Description of giant multipole resonances in spherical nuclei

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Characteristics of isoscalar quadrupole resonances in a number of nuclei are calculated within the framework of the quasiparticle-phonon nuclear model. This model is compared with a nuclear field theory. The influence of the corrections, which arise in the exact inclusion of the Pauli principle in the two-phonon components of the wave functions, on the resonance characteristics is clarified.

INTRODUCTION

At present extensive experimental material on characteristics of giant multipole resonances (GMR) has been accumulated and it is continuously enriched by new data. The most complete systematics of characteristics of the GMR can be found in the reviews of Refs. 1 and 2. During recent years one has achieved a significant progress in describing the widths of the GMR. This is so because of the understanding of the key role played by the inclusion of the correlation between the simple one-phonon states and complicated configurations in describing characteristics of the GMR. In 1968–1971 in Refs. 3 and 4 it was indicated that in the fragmentation of one-phonon states and, thus, in the formation of the width of the GMR in spherical nuclei, an important role is played by the quasiparticle-phonon interaction. Now this point of view is commonly accepted. A review of the main theoretical papers devoted to a description of the GMR is given in Ref. 5. We note that Ref. 6 is one of the first in which quantitative results have been obtained for characteristics of electric GMR with inclusion of the quasiparticle-phonon interaction. The equations with inclusion of the quasiparticle-phonon interaction describing the fragmentation of one-quasiparticle and one-phonon states have been obtained in Refs. 7 and 8. In order to calculate the widths of the GMR one must take into account the widths $\Gamma^{\downarrow}$ related to the inclusion of $2p$–$2h$ configurations and also the widths $\Gamma^{\uparrow}$ related to particle emission. The latter, however, are small in comparison with the former in medium and heavy nuclei, and therefore the main problem is calculation of $\Gamma^{\downarrow}$ with inclusion of the $2p$–$2h$ configurations. In the quasiparticle-phonon model of the nucleus[9] (QPM) this is carried out by introducing two-phonon components in the wave function. General equations of this type are given in Refs. 9 and 10. A detailed study of characteristics of the GMR of many spherical nuclei (magic and not magic) with $58 \leqslant A \leqslant 208$ within the framework of the QPM is performed in Refs. 11–14. In Refs. 15–21 in the description of electric GMR in spherical nuclei the $2p$–$2h$ configurations have been included. As a rule, the calculations are restricted to doubly magic nuclei.

In the present paper we continue the study of the GMR of the electric type: we calculate characteristics of isocalar quadrupole resonances, compare the QPM with nuclear field theory[16,19] (TNP), and clarify the influence of corrections, which arise in the exact inclusion of the Pauli principle in two-phonon components of the wave functions of the GMR, on the probabilities and excitations of the GMR.

1. MAIN FORMULAS AND DETAILS OF CALCULATIONS

In the QPM for the calculation of the fragmentation the one-phonon states, the wave functions of which are calculated in the random-phase approximation, are taken as a basis. Here in the process of construction of the basis all parameters of the model are fixed. The method of strength functions is used and one calculates directly the reduced transition probabilities, the transition densities, the scattering cross sections, and other nuclear characteristics without solving the corresponding secular equations.[9]

The QPM Hamiltonian contains terms which describe the mean field of the nucleus in the form of the Woods-Saxon potential, the interactions leading to pairing, and the multipole-multipole and spin-multipole–spin-multipole isoscalar and isovector interactions, including charge exchange. The general characteristics of the Hamiltonian are given in Ref. 9. In the present paper we confine ourselves to consideration of the particle-hole channel.

