Excitation of states of different multipolarity in large-angle inelastic electron scattering

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The cross sections for electroexcitation of the single-phonon states of the nucleus ^{90}Zr are computed within the framework of the method of distorted waves. The incoming-electron energy is varied in the range from 30 to 140 MeV and the scattering angle $\theta = 160°$. The contributions of states of different multipole orders ($1^\pm \leqslant \lambda^\pi \leqslant 6^\pm$) to the reaction cross section are analyzed.

The reaction of inelastic electron scattering is at present the most productive in terms of provision of information about the excited states of nuclei. A wealth of data has been collected on both the low-lying excitations[1] and the giant resonances of the electric[2,3] and magnetic[4,5] types. The region of low momentum-transfer values ($q \leqslant 0.5$ F^{-1}) is the most thoroughly investigated region; under the conditions of low momentum transfer the states that get excited are mostly the ones with low λ. As q increases, the spectrum of the excited states should become increasingly complicated as a result of the inclusion of states of higher multipole orders. The question naturally arises whether the e–e' scattering cross section in the region of large q will exhibit resonance-like structures due to the preferential excitation of states of a definite multipole order, or the reaction cross section will be formed from a mixture of different multipole orders. A more complicated picture is to be expected for scattering angles close to 180°, since under these conditions there should be excited states of both the electric (on account of the transverse form factor) and the magnetic type. On the other hand, it is precisely large scattering angles that are of greatest interest at present because of the increased attention being given to everything connected with the spin modes of excitation.

In the present paper we consider the changes that occur in the cross section for large-angle e–e' scattering as q increases in the particular case of the ^{90}Zr nucleus. We shall describe the structure of the excited states within the framework of the random-phase approximation for the quasiparticle–phonon model (QPM) of the nucleus, the physical bases of which are expounded in detail in Vdovin and Solov'ev's review article.[6] The model Hamiltonian contains the mean field for the protons and neutrons, the pairing forces (for ^{90}Zr pairing occurs only in the proton system), and the effective residual forces that produce the phonon excitations in nuclei. The electric-type states are produced by separable multipole forces:

$$V_\lambda(\mathbf{r}_1,\mathbf{r}_2)=\frac{1}{2}\left(\varkappa_0^{(\lambda)}+\boldsymbol{\tau}_1\boldsymbol{\tau}_2\varkappa_1^{(\lambda)}\right)\frac{\partial U}{\partial r_1}\frac{\partial U}{\partial r_2}\sum_\mu Y_{\lambda\mu}(\Omega_1)Y_{\lambda\mu}^*(\Omega_2),$$

while the magnetic-type states are produced by separable spin-multipole forces:

$$V_{\sigma,\lambda-1}^{\lambda}(\mathbf{r}_1,\mathbf{r}_2)=\frac{1}{2}\left(\varkappa_0^{(\lambda-1,\lambda)}+\boldsymbol{\tau}_1\boldsymbol{\tau}_2\varkappa_1^{(\lambda-1,\lambda)}\right)\times\frac{\partial U}{\partial r_1}\frac{\partial U}{\partial r_2}\sum_\mu[\boldsymbol{\sigma}_1 Y_{\lambda-1}(\Omega_1)]_{\lambda\mu}[\boldsymbol{\sigma}_2 Y_{\lambda-1}(\Omega_2)]_{\lambda\mu}^+,$$

where U is the central part of the mean-field potential; $\varkappa_0$ and $\varkappa_1$ are respectively the isoscalar and isovector residual-interaction constants. The parameters of the Woods-Saxon potential describing the mean field and the pairing-interaction constants were taken from Ref. 7. The residual-interaction constants are determined with the aid of the principle described in Ref. 8.

To begin with, let us find out how we can estimate the relative contributions made by states of different multipole orders to the total scattering cross section without going through tedious numerical calculations. The e-e' reaction cross section in the plane-wave Born approximation (PWBA) has the form[9]

$$\frac{d\sigma}{d\Omega}=\frac{2Z^2e^4}{q_\mu^4}\frac{p_f}{p_i}\left\{V_L(\theta)\sum_\lambda|F_\lambda^C(q^2)|^2+V_T(\theta)\sum_\lambda(|F_\lambda^M(q^2)|^2+|F_\lambda^E(q^2)|^2)\right\};\qquad(1)$$

the Coulomb $F_\lambda^C(q^2)$ and magnetic $F_\lambda^M(q^2)$ form factors can be expressed respectively in terms of the transition charge density $\rho_\lambda(r)$ and transition current $\rho_{\lambda\lambda}(r)$ density:

$$F_\lambda^C(q^2)=\frac{\sqrt{4\pi}(2\lambda+1)}{Z}\int\rho_\lambda(r)j_\lambda(qr)r^2\,dr,$$
$$F_\lambda^M(q^2)=\frac{\sqrt{4\pi}(2\lambda+1)}{Z}\int\rho_{\lambda\lambda}(r)j_\lambda(qr)r^2\,dr.\qquad(2)$$

