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NUCLEI
Theory

Charge-Exchange Transitions in Hot Nuclei*

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Abstract—A formalism based on the thermo field dynamics and allowing one to treat thermal effects on the strength distribution of charge-exchange transitions in hot nuclei is developed. The strength distributions of the allowed and first-forbidden $p \to n$ transitions are calculated for the neutron-rich nucleus ^{80}Ge at different temperatures. Then the electron capture rates on the same nucleus are calculated at temperatures and densities corresponding to an advanced stage of stellar evolution.

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1. INTRODUCTION

Weak interaction mediated reactions play a decisive role in many astrophysical processes [1]. A reliable determination of the reaction rates is necessary to simulate stellar evolution, nucleosynthesis, etc. Since the famous paper by H.A. Bethe et al. [2], it is generally accepted that electron capture on pf-shell nuclei is one of the most important processes before and during the collapse of a massive star. Via the deleptonization and the size of the outer and inner core, its rate essentially determines the collapse dynamics. Moreover, just the electron capture process is the principal source of neutrinos which carry away most of the energy released in the collapse.

The problem of electron capture by nuclei at high temperature and densities has been tackled by many authors (see, e.g., [3–9]). One of the most interesting and important problems analyzed in these papers was the influence of thermal effects, i.e., a thermal population of nuclear excited states, on the capture rate. It was shown that the thermal population could noticeably increase the capture as well as β^- decay rates.

To date, the most reliable calculations of electron capture rates for hot pf-shell nuclei have been performed in the framework of the shell model with a realistic nucleon–nucleon interaction (see [9] and references therein). However, such calculations require diagonalization of matrices of rank of an order $10^9 \times 10^9$, which is at the limit of computational capabilities of modern computers. Therefore, the shell-model calculations are restricted to nuclei in the mass range $A = 45 - 65$. Moreover, only contributions from the allowed Fermi and Gamow–Teller (GT) transitions were taken into account, which, however, were shown to be quite sufficient for the iron-group region.

During stellar collapse, a relative abundance of nuclei (A, Z) in the stellar matter changes, and the corresponding distribution function depends on the temperature and density. For $T \approx 0.5$ MeV, the ratio Z/A corresponds roughly to neutron-rich Ge isotopes with a completely occupied neutron $1f_{5/2}$-shell [10]. At $T = 0$, the allowed transitions are suppressed in these nuclei and the electron capture rates are dominated by the first-forbidden transitions [11]. Nevertheless, it was demonstrated in [10, 12] that configuration mixing and thermal excitations unblock the GT transitions in neutron-rich Ge isotopes and increase the electron capture rates.

In [13, 14], a novel approach based on the thermo field dynamics (TFD) [15] was suggested to study the GT strength distributions in hot nuclei and to estimate electron capture and β^- decay rates under stellar conditions. Using the ^{54,56}Fe nuclei as an example, it was shown that the results of the new approach agree satisfactorily with the shell-model ones. In the present paper, the approach is applied to

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study charge-exchange transitions (allowed and first-forbidden) in hot neutron-rich nuclei and to analyze thermal effects on the corresponding electron capture rate.

2. GENERAL THEORY

2.1. Fundamentals of the Thermo Field Dynamics

In the standard statistical mechanics, a heated quantum system is described by a density matrix ρ which is the solution to the Liouville–von Neumann equation

$$i\,\frac{\partial \rho}{\partial t} = [H, \rho]. \tag{1}$$

Here, H is the Hamiltonian of the system under consideration with eigenstates $|n\rangle$ and eigenvalues E_n (the chemical potential is included in H). The thermal average of an arbitrary operator A is given by

$$\langle\langle A\rangle\rangle = \mathrm{Tr}[\rho A]/\mathrm{Tr}[\rho] \tag{2}$$
$$= Z^{-1}(T)\sum_n \mathrm{e}^{-E_n/T}\langle n|A|n\rangle,$$

where T is the temperature in units of energy and $Z(T)$ is the partition function. The main idea behind TFD is to define a special state which is named a thermal vacuum $|0(T)\rangle$, so that the thermal average of A equals the expectation value of A with respect to this state

$$\langle\langle A\rangle\rangle = \langle 0(T)|A|\,0(T)\rangle. \tag{3}$$

In this sense, the thermal vacuum describes the system in the thermal equilibrium.

The thermal vacuum $|0(T)\rangle$ cannot be constructed in the Hilbert space of the system [15]. To construct $|0(T)\rangle$, one should double the Hilbert space of the system adding the so-called tilde states $|\widetilde{n}\rangle$. These tilde states are defined as the eigenstates of the tilde Hamiltonian $\widetilde{H}$ with the same eigenvalues E_n, i.e., $\widetilde{H}|\widetilde{n}\rangle = E_n|\widetilde{n}\rangle$. In the doubled Hilbert space, the thermal vacuum is constructed as follows:

$$|0(T)\rangle = Z^{-1/2}(T)\sum_n \mathrm{e}^{-E_n/2T}|n\rangle\otimes|\widetilde{n}\rangle. \tag{4}$$

There are two types of operators acting in the doubled Hilbert space—physical operators which act in the initial Hilbert space and their tilde counterparts acting on tilde states. The one-to-one correspondence between these two operator sets is given by tilde conjugation operations. The rules of the tilde-conjugation are listed in [15]. However, unlike the original TFD formulation [15], the double tilde-conjugation rule suggested in [16] is used in the present study. The reasons for this are given in [17].

In the doubled Hilbert space, the time evolution of a hot system is described by the following Schrödinger equation

$$i\frac{\partial}{\partial t}|\Psi(t,T)\rangle = \mathcal{H}|\Psi(t,T)\rangle, \tag{5}$$

where $\mathcal{H} = H - \widetilde{H}$ is the so-called thermal Hamiltonian. Thus, within TFD, the time-evolution operator is not the initial Hamiltonian H but the thermal Hamiltonian $\mathcal{H}$ whose eigenvalues determine the energies of excited states at $T \neq 0$. The thermal vacuum (4) is the eigenstate of $\mathcal{H}$ corresponding to the zero eigenvalue.

