^{(I)} = \omega_{2_1^+}^{\exp}$. The obtained results are given in Table 1 (all values are given in MeV). The strength of the interaction of the single- and two-phonon components can be judged from the value of the coefficient U. In nuclei in which U is small (as a rule, these are magic and near-magic nuclei) we have the solution $\eta_1^{(I)} \simeq \omega$ and $R \simeq 1$ (and, respectively, $\eta_2^{(I)} \simeq 2\omega$ and $P(2) \approx 1$) – the isotope ^{118}Sn can serve as an example. In other nuclei, the interaction of the single- and two-phonon components leads to appreciable mixing of them and a significant decrease in the energy of the lowest excitation (see, for example, ^{146}Sm). The energy of the second state is increased at the same time, though not so much. Allowance for the phonon correlations in the ground state results in an even larger sinking of the lowest excited state. The energy of the second state is also decreased compared with $\eta_2^{(I)}$ (at the same time, for all nuclei we obtain $2\omega < \eta_2^{(II)} < \eta_2^{(I)}$). The influence of the phonon correlations on the position of the lowest levels is most important in nuclei with large values of the coefficients U and V. The values of these coefficients are determined by the extent to which the phonons are collectivized. Therefore, in spherical nuclei far in N and Z from the magic numbers, whose 2_1^+ and 3_1^- states have a low energy and are accordingly strongly collectivized, the phonon correlations in the ground state play a significant part. On the other hand, in magic and near-magic nuclei their influence can be ignored.

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