The explicit forms of the transition densities within the framework of the *QPM* are given in Ref. 6. The expression for the transition current densities contains the effective gyromagnetic ratios $g_s^{\rm eff}$ and $g_l^{\rm eff}$. In the numerical computations we shall use the value $g_s^{\rm eff}=0.8g_s^{\rm free}$ and take the orbital ratios $g_l^{\rm eff}$ to be equal to the free ratio.

If we go over to the long-wave approximation in the formulas (2), i.e., if we limit ourselves in the expansion of the

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Bessel function to the first term, then the square of the form factor is proportional to the reduced probability $B(E\lambda)$ (or $B(M\lambda)$):

$$|F_\lambda{}^C(q^2)|^2=\frac{4\pi}{Z^2}\frac{q^{2\lambda}}{[(2\lambda-1)!!]^2}\frac{B(E\lambda)}{e^2(2\lambda+1)},$$
$$|F_\lambda{}^M(q^2)|^2=\frac{4\pi}{Z^2}\frac{q^{2\lambda}}{[(2\lambda-1)!!]^2}\frac{B(M\lambda)}{e^2(2\lambda+1)}\frac{\lambda+1}{\lambda}. \quad (3)$$

On the other hand, using the Segert formula, which is valid when $qR \lesssim 1$ (R is the radius of the nucleus), we can express the electric (or transverse) form factor $F_\lambda^E(q^2)$ in terms of the Coulomb form factor:

$$F_\lambda{}^E(q^2)\simeq\frac{k}{q}\sqrt{\frac{\lambda+1}{\lambda}}F_\lambda{}^C(q^2), \quad (4)$$

where k is the nuclear excitation energy expressed in the same units as q. Substituting (3) and (4) into the formula (1), we obtain an expression for the e–e' scattering cross section in terms of the quantities $B(E\lambda)$ and $B(M\lambda)$:

$$\left(\frac{d\sigma}{d\Omega}\right)_{E\lambda}=\frac{8\pi e^2}{q_\mu{}^4}\frac{p_f}{p_i}\frac{q^{2\lambda}}{(2\lambda+1)[(2\lambda-1)!!]^2}\left\{V_L(\theta)+\frac{k}{q}\sqrt{\frac{\lambda+1}{\lambda}}V_T(\theta)\right\}B(E\lambda), \quad (5)$$

$$\left(\frac{d\sigma}{d\Omega}\right)_{M\lambda}=\frac{8\pi e^2}{q_\mu{}^4}\frac{p_f}{p_i}\frac{q^{2\lambda}(\lambda+1)}{(2\lambda+1)[(2\lambda-1)!!]^2\lambda}V_T(\theta)B(M\lambda). \quad (6)$$

Now it is quite easy to estimate the contributions of states of different multipole orders to the e–e' scattering cross section. To do this, let us compare the integrated quantities $\sigma_{E\lambda}(\theta)$ and $\sigma_{M\lambda}(\theta)$, computing the cross section with the aid of the formulas (5) and (6), and summing over all the single-phonon states from the energy range from $E_{\min}=5$ MeV to $E_{\max}=20$ MeV. The cross section $\sigma_{\lambda\pi}(\theta)$ is accordingly defined as follows:

$$\sigma_{\lambda\pi}(\theta)=\int_{E_{min}}^{E_{max}}\left(\frac{d^2\sigma}{d\Omega\, dE_x}\right)_{\lambda^\pi}dE_x. \quad (7)$$

The ratios of the total excitation intensities of the 1^-, 2^+, 1^+, and 2^- states for inelastic scattering of electrons with energies $E_0=20$, 30, and 50 MeV through an angle of $\theta=160°$ are presented in Table I. On the basis of these qualitative estimates we can draw the following conclusions. First, in reactions involving large-angle inelastic scattering of slow electrons the most intensely excited states are the 1^- states; as E_0 increases, the relative contribution of the 1^- states to the total reaction cross section decreases. Second, the states of the magnetic type are, on the whole, excited much more feebly than the states of the electric type. Third, at low incoming-electron energies and for large scattering angles the cross section for electroexcitation of the $E\lambda$ states is almost completely determined by the electric form factor $F_\lambda{}^E(q^2)$: this can be judged from the magnitude of $(k/q)(V_T/V_L)$ [see the formula (5)]. As E_0 increases, this quantity decreases, but remains greater than unity. In the limiting case $\theta=180°$ the quantity in question in the PWBA tends to infinity, since $V_L(\theta)\to 0$, while $V_T(\theta)\to 2p_ip_f$.