We write the wave function of the excited state of the even-even spherical nucleus in the form

$$\Psi_\nu(JM)=\Big\{\sum_i R_i(J\nu)Q^+_{JMi}+\sum_{\substack{\lambda_1 i_1\\ \lambda_2 i_2}} P^{\lambda_1 i_1}_{\lambda_2 i_2}(J\nu)\,[Q^+_{\lambda_1 i_1}Q^+_{\lambda_2 i_2}]_{JM}\Big\}\Psi_0, \tag{1}$$

where Ψ_0 is the phonon vacuum and the wave function of the ground state. Calculating the average value of the QPM Hamiltonian over the states (1) and using the variational principle, one can obtain a system of equations for determination of the coefficients R and P. In Ref. 10 a secular equation in the space of two-phonon states was obtained and it was shown that in this case a large set of diagrams is summed. Here many diagrams are taken into account which weakly influence the fragmentation of the one-phonon states. If one discards them one is able to go over to approximate equations. In this case the secular equation is written in the space of one-phonon states in the form

$$\mathscr{F}(\eta_\nu)=\det\Big\|(\omega-\eta_\nu)\delta_{ii'} -\frac{1}{2}\sum_{\substack{\lambda_1 i_1\\ \lambda_2 i_2}} \frac{U^{\lambda_1 i_1}_{\lambda_2 i_2}(Ji)\,U^{\lambda_1 i_1}_{\lambda_2 i_2}(Ji')\,\{1+{}^1/_2\mathscr{K}^J(\lambda_1 i_{1'},\lambda_2 i_2)\}}{\omega_{\lambda_1 i_1}+\omega_{\lambda_2 i_2}+\Delta\omega(\lambda_1 i_1,\lambda_2 i_2)-\eta_\nu}\Big\|=0, \tag{2}$$

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where

$$\mathcal{K}^J(\lambda_1 i_1, \lambda_2 i_2) = \sum_{j_1 j_2 j_3 j_4} (-1)^{j_2+j_4-J}(2\lambda_1+1)(2\lambda_2+1)$$

$$\times \begin{Bmatrix} j_1 & j_2 & \lambda_2 \\ j_4 & j_1 & \lambda_1 \\ \lambda_1 & \lambda_2 & J \end{Bmatrix} [\psi_{j_3 j_4}^{\lambda_1 i_1} \psi_{j_1 j_4}^{\lambda_1 i_1} \psi_{j_3 j_2}^{\lambda_2 i_2} \psi_{j_1 j_3}^{\lambda_2 i_2} - \varphi_{j_3 j_4}^{\lambda_1 i_1} \varphi_{j_1 j_4}^{\lambda_1 i_1} \varphi_{j_3 j_2}^{\lambda_2 i_2} \varphi_{j_1 j_2}^{\lambda_2 i_2}], \quad (3)$$

and $\psi_{j_1 j_2}^{\lambda_1 i_1}$ and $\varphi_{j_1 j_2}^{\lambda_1 i_1}$ are phonon amplitudes determined from the solution of the equation of the random-phase approximation: $\Delta\omega(\lambda_1 i_1, \lambda_2 i_2)$ is the shift of the two-phonon pole, the explicit form of which is given in Ref. 10. In Ref. 22 it is shown that one must include this shift in the calculations of the low-lying states. In the calculation of the GMR the shift is small and can be neglected.

The wave function (1) is normalized as follows:

$$\langle \Psi_\nu^*(JM)\Psi_\nu(JM)\rangle = \sum_i (R_i(J\nu))^2$$

$$+2\sum_{\substack{\lambda_1 i_1 \\ \lambda_2 i_2}} (P_{\lambda_2 i_2}^{\lambda_1 i_1}(J\nu))^2 \{1 + {}^1/_2 \mathcal{K}^J(\lambda_1 i_1, \lambda_2 i_2)\} = 1. \quad (4)$$

In the space of one-phonon states in the QPM the diagrams shown in Figs. 1a, 1b, and 1c are included. Here the diagrams in Fig. 1b and 1c correspond to corrections arising in the exact inclusion of the Pauli principle in the two-phonon components of the wave function (1). As is shown in Ref. 19, in numerical calculations of the TNP one takes into account only those cases of the diagram in Fig. 1a, show in Fig. 1c, when one of the intermediate phonons is replaced by two-quasiparticle states and the other one is represented by strongly collectivized phonons. We note that in all contemporary theoretical calculations with inclusion of the $2p$–$2h$ configurations[15–20] in practical evaluations one includes only the diagrams of the type of Fig. 1a (see the discussion of this point in Refs. 5 and 19).