Generally, excited state energies appear to be dependent on temperature. Moreover, any eigenstate of $\mathcal{H}$ with positive energy has its counterpart—a tilde-conjugated eigenstate with negative energy. Within TFD, transitions from the thermal vacuum to states with positive energy correspond to the excitation of the heated system, whereas transitions to negative energy states correspond to deexcitation of the heated system.

Obviously, in most cases, one cannot find the exact spectrum of the thermal Hamiltonian and construct the exact thermal vacuum (4). Some approximations should be applied to diagonalize the thermal Hamiltonian. The merit of TFD allows one to resort to approximations valid at zero temperature. As a result, the thermal vacuum can be constructed in the Hartree–Fock–Bogoliubov approximation, or in the random phase approximation [18]. In the case that there appear several solutions in the given approximation, one should choose the solution providing a minimum of the thermodynamic potential.

2.2. Charge-Exchange Excitations in Hot Nuclei

Here, the TFD formalism is applied to study thermal effects on charge-exchange transitions in hot nuclei.

As a model Hamiltonian we use the Hamiltonian of the quasiparticle-phonon nuclear model (QPM) [19]. The QPM Hamiltonian includes phenomenological mean fields of protons and neutrons

$$H_{\mathrm{sp}} = \sum_\tau \sum_{jm}^{\tau} (E_j - \lambda_\tau) a^\dagger_{jm} a_{jm}, \tag{6}$$

pairing interactions of the BCS type with the coupling constants G_n and G_p

$$H_{\mathrm{pair}} = -\frac{1}{4} \tag{7}$$
$$\times \sum_\tau G_\tau \sum_{\substack{j_1 m_1 \\ j_2 m_2}}^{\tau} a^\dagger_{j_1 m_1} a^\dagger_{\overline{j_1 m_1}} a_{\overline{j_2 m_2}} a_{j_2 m_2},$$
$$(a_{\overline{jm}} = (-1)^{j-m} a_{j-m}),$$

and a separable residual interaction $H_{\rm int}$ in particle-hole channel.

In the present paper, we restrict ourselves to the random phase approximation. Therefore, it is sufficient to take into account only that part of the particle-hole interaction which is responsible for charge-exchange excitations [19–21]

$$H_{\rm int} = -2\sum_{\lambda\mu}\kappa_1^{(\lambda)} M^\dagger_{\lambda\mu} M_{\lambda\mu} - 2\sum_{L\lambda\mu}\kappa_1^{(L\lambda)} S^\dagger_{L\lambda\mu} S_{L\lambda\mu}. \quad (8)$$

Here, $\kappa_1^{(\lambda)}$ and $\kappa_1^{(L\lambda)}$ are the constants of isovector multipole and spin-multipole interactions, respectively, and the operators $M^\dagger_{\lambda\mu}$ and $S^\dagger_{L\lambda\mu}$ are defined as follows:

$$M^\dagger_{\lambda\mu} = \sum_{\substack{j_p m_p \\ j_n m_n}} \langle j_p m_p | i^\lambda r^\lambda Y_{\lambda\mu}(\theta,\phi) t^{(-)} | j_n m_n\rangle a^\dagger_{j_p m_p} a_{j_n m_n},$$

$$S^\dagger_{L\lambda\mu} = \sum_{\substack{j_p m_p \\ j_n m_n}} \langle j_p m_p | i^L r^L [Y_L(\theta,\phi)\sigma]^\lambda_\mu\, t^{(-)} | j_n m_n\rangle a^\dagger_{j_p m_p} a_{j_n m_n},$$

$$\left[Y_L(\theta,\phi)\sigma\right]^\lambda_\mu = \sum_{M,m} \langle LM\, 1m | \lambda\mu\rangle Y_{LM}(\theta,\phi)\sigma_m. \quad (9)$$

The structure of charge-exchange excitations of normal parity is determined by the multipole λ and spin-multipole $L = \lambda$ components of $H_{\rm int}$, while the spin-multipole components with $L = \lambda - 1$ and $L = \lambda + 1$ are responsible for charge-exchange excitations of abnormal parity. Hereinafter, the operator $a^\dagger_{jm}$ (a_{jm}) is the creation (annihilation) operator of a nucleon in the single-particle state with quantum numbers $nljm \equiv jm$ and energy E_j. An index $\tau = n, p$ is the isotopic one and it refers to proton or neutron variables. The notation $\sum^\tau$ implies a summation over neutron $(\tau = n)$ or proton $(\tau = p)$ single-particle states only. The value λ_τ is the neutron or proton chemical potential (the Fermi level). The isospin lowering operator $t^{(-)}$ transforms neutrons to protons.

The thermal Hamiltonian $\mathcal{H}$ corresponding to the QPM Hamiltonian reads

$$\mathcal{H} = H - \widetilde{H} = \mathcal{H}_{\rm sp} + \mathcal{H}_{\rm pair} + \mathcal{H}_{\rm int}. \quad (10)$$

The diagonalization of $\mathcal{H}$ begins with introducing a thermal quasiparticle basis which diagonalizes the BCS part $\mathcal{H}_{\rm sp} + \mathcal{H}_{\rm pair}$ of the thermal Hamiltonian (see also [22, 23]). To this aim, two canonical transformations are performed. The first one is the standard Bogolyubov transformation to quasiparticle operators $\alpha^\dagger_{jm}$, α_{jm}

$$\alpha^\dagger_{jm} = u_j a^\dagger_{jm} - v_j a_{\overline{jm}},$$
$$\alpha_{jm} = u_j a_{jm} - v_j a^\dagger_{\overline{jm}}, \qquad (u_j^2 + v_j^2 = 1). \quad (11)$$

The same transformation with the same coefficients is applied to tilde operators $\widetilde{a}^\dagger_{jm}$, $\widetilde{a}_{jm}$.

The second transformation, the so-called thermal rotation or the thermal Bogolyubov transformation, mixes the ordinary and tilde quasiparticle operators and calls into play thermal (nontilde and tilde) quasiparticles

$$\beta^\dagger_{jm} = x_j \alpha^\dagger_{jm} - i y_j \widetilde{\alpha}_{jm},$$
$$\widetilde{\beta}^\dagger_{jm} = x_j \widetilde{\alpha}^\dagger_{jm} + i y_j \alpha_{jm}, (x_j^2 + y_j^2 = 1). \quad (12)$$

Note that the thermal transformation (12) differs from the one used before (see, e.g., [15, 18, 22, 23]). The presence of the imaginary unit i in (12) is due to the other double tilde-conjugation rule definition than in the cited papers (as was pointed out above, we follow [16]; see also [17]).