In Table I we give in brackets for comparison the analogous values obtained on the basis of microscopic computations carried out within the framework of the distorted-wave Born approximation (DWBA); we shall discuss such computations below. The agreement obtained is quite satisfactory.

The formulas (5) and (6) are very simple and convenient for computations, but the region of their applicability, $qR\lesssim 1$, is quite narrow, and at large q values we can use neither the long-wave approximation nor the Segert theorem. In the case of back scattering the condition $qR=1$ is attainable even at low values of the incoming-electron energy E_0. Thus, for example, in the case of inelastic electron scattering by the nucleus ^{90}Zr through an angle of $\theta=160°$, $qR=1$ for $E_0=24$ MeV. Therefore, below we shall compute the form factors of the excited states directly in terms of their transition densities.

We shall describe the process of inelastic electron scattering within the framework of the DWBA with allowance for the electron energy loss in the exit channel resulting from excitation of the target nucleus; the corresponding formulas are given in Ref. 10. For the states of the electric type we shall take both the Coulomb and the electric form factors into account. The results of the computations are presented in Figs. 1 and 2 in the form of histograms and in Table II.

Figure 1 shows the differential cross sections for inelastic scattering through 160° of electrons with energies in the range from 30 to 140 MeV. The scattering angle was chosen on the basis of arguments of the following nature. On the one

TABLE I. Ratios of the total intensities of excitation of the 1^-, 2^+, 1^+, and 2^- states in the inelastic scattering of electrons with energy E_0 through an angle of $\theta=160°$.

E_0, MeV	$\frac{\sigma_{1^-}(\theta)}{\sigma_{2^+}(\theta)}$	$\frac{\sigma_{1^+}(\theta)}{\sigma_{2^-}(\theta)}$	$\frac{\sigma_{1^-}(\theta)}{\sigma_{1^+}(\theta)}$	$\frac{k}{q}\frac{V_T}{V_L}$
20	9.96	3.19	51.32	14.04
30	3.59 (4.04)	1.15 (1.35)	30.83 (11.30)	7.21
50	1.11 (0.70)	0.36 (0.39)	17.14 (8.61)	3.78

Footnote. The quantities $\sigma_{\lambda^\pi}(\theta)$ were determined with the aid of the formula (7): $E_{\min}=5$ MeV and $E_{\max}=20$ MeV. The calculations were performed in the long-wave approximation to the PWBA. Presented in the brackets are the analogous values computed in the DWBA.

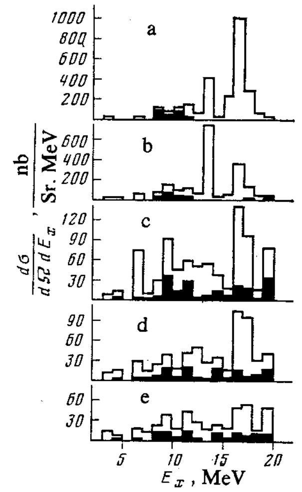

FIG. 1. Cross section for inelastic scattering through an angle $\theta = 160°$ of electrons with energies $E_0 = 30$ (a), 50 (b), 80 (c), 110 (d), and 140 (e) MeV by the ^{90}Zr nucleus. The solid histograms represent the contributions of the magnetic-type states to the reaction cross section.

hand, this should be an angle close to 180°, which is the condition that allows us to reduce the contribution of the Coulomb form factor to a minimum. On the other hand, at $\theta \simeq 180°$ the calculation of $F_\lambda^E(q^2)$ in the DWBA meets with difficulties of a computational nature. Notice that the majority of the published experiments on large-angle e–e' scattering were performed for θ lying in the range from 160° to 165°.

Each column of the open histograms in Figs. 1a–1e is the sum of the cross sections for electroexcitation of the states of different multipole orders from a region of width 1 MeV. Here all the single-phonon states (with 3 MeV $\leqslant E_x \leqslant 20$ MeV), electric and magnetic, of multipole order λ^π ranging from $1^\pm$ to $6^\pm$ were taken into account. The states with $\lambda > 7$ make a small contribution to the reaction cross section even in the case when $E_0 = 140$ MeV, and they can be neglected entirely. The solid histograms in these figures represent the cross sections for electroexcitation of the states of the magnetic type (specifically, the $M\,1$–$M\,6$ states).