We now consider Eq. (2). The rank of the determinant is equal to the number of one-phonon states in the first term of the wave function (1). The factor $\{1 + (1/2)\mathcal{K}^J(\lambda_1, i_1, \lambda_2, i_2)\}$ originates from the inclusion of the Pauli principle in two-phonon components of the wave function (1). For the components which are strictly forbidden by the Pauli principle we have $\mathcal{K}^J = -2$ and the corresponding terms are excluded from the sums over $\lambda_1 i_1 \lambda_2 i_2$ in Eqs. (2) and (4). If one puts $\mathcal{K}^J = 0$, then the secular equation (2) goes over into the well known equation used in Refs. 11–14 in the study of fragmentation of the one-phonon and two-quasiparticle states. Here we used the program GIRES,[23] in which, when numerically solving Eq. (2) with $\mathcal{K}^J = 0$, the two-phonon components with two noncollective phonons, which violate the Pauli principle, were excluded. In this procedure the renormalization of the two-phonon poles, which can be significant for some low-lying states, has not been taken into account and a part of components allowed by the Pauli principle was excluded. In what follows we shall compare, using a numerical example, the results of aproximate and exact inclusion of the Pauli principle.

In the QPM calculations one often uses the method of strength functions, which allows one to calculate directly without solving Eq. (2) the distribution of the strength of certain physical quantities averaged over the energy interval Δ. In general the strength function is

$$b(\lambda, \eta) = \frac{1}{\pi} \mathrm{Im}\left\{ \sum_{ii'} \mathcal{A}_{ii'}{}' \eta + i\frac{\Delta}{2}\Big) \mathcal{M}_{Ji} \mathcal{M}_{Ji'} / \mathcal{F}\left(\eta + i\frac{\Delta}{2}\right)\right\}, \quad (5)$$

where $\mathcal{A}_{ii'}$, is the algebraic adunct of the determinant (2) for complex values of energy. The specific form of the matrix elements $\mathcal{M}_{Ji}$ depends on the process considered.[9] In the present paper $\mathcal{M}_{Ji}$ correspond to matrix elements of the $E\lambda$ transitions from the ground state to the one-phonon states.

The method of choice of the constants of the QPM Hamiltonian is described in Refs. 9 and 11–14. In the present paper we use the same set of parameters as in Refs. 12, 14, and 24.

2. RESULTS OF THE CALCULATIONS

We now study the difference between the exact inclusion of the Pauli principle in the two-phonon components of the wave function (1) and the approximate procedure used in the QPM.[10–14] The strength distribution of the isoscalar giant quadrupole resonance in ^{118}Sn is shown in Fig. 2. In the calculations of the strength functions $b(E2,\eta)$ with exact in-

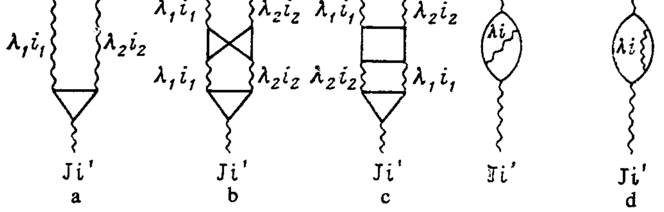

FIG. 1. Diagrams included in the QPM (a, b, and c) and in the TNP (d).

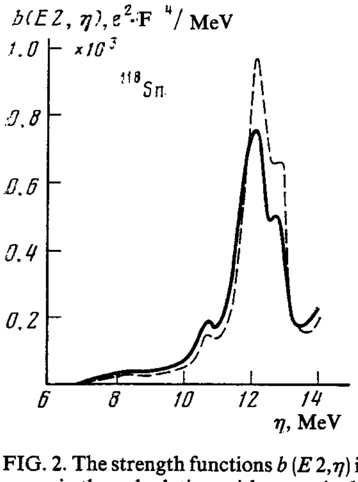

FIG. 2. The strength functions $b(E2,\eta)$ in ^{118}Sn ($\Delta = 0.2$ MeV). The solid curve is the calculation with exact inclusion of the Pauli principle, the dashed one is the calculation with approximate inclusion of the Pauli principle.