To find the Bogolyubov transformation coefficients, one expresses $\mathcal{H}_{\rm sp} + \mathcal{H}_{\rm pair}$ in terms of thermal quasiparticle operators and then requires that the one-body part of the thermal BCS Hamiltonian has to be diagonal. This leads to the following equations for the u_j, v_j coefficients:

$$\begin{pmatrix} u_j \\ v_j \end{pmatrix} = \frac{1}{\sqrt{2}} \left(1 \pm \frac{E_j - \lambda_\tau}{\sqrt{(E_j - \lambda_\tau)^2 + \Delta_\tau^2}} \right)^{1/2}. \quad (13)$$

The gap parameter Δ_τ and the chemical potential λ_τ depend on the coefficients of the thermal transforma-

tion

$$\Delta_\tau = \frac{G_\tau}{2}\sum_j{}^{\tau}(2j+1)(x_j^2-y_j^2)u_jv_j, \qquad (14)$$
$$N_\tau = \sum_j{}^{\tau}(2j+1)(v_j^2x_j^2+u_j^2y_j^2),$$

where N_τ is the number of neutrons or protons in a nucleus.

The coefficients x_j, y_j are determined from the demand that the variation of the expectation value of the thermodynamic potential with respect to the thermal quasiparticle vacuum $|0(T);qp\rangle$ vanishes

$$\delta\langle 0(T);qp|(H_{\rm sp}+H_{\rm pair})-TK_{\rm q}|0(T);qp\rangle = 0, \qquad (15)$$

where

$$K_{\rm q} = -\sum_\tau\sum_{jm}{}^{\tau}\{\alpha_{jm}^\dagger\alpha_{jm}\ln y_j^2+\alpha_{jm}\alpha_{jm}^\dagger\ln x_j^2\} \qquad (16)$$

is the entropy operator for the system of noninteracting Bogolyubov quasiparticles.

As a result, we obtain

$$\begin{pmatrix}x_j\\y_j\end{pmatrix} = \left[1+\exp\left(\mp\frac{\varepsilon_j}{T}\right)\right]^{-1/2}, \qquad (17)$$
$$\varepsilon_j = \sqrt{(E_j-\lambda_\tau)^2+\Delta_\tau^2}.$$

The vacuum $|0(T);qp\rangle$ of thermal quasiparticles corresponding to x_j, y_j given by (17) is the thermal vacuum in the BCS approximation.

The quantity y_j^2 is nothing else but the Fermi–Dirac thermal occupation number which determines the number of Bogolyubov quasiparticles with energy ε_j in the BCS thermal vacuum

$$\langle 0(T);qp|\alpha_{jm}^\dagger\alpha_{jm}|0(T);qp\rangle \qquad (18)$$
$$= y_j^2 \equiv \left[1+\exp\left(\frac{\varepsilon_j}{T}\right)\right]^{-1}.$$

Equations (13), (14), and (17) are the well-known BCS equations at finite temperatures [24, 25]. It is well known that pair correlations diminish with increasing temperature and the pairing gap vanishes at a relatively small critical temperature ($T_{\rm cr}\approx\Delta_\tau(T=0)/2$) within the thermal BCS approximation.

After the diagonalization, the BCS part of the thermal Hamiltonian takes the form

$$\mathcal{H}_{\rm sp}+\mathcal{H}_{\rm pair} \qquad (19)$$
$$\simeq \sum_\tau\sum_{jm}{}^{\tau}\varepsilon_j(\beta_{jm}^\dagger\beta_{jm}-\widetilde{\beta}_{jm}^\dagger\widetilde{\beta}_{jm}).$$

It describes the system of independent thermal quasiparticles. Since the thermal vacuum $|0(T);qp\rangle$ contains a certain number of Bogolyubov quasiparticles, the excited states can be built on the top of $|0(T);qp\rangle$ by either adding or eliminating a Bogolyubov quasiparticle. The relations

$$\alpha_{jm}^\dagger|0(T);qp\rangle = x_j\beta_{jm}^\dagger|0(T);qp\rangle, \qquad (20)$$

$$\alpha_{\overline{jm}}|0(T);qp\rangle = -iy_j\widetilde{\beta}_{\overline{jm}}^\dagger|0(T);qp\rangle$$

demonstrate that the first process can be associated with a creation of a nontilde thermal quasiparticle with a positive energy, whereas the second process can be considered as creation of a thermal tilde quasiparticle with a negative energy.

The next step in the diagonalization of the thermal Hamiltonian (10) is to take into account the proton–neutron particle–hole interaction $H_{\rm int}$. To this aim transformations (11) and (12) with the coefficients (13) and (17) are applied to the p-h part of Eq. (8): $\mathcal{H}_{\rm int}=H_{\rm int}-\widetilde{H}_{\rm int}$.

Then the operator of a charge-exchange phonon is introduced

$$Q_{\lambda\mu i}^\dagger = \sum_{j_pj_n}\left(\psi_{j_pj_n}^{\lambda i}[\beta_{j_p}^\dagger\beta_{j_n}^\dagger]_\mu^\lambda+\widetilde{\psi}_{j_pj_n}^{\lambda i}[\widetilde{\beta}_{\overline{j_p}}^\dagger\widetilde{\beta}_{\overline{j_n}}^\dagger]_\mu^\lambda+i\eta_{j_pj_n}^{\lambda i}[\beta_{j_p}^\dagger\widetilde{\beta}_{\overline{j_n}}^\dagger]_\mu^\lambda+i\widetilde{\eta}_{j_pj_n}^{\lambda i}[\widetilde{\beta}_{\overline{j_p}}^\dagger\beta_{j_n}^\dagger]_\mu^\lambda\right) \qquad (21)$$
$$+\left(\phi_{j_pj_n}^{\lambda i}[\beta_{\overline{j_p}}\beta_{\overline{j_n}}]_\mu^\lambda+\widetilde{\phi}_{j_pj_n}^{\lambda i}[\widetilde{\beta}_{j_p}\widetilde{\beta}_{j_n}]_\mu^\lambda+i\xi_{j_pj_n}^{\lambda i}[\beta_{\overline{j_p}}\widetilde{\beta}_{j_n}]_\mu^\lambda+i\widetilde{\xi}_{j_pj_n}^{\lambda i}[\widetilde{\beta}_{j_p}\beta_{\overline{j_n}}]_\mu^\lambda\right)$$

and a new thermal vacuum is defined as the vacuum state $|0(T);ph\rangle$ for thermal phonons

$$Q_{\lambda\mu i}|0(T);ph\rangle = 0, \quad \widetilde{Q}_{\lambda\mu i}|0(T);ph\rangle = 0. \qquad (22)$$