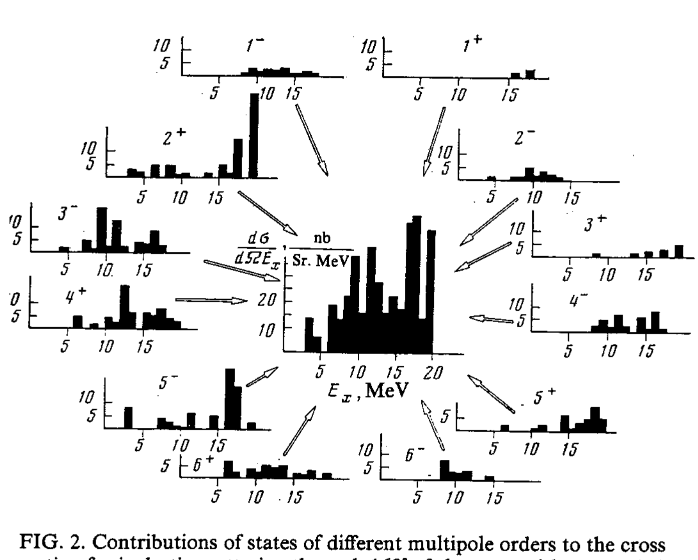

FIG. 2. Contributions of states of different multipole orders to the cross section for inelastic scattering through 160° of electrons with energy 140 MeV by the ^{90}Zr nucleus.

Figure 1 gives an idea about the shape of the plot of the e–e' scattering cross section as a function of the energy E_x transferred to the nucleus. The relative contributions of the states with different multipole orders to these curves can be extracted from Table II, where we give the values of the quantity $\sigma_{\lambda\pi}(\theta)/\Sigma\sigma_{\lambda\pi}(\theta)\cdot 100\%$ for all the λ^π values under consideration. Let us now proceed to analyze the results obtained.

As the calculations show, electrons with energy $E_0 = 30$ MeV most intensely excite the isovector $E\,1$ resonance (the 1^- states from the 15–18-MeV energy region); the isoscalar $E\,2$ resonance (the 2^+ states from the 12–14-MeV region) also makes a large contribution to the cross section (Fig. 1a). As E_0 increases, the absolute value of the cross section for excitation of the $E\,1$ (as well as the $M\,1$) resonance decreases, while the cross section for excitation of the resonances with $\lambda > 1$ increases, with the result that the highest peak in the reaction cross section for $E_0 = 50$ MeV is due to the $E\,2$ resonance (Fig. 1b). Similar conclusions were drawn above on the basis of qualitative arguments.

Let us turn our attention to the energy region $8 \leqslant \Delta E_x \leqslant 11$ MeV for the case of small q values. This is in fact a unique region, where the states that get excited are mainly of the magnetic type (specifically, the 1^+ and 2^- states); besides them, the individual 1^-, 2^+, and 3^- states make contributions to the cross section. These results are in qualitative agreement with the experimental data reported in Ref. 11. In the experiment performed by Richter's group the excitation-energy region $8 \leqslant \Delta E_x \leqslant 10$ MeV was investigated in ^{90}Zr; the incoming-electron energy wa varied from 24 to 66 MeV and the scattering angle $\theta = 165°$. The spin and parity of the states were determined from the behavior of the form factors. Three 1^- and, possibly, two 3^- levels were detected in the background of a large number of 2^- and 1^+ levels.

Table II demonstrates how, as q increases, states of higher multipole orders begin to be intensely excited. For $E_0 = 80$ MeV a large contribution to the reaction cross section is made by the 3^- states—mainly the 3^- states that produce the low-lying isoscalar octupole resonance (LIOR): in Fig. 1c the cross-section peak in the region $6 \leqslant E_x \leqslant 7$ MeV is due to this contribution. Electrons with energy 110 MeV intensely excite the 4^+ states (the excitation spectrum for the 4^+ states does not have a resonance character); notice the strong suppression under these conditions of the contribution of the 1^- levels. When $E_0 = 140$ MeV, we can no longer easily indicate the preferentially-excitable states, since many multipole orders make appreciable contributions to the reaction cross section. In this case the relative contribution of the most intensely excited multipole order is significantly smaller: thus, while for $E_0 = 30$ MeV the contribution of the 1^- levels constitutes 71%, for $E_0 = 140$ MeV the contribution of the 2^+ levels constitutes only 16%. As a result of the increase in the absolute number of intensely excited levels, the e–e' reaction cross section—E_x curve becomes smoother as q increases. The cross section for $E_0 = 140$ MeV no longer exhibits the sharp peaks occurring in the cross-section curves for $E_0 = 30$, 50, and 80 MeV.