clusion of the Pauli principle we use the function $\mathcal{F}(\eta)$ defined by Eq. (2) with $\Delta\omega = 0$, because for the GMR the renormalization of the two-phonon poles is small.[22] As is evident from Fig. 2, the exact inclusion of the Pauli principle slightly reduces the quantity $b(E2,\eta)$ in the region of the maximum. Here the integrated strength in the interval from 5 to 14 MeV is reduced by only 5%. On the whole the approximate procedure of exclusion of the states forbidden by the Pauli principle realized in Ref. 23 essentially turns out to be equivalent to the exact inclusion of the diagrams 1b and 1c; however in the former case the calculations are much simpler. For the states not strictly forbidden by the Pauli principle the renormalization of the interaction, because of the corrections arising when taking into account the fermion structure of the two-phonon states, turns out to be weak. Disregarding the requirements following from the Pauli principle can lead to arising of a large number of false two-phonon components in the wave function (1).

We shall now consider how strongly the results of the QPM calculations with summation of the diagrams of the type of Figs. 1a, 1b, and 1c differ from the results of the TNP calculations[16] when the diagrams shown in Fig. 1d are summed. In the TNP in the diagrams of Fig. 1a one of the intermediate phonons is represented by a collective phonon and the other one by a noncollective phonon (essentially a two-quasiparticle state). There is a certain arbitrariness in the division of the phonons into collective and noncollective or weakly collective ones. We assume the one-phonon state to be collective if in the normalization of its wave function there are no components which make a contribution larger than 50%. The states not satisfying this criterion are regarded as weakly collective ones. The results of the calculations in the QPM and TNP for the giant isovector dipole resonance in ^{116}Sn ($\Delta = 0.2$ MeV) are shown in Fig. 3 and for the isoscalar quadrupole resonance in ^{118}Sn ($\Delta = 0.5$ MeV) and in ^{208}Pb ($\Delta = 0.2$ MeV) in Fig. 4. It is seen from these figures that the QPM and TNP give close results, although in the QPM calculations the strength of the GMR is fragmented

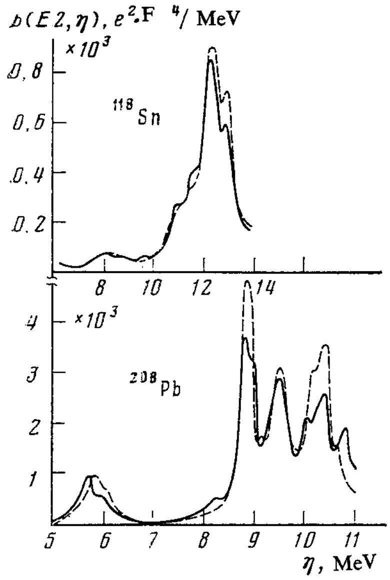

FIG. 4. The strength functions for the isoscalar quadrupole resonance in ^{118}Sn and ^{208}Pb. The designation of curves is the same as in Fig. 3.

slightly more strongly. Although the amplitude of the peaks obtained in inclusion of the diagrams of Fig. 1d is slightly larger, on the whole the two calculations give the same gross structure in the distribution of the $E2$ strength. This shows that in the TNP the most important diagrams are taken into account. We note that the conclusions drawn remain valid if for the criterion of the collectivity of the phonon one takes the value 30%. Here the values of the strength functions change very little.