By the analogy with the quasiparticle RPA at zero temperature, it is assumed that thermal phonon creation and annihilation operators obey a bosonic commutation rules.[4)] This imposes constraints on the thermal phonon

[4]It means a small number of thermal quasiparticles in $|0(T);ph\rangle$.

amplitudes $\psi^{\lambda i}_{j_p j_n}$, $\phi^{\lambda i}_{j_p j_n}$, etc. In particular, from the condition $[Q_{\lambda\mu i}, Q^\dagger_{\lambda'\mu' i'}] = \delta_{\lambda\lambda'}\delta_{\mu\mu'}\delta_{ii'}$, it follows that

$$\sum_{j_p j_n}\Big\{\psi^{\lambda i}_{j_p j_n}\psi^{\lambda i'}_{j_p j_n} + \widetilde{\psi}^{\lambda i}_{j_p j_n}\widetilde{\psi}^{\lambda i'}_{j_p j_n} + \eta^{\lambda i}_{j_p j_n}\eta^{\lambda i'}_{j_p j_n} + \widetilde{\eta}^{\lambda i}_{j_p j_n}\widetilde{\eta}^{\lambda i'}_{j_p j_n} \\ -\phi^{\lambda i}_{j_p j_n}\phi^{\lambda i'}_{j_p j_n} - \widetilde{\phi}^{\lambda i}_{j_p j_n}\widetilde{\phi}^{\lambda i'}_{j_p j_n} - \xi^{\lambda i}_{j_p j_n}\xi^{\lambda i'}_{j_p j_n} - \widetilde{\xi}^{\lambda i}_{j_p j_n}\widetilde{\xi}^{\lambda i'}_{j_p j_n}\Big\} = \delta_{ii'}. \tag{23}$$

To find structures and energies of thermal phonons, the variational principle can be used; i.e., one should find the minimum of the expectation value of the thermal Hamiltonian over the thermal one-phonon state under constraints (23)

$$\delta\Big\{\langle 0(T); ph|Q_{\lambda\mu i}\mathcal{H}Q^\dagger_{\lambda\mu i}|0(T); ph\rangle \\ -\frac{\omega_{\lambda i}}{2}\left[\langle 0(T); ph|[Q_{\lambda\mu i}, Q^\dagger_{\lambda\mu i}]|0(T); ph\rangle - 1\right]\Big\} = 0. \tag{24}$$

The Lagrangian factor $\omega_{\lambda i}$ is the energy of a thermal one-phonon state. The system of linear equations resulting from the variation has a nontrivial solution if $\omega_{\lambda i}$ obeys the following secular equation:

$$\mathcal{A}(\omega) = \begin{vmatrix} X^{(+)}_{aa} - \dfrac{1}{\kappa^{(a)}_1} & X^{(+-)}_{aa} & X^{(+)}_{ab} & X^{(+-)}_{ab} \\ X^{(-+)}_{aa} & X^{(-)}_{aa} - \dfrac{1}{\kappa^{(a)}_1} & X^{(-+)}_{ab} & X^{(-)}_{ab} \\ X^{(+)}_{ab} & X^{(+-)}_{ab} & X^{(+)}_{bb} - \dfrac{1}{\kappa^{(b)}_1} & X^{(+-)}_{bb} \\ X^{(+-)}_{ab} & X^{(-)}_{ab} & X^{(-+)}_{ab} & X^{(-)}_{bb} - \dfrac{1}{\kappa^{(b)}_1} \end{vmatrix} = 0, \tag{25}$$

where for charge-exchange excitations of normal parity $a \equiv \lambda$, $b \equiv \lambda\lambda$, while for charge-exchange excitations of abnormal parity $a \equiv (\lambda-1)\lambda$ and $b \equiv (\lambda+1)\lambda$. Functions $X^{(\pm)}_{cd}$, $X^{(\pm\mp)}_{cd}$ ($c = a, b$ and $d = a, b$) in (25) are defined as

$$X^{(\pm)}_{cd}(\omega) = \frac{2}{\hat{\lambda}^2} \\ \times \sum_{j_p j_n} f^{(c)}_{j_p j_n} f^{(d)}_{j_p j_n}\left\{\frac{\varepsilon^{(+)}_{j_p j_n}(u^{(\pm)}_{j_p j_n})^2}{(\varepsilon^{(+)}_{j_p j_n})^2 - \omega^2}(1 - y^2_{j_p} - y^2_{j_n}) \right. \\ \left. - \frac{\varepsilon^{(-)}_{j_p j_n}(v^{(\mp)}_{j_p j_n})^2}{(\varepsilon^{(-)}_{j_p j_n})^2 - \omega^2}(y^2_{j_p} - y^2_{j_n})\right\},$$

$$X^{(\pm\mp)}_{cd}(\omega) = \frac{2}{\hat{\lambda}^2}\,\omega \\ \times \sum_{j_p j_n} f^{(c)}_{j_p j_n} f^{(d)}_{j_p j_n}\left\{\frac{u^{(\pm)}_{j_p j_n} u^{(\mp)}_{j_p j_n}}{(\varepsilon^{(+)}_{j_p j_n})^2 - \omega^2}(1 - y^2_{j_p} - y^2_{j_n}) \right. \\ \left. - \frac{v^{(\pm)}_{j_p j_n} v^{(\mp)}_{j_p j_n}}{(\varepsilon^{(-)}_{j_p j_n})^2 - \omega^2}(y^2_{j_p} - y^2_{j_n})\right\},$$

where $f^{(\lambda)}_{j_p j_n}$, $f^{(L\lambda)}_{j_p j_n}$ are the reduced single-particle matrix elements of the multipole and spin-multipole operators (9); $u^{(\pm)}_{j_p j_n} = u_{j_p}v_{j_n} \pm v_{j_p}u_{j_n}$, $v^{(\pm)}_{j_p j_n} = u_{j_p}u_{j_n} \pm v_{j_p}v_{j_n}$; $\varepsilon^{(\pm)}_{j_p j_n} = \varepsilon_{j_p} \pm \varepsilon_{j_n}$; $\hat{\lambda} = \sqrt{2\lambda+1}$. One gets the same secular equations applying the variational principle to the tilde thermal one-phonon state $\widetilde{Q}^\dagger_{\lambda\mu i}|0(T); ph\rangle$.