TABLE II. Relative contributions of states of different multipole orders to the e–e' scattering cross section (in %).

$\lambda\pi$		$\sigma_{\lambda\pi}(\theta)/\Sigma\sigma_{\lambda\pi}(\theta)$				
		$E_0=30$ MeV	50	80	110	140
		$q=0,25$ F^{-1}	0,45	0,75	1,05	1,35
$E\lambda$	1−	71	34	18	5	4
	2+	17	47	25	21	16
	3−	<1	5	24	20	14
	4+	<1	<1	6	19	14
	5−	<1	<1	1	9	15
	6+	<1	<1	<1	2	9
$M\lambda$	1+	7	4	2	<1	1
	2−	5	10	13	6	3
	3+	<1	<1	7	7	2
	4−	<1	<1	2	6	8
	5+	<1	<1	<1	4	9
	6−	<1	<1	<1	<1	5
$\overset{M\lambda}{\Sigma\sigma_{\lambda\pi}(\theta)}$		12%	14%	25%	24%	28%

Footnote. The quantities $\sigma_{\lambda^\pi}(\theta)$ were determined from the formula (7): $E_{\min} = 5$ MeV and $E_{\max} = 20$ MeV. The calculations were performed in the DWBA. The scattering angle is $\theta = 160°$. The momentum transfer q was computed for $E_x = 10$ MeV.

Let us now consider how the reaction cross section at values of $q \geqslant 1$ F^{-1} is formed. In Fig. 2, besides the histograms representing the cross section for inelastic scattering of electrons with energy 140 MeV, we present histograms corresponding to the electroexcitation of states with definite λ^π. It is noteworthy that the e–e' scattering-cross-section pattern differs greatly from the distribution pattern for the quantity $B(E\lambda)$ or $B(M\lambda)$, especially in the case of small values of λ. We do not find in the electroexcitation spectra for $q \geqslant 1 F^{-1}$ a giant dipole resonance, or an isoscalar quadrupole resonance, or a LIOR, resonance which were easily seen at lower q values. This is explained by the fact that in the region beyond the first peak the square of the form factor is no longer proportional to the quantity $B(E\lambda)$ [or $B(M\lambda)$], but is determined solely by the individual features of the transition densities of specific states, as a result of which the states characterized by small $B(E\lambda)$ [or $B(M\lambda)$] values begin at definite q values to be excited very intensely, and vice versa. As applied to the 1^+ and 2^- states, similar questions are discussed in detail in Ref. 12.

Figure 2 shows that, for $E_0 = 140$ MeV, not only do many multipoles make significant contributions to the reaction cross section (this can be judged also from the last column in Table II), but we also have in each energy region with $\Delta E_x = 1$ MeV a mixture of contributions from several multipoles. Therefore, we can, in the case when $q \geqslant 1$ F^{-1}, hardly expect the occurrence of energy regions in which states of only one multipole order would be preferentially excited.

Another interesting question arises in connection with the relative contribution to the reaction cross section of the magnetic-type states as a whole. Above we concluded on the basis of qualitative estimates that in the region of low q values the states of the magnetic type are excited much more feebly than the states of the electric type. Microscopic calculations confirm this conclusion. In the last row in Table II we present the relative contribution made by all the magnetic-type states for different values of E_0. It can be seen from this row that, as q increases, the contribution of the magnetic-type states increases, but that it does not exceed 30%. Thus, the present calculations show that, even for scattering angles close to 180°, the $E\lambda$ states are, on the whole, excited much more intensely than the $M\lambda$ states.

To sum up the investigation carried out above, let us note the following main points. As q increases, the e–e' scattering cross section—versus—E_x curve becomes smoother, and there occurs strong intermixing of those states of different multipole orders which make appreciable contributions to the reaction cross section. No energy regions in which states of only one multipole order are preferentially excited were found in the case when $q \geqslant 1$ F^{-1}. As to the magnetic-type states, even for scattering angles close ot 180° their total contribution to the reaction cross section is not large, being not greater than 30%. The 8–11-MeV energy region, where the 1^+ and 2^- states ar found at[11] $q \leqslant 0.5$ F^{-1}, is the only region in ^{90}Zr where we can expect preferential excitation of the magnetic resonances; as q increases, their contribution to the e–e' scattering cross section gets "smeared out." Thus, it is to be acknowledged that, for study of the spin modes of excitation of a nucleus, the choice of the experimental conditions in the investigations reported in Refs. 4, 5, and 11 was a good one.

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