One of the advantages of the QPM is a possibility to perform calculations in open-shell nuclei. This is demonstrated in Ref. 12 in the calculation of the total cross sections for dipole photoabsorption in ^{124}Te and ^{140}Ce. The results of the calculation of the strength function $b(E2,\eta)$ in ^{114}Nd are presented in Fig. 5. The calculations in the random-phase approximation shown in the same figure demonstrate that the total strength of the isoscalar $E2$ resonance is concentrat-

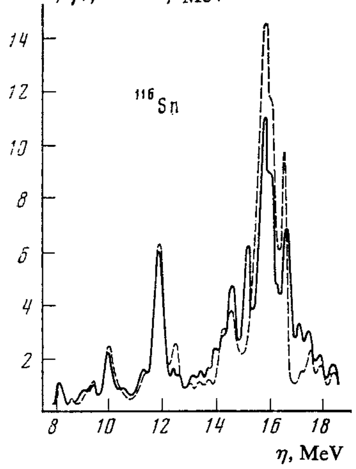

FIG. 3. The strength function for the giant dipole resonance in ^{116}Sn. The solid curve is the calculation with inclusion of the diagrams 1a, 1b, and 1c, and the dashed one is the calculation with inclusion of the diagrams 1d.

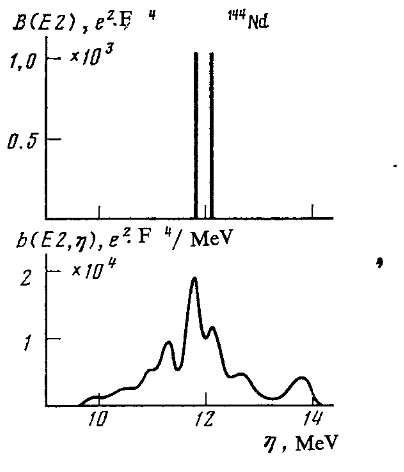

FIG. 5. The strength function $b(E2,\eta)$ and the quantities $B(E2)$, calculated in the random-phase approximation for ^{144}Nd.

TABLE I. Characteristics of giant quadrupole resonances.

Nucleus	Experiment				Calculation		
	E_x, MeV	Γ, MeV	EWSR(%)	Ref.	E_x, MeV	Γ, MeV	EWSR(%)
^{118}Sn	~12	–	–	[25]	12.1	2.1	51.7
^{144}Nd	–	–	–	–	11,9	2.1	49,4
^{208}Pb	10.9±0,3	2,4±0.4	80	[26]	9,6	1.8	66

ed in two solutions at excitation energies around 12 MeV. The inclusion of the correlation with two-phonon states leads to a redistribution of the resonance strength in the energy interval 10 to 14 MeV. This figure clearly demonstrates the arising of the width of the $E2$ resonance as the result of the fragmentation of the one-phonon states. The integrated characteristics of the isoscalar giant quadrupole resonances are presented in Table I. The width of the $E2$ resonances have been calculated with the standard formula for a Gaussian distribution (see Refs. 5 and 6). In the nuclei considered in the region of the $E2$ resonance 50 to 70% of the isoscalar energy-weighted sum rule (EWSR) is saturated. The calculated values of the energies are rather close to the experimental systematics for energies of the isoscalar $E2$ resonance $E_x \sim 65A^{-1/3}$ MeV. The isoscalar $E2$ resonance in ^{208}Pb is the one which has been best studied. However, at present it is not finally established which part of the model-independent EWSR is saturated in the region of the resonance. The experimental papers[27,28] devoted to study of the excitation of the $E2$ resonance in (α,α') and (d,d') reactions in ^{208}Pb give $E_x = 10.5$–10.9 MeV and the (60–80)% saturation of the EWSR. In electron scattering by ^{208}Pb a large number of 2^+ states in the interval of excitation energies 8–12 MeV have been discovered.[29] The states of the fine structure are concentrated in the groups around the energies $E_x \approx 8.9$, 10.2, and 10.6 MeV. The experimentally measured strength of the $E2$ resonance constitutes (29^{+11}_{-8})% of the EWSR. As is evident from Fig. 4, the strength function in ^{208}Pb calculated by us also exhibits substructures at the energies 8.8, 9.5, 10.4, and 10.8 MeV. However, one should note that in the calculations with $\Delta = 0.2$ MeV the fine structure of the peaks is washed out. In Fig. 6 the strength function $b(E2,\eta)$ calculated with $\Delta = 0.05$ MeV is shown. As is evident from the figure, the quadrupole resonance in ^{208}Pb has a rich fine structure. The gross structure of the $E2$ resonance is caused by the correlation between the one-phonon state and the two-phonon states whch include collective phonons. The existence of the fine structure is to a large extent caused by the two-phonon states constructed from the noncollective phonons. The theoretical calculations[14–20] predict an EWSR saturation of the order of 70%. Recently new data[30] have appeared on the excitation of the GMR in ^{208}Pb in inelastic ^{3}He scattering. In these experiments it was discovered that in the region of the $E2$ resonance also the hexadecapole resonance is located; this exhausts the EWSR by (23–29)%. For the quadrupole resonance the saturation of the EWSR is estimated as (32–50)%. Our calculations show that for energies around $E_x = 10.2$ MeV several 4^+ states are concentrated, saturating the EWSR by 18%. Similar results have been obtained also in the TNP calculations.[19] Thus, the experimental data on the $E4$ resonance agree reasonably with theoretical calculations; however, the question of the strength of the $E2$ resonance remains open.