Let us consider the secular equation (25) in detail. The poles $\varepsilon^{(-)}_{j_p j_n}$ which do not exist in the pnQRPA equations at zero temperature arise from crossover terms $\beta^\dagger\widetilde{\beta}^\dagger$ in the thermal phonon operator (21). Owing to these poles, new states appear in a low-energy part of the charge-exchange excitation spectrum at $T \neq 0$. Moreover, in contrast to the zero temperature case, the negative solutions of the secular equation (25) have a physical meaning. They correspond to the tilde thermal one-phonon

states $\widetilde{Q}^{\dagger}_{\lambda\mu i}|0(T);ph\rangle$ and arise due to $\widetilde{\beta}^{\dagger}\widetilde{\beta}^{\dagger}$ terms in the thermal phonon operator. As was noted above, creation of a tilde thermal quasiparticle corresponds to annihilation of a thermally excited Bogolyubov quasiparticle. Consequently, excitation of low- and negative-energy thermal phonons corresponds to charge-exchange transitions from thermally excited nuclear states.

The phonon amplitudes are the following:

$$\begin{aligned}
\begin{pmatrix}\psi\\ \phi\end{pmatrix}^{\lambda i}_{j_p j_n} &= \frac{\mathcal{N}_{\lambda i}}{\varepsilon^{(+)}_{j_p j_n} \mp \omega_{\lambda i}} \\
&\times \Big\{\kappa_1^{(a)} f^{(a)}_{j_p j_n}\big(u^{(+)}_{j_p j_n} A_{11} \pm u^{(-)}_{j_p j_n} A_{12}\big) \\
&+ \kappa_1^{(b)} f^{(b)}_{j_p j_n}\big(u^{(+)}_{j_p j_n} A_{13} \pm u^{(-)}_{j_p j_n} A_{14}\big)\Big\} \\
&\times \big(x_{j_p} x_{j_n} X_{\lambda i} - y_{j_p} y_{j_n} Y_{\lambda i}\big), \\
\begin{pmatrix}\widetilde{\psi}\\ \widetilde{\phi}\end{pmatrix}^{\lambda i}_{j_p j_n} &= -\frac{\mathcal{N}_{\lambda i}}{\varepsilon^{(+)}_{j_p j_n} \pm \omega_{\lambda i}} \\
&\times \Big\{\kappa_1^{(a)} f^{(a)}_{j_p j_n}\big(u^{(+)}_{j_p j_n} A_{11} \mp u^{(-)}_{j_p j_n} A_{12}\big) \\
&+ \kappa_1^{(b)} f^{(b)}_{j_p j_n}\big(u^{(+)}_{j_p j_n} A_{13} \mp u^{(-)}_{j_p j_n} A_{14}\big)\Big\} \\
&\times \big(y_{j_p} y_{j_n} X_{\lambda i} - x_{j_p} x_{j_n} Y_{\lambda i}\big), \\
\begin{pmatrix}\eta\\ \xi\end{pmatrix}^{\lambda i}_{j_p j_n} &= \frac{\mathcal{N}_{\lambda i}}{\varepsilon^{(-)}_{j_p j_n} \mp \omega_{\lambda i}} \\
&\times \Big\{\kappa_1^{(a)} f^{(a)}_{j_p j_n}\big(v^{(-)}_{j_p j_n} A_{11} \pm v^{(+)}_{j_p j_n} A_{12}\big) \\
&+ \kappa_1^{(b)} f^{(b)}_{j_p j_n}\big(v^{(-)}_{j_p j_n} A_{13} \pm v^{(+)}_{j_p j_n} A_{14}\big)\Big\} \\
&\times \big(x_{j_p} y_{j_n} X_{\lambda i} - y_{j_p} x_{j_n} Y_{\lambda i}\big), \\
\begin{pmatrix}\widetilde{\eta}\\ \widetilde{\xi}\end{pmatrix}^{\lambda i}_{j_p j_n} &= -\frac{\mathcal{N}_{\lambda i}}{\varepsilon^{(-)}_{j_p j_n} \pm \omega_{\lambda i}} \\
&\times \Big\{\kappa_1^{(a)} f^{(a)}_{j_p j_n}\big(v^{(-)}_{j_p j_n} A_{11} \mp v^{(+)}_{j_p j_n} A_{12}\big) \\
&+ \kappa_1^{(b)} f^{(b)}_{j_p j_n}\big(v^{(-)}_{j_p j_n} A_{13} \mp v^{(+)}_{j_p j_n} A_{14}\big)\Big\} \\
&\times \big(y_{j_p} x_{j_n} X_{\lambda i} - x_{j_p} y_{j_n} Y_{\lambda i}\big),
\end{aligned} \tag{26}$$

where $A_{j\,k}$ are the algebraic supplements of the determinant $\mathcal{A}(\omega_{\lambda i})$; the factor $\mathcal{N}_{\lambda i}$ is defined by the normalization condition (23), while unknown variables $X_{\lambda i},\ Y_{\lambda i}$ obey the condition $X^2_{\lambda i} - Y^2_{\lambda i} = 1$.

Thus, only the energies of thermal phonons can be found unambiguously by the thermal Hamiltonian diagonalization, while their structure is determined up to the unitary transformation

$$Q^{\dagger}_{\lambda\mu i} \to X_{\lambda i} Q^{\dagger}_{\lambda\mu i} - Y_{\lambda i}\widetilde{Q}_{\lambda\mu i}, \tag{27}$$

$$\widetilde{Q}^{\dagger}_{\lambda\mu i} \to X_{\lambda i}\widetilde{Q}^{\dagger}_{\lambda\mu i} - Y_{\lambda i} Q_{\lambda\mu i}.$$

This uncertainty arises due to the invariance of the thermal Hamiltonian with respect to the unitary transformation (27)

$$\mathcal{H} = \sum_{\lambda\mu i} \omega_{\lambda i}\big(Q^{\dagger}_{\lambda\mu i} Q_{\lambda\mu i} - \widetilde{Q}^{\dagger}_{\lambda\mu i}\widetilde{Q}_{\lambda\mu i}\big). \tag{28}$$

The uncertainty of the thermal phonon structure is reflected in the corresponding uncertainty in the thermal vacuum determination. To remove the uncertainty, additional constraints should be applied. One should demand that the thermodynamic potential $\Omega_{\rm ph}$ for a system of noninteracting phonons reaches the minimal value in the "true" thermal phonon vacuum. The thermodynamic potential $\Omega_{\rm ph}$ is given by

$$\begin{aligned}
&\Omega_{\rm ph} \\
&= \left\langle 0(T);ph \left| \sum_{\lambda\mu i} \omega_{\lambda i}\, q^{\dagger}_{\lambda\mu i} q_{\lambda\mu i} - T\hat{K}_{\rm ph} \right| 0(T);ph \right\rangle,
\end{aligned} \tag{29}$$

where $q^{\dagger}_{\lambda\mu i},\ q_{\lambda\mu i}$ is the thermal phonon creation and annihilation operators with the amplitudes (26) corresponding to $X_{\lambda i} = 1, Y_{\lambda i} = 0$; $K_{\rm ph}$ is the entropy operator for a system of noninteracting phonons [15]

$$K_{\rm ph} = -\sum_{\lambda\mu i}\{q^{\dagger}_{\lambda\mu i} q_{\lambda\mu i} \ln Y^2_{\lambda i} - q_{\lambda\mu i} q^{\dagger}_{\lambda\mu i} \ln X^2_{\lambda i}\}. \tag{30}$$

Minimization of $\Omega_{\rm ph}$ yields

$$Y_{\lambda i} = \left[\exp\left(\frac{\omega_{\lambda i}}{T}\right) - 1\right]^{-1/2}, \quad X^2_{\lambda i} = 1 + Y^2_{\lambda i}. \tag{31}$$

The coefficients $Y^2_{\lambda i}$ are the thermal occupation factors of the Bose–Einstein statistics. They determine the average number of $q_{\lambda\mu i}$ phonons in the "true" thermal phonon vacuum

$$\langle 0(T);ph|q^{\dagger}_{\lambda\mu i} q_{\lambda\mu i}|0(T);ph\rangle = Y^2_{\lambda i}. \tag{32}$$

It should be noted that any thermal one-phonon state $|Q_{\lambda\mu i}\rangle = Q^{\dagger}_{\lambda\mu i}|0(T);\ ph\rangle$ (or $|\widetilde{Q}_{\lambda\mu i}\rangle = \widetilde{Q}^{\dagger}_{\lambda\mu i}|0(T);ph\rangle$) is a superposition of excitations in daughter nuclei $(Z+1, N-1)$ and $(Z-1, N+1)$. This mixing is a result of the BCS approximation for pairing correlations. Only at temperatures higher than the critical one $T_{\rm cr}$ when pairing correlations disappear does each thermal one-phonon state belong to one definite daughter nucleus. The interesting point is that, if $|Q_{\lambda\mu i}\rangle$ belongs to the $(Z+1, N-1)$ nucleus, then $|\widetilde{Q}_{\lambda\mu i}\rangle$ belongs to the $(Z-1, N+1)$ nucleus.

Charge-exchange transition probabilities (transition strengths) from the thermal vacuum to thermal

one-phonon states are given by the squared reduced matrix elements of the corresponding transition operator

$$\Phi_{\lambda i}^{(\pm)} = \left|\langle 0(T); ph \| D_{\lambda\mu}^{(\pm)} \| Q_{\lambda\mu i}\rangle\right|^2,$$
$$\widetilde{\Phi}_{\lambda i}^{(\pm)} = \left|\langle 0(T); ph \| D_{\lambda\mu}^{(\pm)} \| \widetilde{Q}_{\lambda\mu i}\rangle\right|^2. \quad (33)$$

Hereinafter $(-)$ labels the $n \to p$ transition operators, and $(+)$ labels the $p \to n$ transition operators. The explicit expressions for $\Phi_{\lambda i}^{(\pm)}$ and $\widetilde{\Phi}_{\lambda i}^{(\pm)}$ are given in the Appendix. Furthermore, in the Appendix, the fulfillment of the Ikeda sum rule for Fermi and Gamow–Teller transitions in hot nuclei is proved.

The energies of charge exchange-transitions from the thermal vacuum to thermal one-phonon states are

$$E_{\lambda i}^{(\pm)} = \omega_{\lambda i} \pm (\Delta\lambda_{np} + \Delta m_{np}),$$
$$\widetilde{E}_{\lambda i}^{(\pm)} = -\omega_{\lambda i} \pm (\Delta\lambda_{np} + \Delta m_{np}), \quad (34)$$

where $\Delta\lambda_{np} = \lambda_n - \lambda_p$ is the difference between neutron and proton chemical potentials and $\Delta m_{np} = m_n - m_p = 1.29$ MeV is the neutron–proton mass difference. In (34), one takes into account that, before and after charge-exchange transition, the nucleon is attached to different subsystems.

Expressions (33) and (34) determine charge-exchange strength distribution in a hot nucleus within the thermal pnQRPA. At finite temperatures, some amount of transition strength is always located in the region of negative transition energies. Because of this, β-stable nuclei become unstable at finite temperatures, decaying by electron or positron emission from thermally populated excited states. The inverse reactions, electron or positron captures, make possible charge-exchange transitions with negative energies also. These transitions correspond to the processes of nuclear deexcitation.