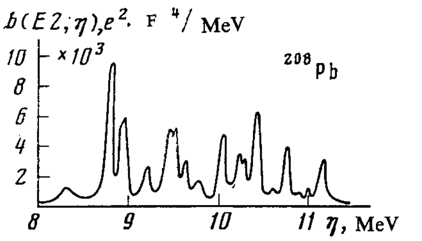

FIG. 6. The strength function $b(E2,\eta)$ for ^{208}Pb calculated with $\Delta = 0.05$ MeV.

CONCLUSIONS

In the present paper we have demonstrated the possibilities of the QPM in describing the characteristics of the GMR in magic and nonmagic nuclei. It is shown that the calculations within the framework of nuclear field theory and of the QPM give close results, although in the latter a more extended class of diagrams is included. Our calculations have also shown that the approximate procedure of inclusion of the Pauli principle extensively used in the QPM turns out to be essentially equivalent to the exact inclusion of the Pauli principle for the two-phonon components. In the description of the characteristics of the GMR different theoretical schemes give similar results; however, in a number of cases there still exists a noticeable discrepancy between the data from various reactions and theoretical calculations.

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[1)]Institute of Nuclear Research and Nuclear Energy, Bulgarian Academy of Sciences, Sofia.

[1]B. L. Berman and S. C. Fultz, Rev. Mod. Phys. **47**, 713 (1975).
[2]J. Speth and A. Van der Woude, Rep. Prog. Phys. **44**, 719 (1981). K. Goeke and J. Speth, Ann. Rev. Nucl. Part. Sci. **32**, 65 (1982).
[3]V. G. Soloviev, Nuclear Structure, Dubna, Symp. 1968. IAEA, Vienna, 1968, p. 101. V. G. Soloviev, Izv. Akad. Nauk SSSR, Ser. Fiz. **35**, 666 (1971) [Bull. Acad. Sci. USSR, Phys. Ser.].
[4]V. G. Soloviev, Proc. Intern. Conf. on Neutron Capture Gamma Ray Spectroscopy. Reactor Centrum Netherlands, Petten, Netherlands, 1975, p. 99.
[5]G. F. Bertsch, P. F. Bortignon, and R. A. Broglia, Rev. Mod. Phys. **55**, 287 (1983).
[6]A. I. Vdovin, V. G. Soloviev, and Ch. Stoyanov, JINR R4-10033, Dubna, 1976.