2.3. Electron Capture Rates

Given the strength distribution of charge-exchange $p \to n$ transitions, electron capture rates can be estimated in stellar interior at presupernova conditions. In the present calculations, the following main assumptions are made: (1) The temperature in stellar interior is so high that atoms are totally ionized, and the surrounding electron gas is described by a Fermi–Dirac distribution, with temperature T and chemical potential μ_e. Neutrino capture is not taken into account. (2) The parent nucleus is in a thermal equilibrium state treated as the thermal (phonon) vacuum. (3) Electron capture leads to charge-exchange transitions from the thermal vacuum to thermal one-phonon states. Under these assumptions, the electron capture rate is given by

$$\lambda = \frac{\ln 2}{6150\,\mathrm{s}} \times \sum_J \sum_i \left[\Phi_{Ji}^{(+)} F^{\mathrm{ec}}(E_{Ji}) + \widetilde{\Phi}_{Ji}^{(+)} F^{\mathrm{ec}}(\widetilde{E}_{Ji})\right]. \quad (35)$$

The phase space integral $F^{\mathrm{ec}}(E)$ depends on the transition energy E and the chemical potential μ_e (see, e.g., [9]). The chemical potential, μ_e, is determined from density ρY_e of the electron gas [9].

The operator of allowed transitions is

$$D = (g_V + g_A^* \boldsymbol{\sigma}) t^{(+)}. \quad (36)$$

It consists of Fermi ($\Delta J^\pi = 0^+$) and Gamow–Teller ($\Delta J^\pi = 1^+$) components. The quantities $g_V = 1$, $g_A^* = 0.75 g_A^{\mathrm{free}}$ are the vector and axial coupling constants. (The star in g_A^* means that we used the quenched value of the axial coupling constants, $g_A^* = -1.25 \times 0.74$).

In the nonrelativistic limit, the operators of first-forbidden transitions read [11, 12]

$$\Delta J^\pi = 0^-, \quad D = g_A^* \Big(\frac{\boldsymbol{\sigma}\cdot\mathbf{p}}{m} + \frac{\alpha Z}{2R}\, i\boldsymbol{\sigma}\cdot\mathbf{r}\Big) t^{(+)},$$
$$\Delta J^\pi = 1^-,$$
$$D = \Big(g_V \frac{\mathbf{p}}{m} - \frac{\alpha Z}{2R}(g_A^* \boldsymbol{\sigma}\times\mathbf{p} - g_V \mathbf{r})\Big) t^{(+)},$$
$$\Delta J^\pi = 2^-, \quad D = i\frac{g_A^*}{\sqrt{3}} [\boldsymbol{\sigma}\cdot\mathbf{r}]_\mu^2 \sqrt{p_e^2 + q_\nu^2}\, t^{(+)}. \quad (37)$$

Here Z, R are the charge and radius of the nucleus; α is the fine structure constant; $\boldsymbol{\sigma}$, $\mathbf{r}$, and $\mathbf{p}$ are the nucleon-spin, nucleon-coordinate, and nucleon momentum operators, respectively; and m is a nucleon mass. In the "unique" first-forbidden case ($\Delta J^\pi = 2^-$), the operator D depends explicitly on the momenta of electron p_e and neutrino q_ν.

3. RESULTS OF CALCULATIONS

Numerical calculations are performed for the neutron-rich ^{80}Ge nucleus. This is a "soft" nucleus; i.e., its low-lying quadrupole states are strongly collective. In the present consideration, ^{80}Ge is treated as a spherically symmetric nucleus. The single-particle wave functions and energies are obtained using the Woods–Saxon potential with Chepurnov parameterization [26]. Only the potential well depth is adjusted to reproduce the experimental proton and neutron separation energies.

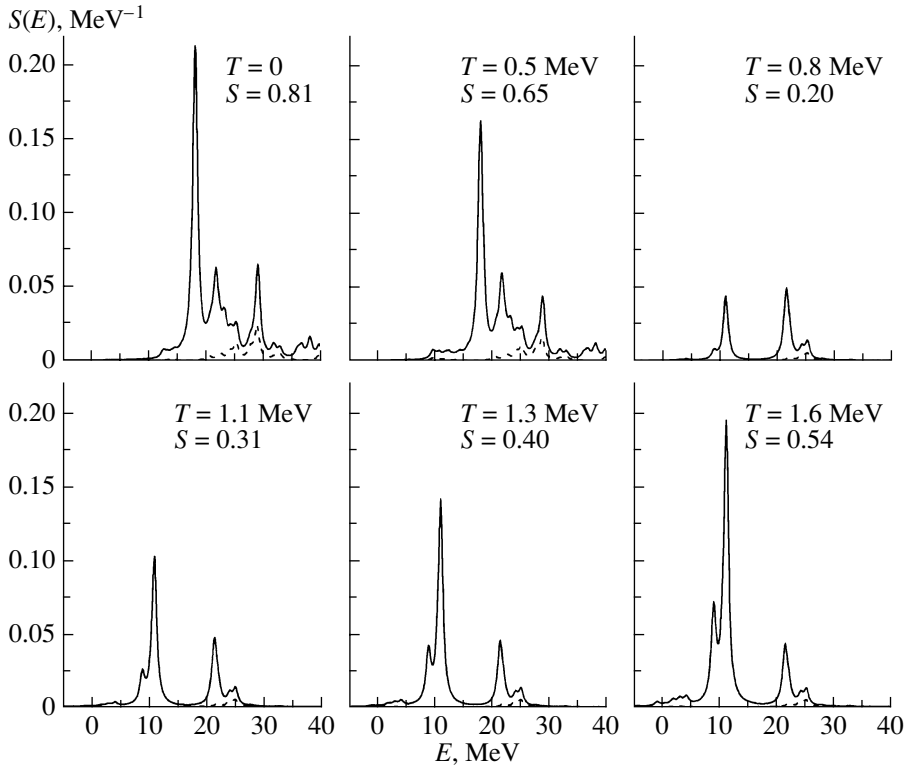

Fig. 1. Strength distribution $S(E)$ of allowed (0^+ and 1^+) $p \to n$ transitions from ^{80}Ge for different temperatures T. E is the transition energy. The contribution of 0^+ transitions is shown by the dashed line. S is the total transition strength at given T. Recall that 1 MeV $\approx 11.6 \times 10^9$ K.

The constants G_p and G_n of the pairing interaction are fitted to reproduce pair energies extracted from the experimental binding energies [27]. The proton and neutron energy gaps at $T = 0$ are $\Delta_p = 1.39$ MeV, $\Delta_n = 1.35$ MeV. The constants $\kappa_1^{(\lambda)}$, $\kappa_1^{(L\lambda)}$ of the isovector multipole and spin-multipole interactions are determined according to the formulas from [28, 29] for separable forces with radial form factors r^λ.