[7]V. G. Soloviev, Theory of Complex Nuclei, Pergamon Press, Oxford, 1976. V. G. Soloviev and L. A. Malov, Nucl. Phys. **A196**, 433 (1972). V. G. Solov'ev, Teor. Mat. Fiz. **17**, 90 (1973) [Theor. Math. Phys. (USSR)]. A. I. Vdovin and V. G. Solov'ev, Teor. Mat. Fiz. **19**, 275 (1974) [Theor. Math. Phys. (USSR)].
[8]A. I. Vdovin, G. Kyrchev, and Ch. Stoyanov, Teor. Mat. Fiz. **21**, 137 (1974) [Theor. Math. Phys. (USSR)]. G. Kyrchev and V. G. Solov'ev, Teor. Mat. Fiz. **22**, 244 (1975) [Theor. Mat. Phys. (USSR)].
[9]V. G. Solov'ev, Fiz. Elem. Chastits At. Yadra **9**, 860 (1978) [Sov. J. Part. Nucl. **9**, 343 (1978)]; Nucleonika **23**, 1149 (1979). A. I. Vdovin and V. G. Solov'ev, Fiz. Elem. Chastits At. Yadra **14**, 237 (1983) [Sov. J. Part. Nucl. **14** 99 (1983)]. V. V. Voronov and V. G. Solov'ev, Fiz. Elem. Chastits At. Yadra **14**, 1380 (1983) [Sov. J. Part. Nucl. **14**, 583 (1983)].
[10]V. G. Soloviev and V. V. Voronov, JINR E4-83-58, Dubna, 1983.
[11]V. G. Soloviev, Ch. Stoyanov, and A. I. Vdovin, Nucl. Phys. **A288**, 376 (1977).
[12]V. G. Soloviev, Ch. Stoyanov, and V. V. Voronov, Nucl. Phys. **A304**, 503 (1978).
[13]V. G. Soloviev, Proc. Intern. Conf. on Nucl. Structure, Tokyo, 1977; J. Phys. Soc. Japan **44**, 324 (1978). V. G. Soloviev, Proc. Sendai Conf. on Electro- and Photoexcitations, Ed. by Y. Kawazoe, Tomizawa Sendai, Japan, 1977, p. 111. V. G. Solov'ev, Tr. IV seminara "Élektromagnitnye vzaimodeĭstviya yader pri malykh i srednikh énergiyakh" [Proc. IV Seminar "Electromagnetic Interactions of Nuclei at Low and Medium Energies], 1977. Moscow, Nauka, 1979, p. 22.
[14]V. G. Soloviev, Ch. Stoyanov, and V. V. Voronov, Nucl. Phys. **A399**, 141 (1983).
[15]J. S. Dehesa *et al.*, Phys. Rev. **C15**, 1858 (1977).
[16]G. F. Bertsch *et al.*, Phys. Lett. **80B**, 161 (1979). R. A. Broglia and P. F. Bortignon, Phys. Lett. **101B**, 135 (1981).
[17]S. Adachi and S. Yoshida, Nucl. Phys. **A306**, 53 (1978).
[18]R. De Haro, S. Krewald, and J. Speth, Nucl. Phys. **A388**, 265 (1982).
[19]P. F. Bortignon and R. A. Broglia, Nucl. Phys. **A371**, 405 (1981).
[20]J. Wambach, V. K. Mishra, and Li Chu-Hsia, Nucl. Phys. **A380**, 285 (1982).
[21]S. P. Kamerdzhiev and V. I. Tselyaev, Izv. Akad. Nauk SSSR, Ser. Fiz. **47**, 917 (1983) [Bull. Acad. Sci. USSR, Phys. Ser.].
[22]V. G. Soloviev, Ch. Stoyanov, and R. Nikolayeva, JINR E4-83-166, Dubna, 1983.
[23]V. Yu. Ponomarev, Ch. Stoyanov, and O. Stoyanova, JINR R4-81-704, Dubna, 1981.
[24]V. G. Soloviev, Ch. Stoyanov, and A. I. Vdovin, Nucl. Phys. **A342**, 261 (1980).
[25]T. Yamagata *et al.*, Phys. Rev. **C23**, 937 (1981).
[26]F. E. Bertrand *et al.*, Phys. Rev. **C22**, 1832 (1980).
[27]N. Marty *et al.*, Proc. Intern. Symp. Highly Excited States, Jülich, 1975, IPNO PhN-76-03, Orsay, 1976.
[28]H. P. Morsch *et al.*, Phys. Rev. **C22**, 489 (1980); Phys. Rev. Lett. **45**, 337 (1980).
[29]G. Kühner *et al.*, Phys. Lett. **104B**, 189 (1981).
[30]T. Yamagata *et al.*, Phys. Lett. **123B**, 169 (1983).

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