The calculated set of single-particle levels is close to that used in [12] in calculations of electron capture rates for ^{82}Ge. In the independent-particle model (IPM), the 12 protons completely occupy the $1f_{7/2}$ and $2p_{3/2}$ subshells, while the $1p_{1/2}$, $1f_{5/2}$, and $1g_{9/2}$ subshells are empty. For the neutrons, the IPM predicts a fully occupied pf shell and eight neutrons in the $1g_{9/2}$ subshell. Clearly, within the IPM, the Pauli principle blocks all allowed $p \to n$ transitions from the ^{80}Ge ground state and electron capture can proceed only via forbidden transitions at $T = 0$. However, pairing correlations smear the neutron and proton Fermi surfaces, thus weakening the blocking.

In Fig. 1, the strength distribution of allowed $p \to n$ transitions from ^{80}Ge is shown for different values of temperature T. For each value of T, the total transition strength S is displayed as well.

At temperatures below $T_{\rm cr} \approx 0.9$–0.8 MeV, the strength distribution has a peak in the energy interval 18–19 MeV. The peak is mainly formed by two single-particle transitions: $\pi(1f_{7/2}) \to \nu(1f_{5/2})$ and $\pi(1g_{9/2}) \to \nu(1g_{7/2})$. The first transition is of the hole–hole (h–h) type, while the second one is of the particle–particle (p–p) type. It means that the peak is formed due to pair correlations. Because of pairing, the proton subshell $1g_{9/2}$ is partially occupied, while the neutron subshell $1f_{5/2}$ is partially empty. The corresponding thermal one-phonon states can be approximately treated as superpositions of the two thermal quasiparticle states of the type $\beta^\dagger_{j_p}\beta^\dagger_{j_n}|0(T)\rangle$. Therefore, neglecting the values $\Delta\lambda_{np}$ and Δm_{np}, one can say that the transition energies are determined by the $\varepsilon^{(+)}_{j_p j_n}$ poles of the secular equation (25).

At $T = T_{\rm cr}$, i.e., when the pair correlations vanish, the peak completely disappears, and the total transition strength is noticeably reduced. However, at higher temperatures, $p \to n$ transitions from thermally excited nuclear states come into play. As is seen in Fig. 1, this leads to an increase of the

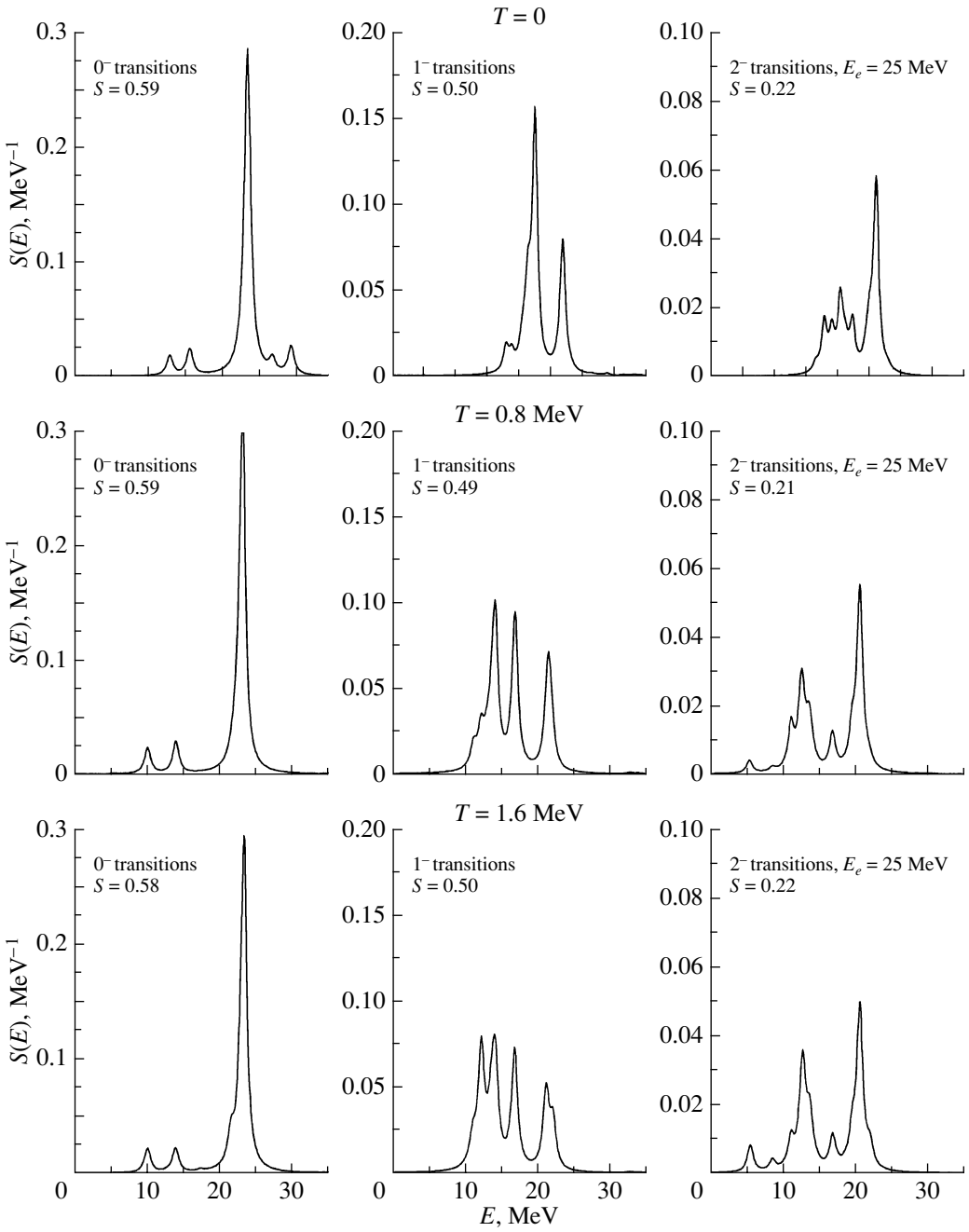

Fig. 2. Strength distributions $S(E)$ for the first-forbidden $\Delta J^\pi = 0^-$, 1^- and 2^- $p \to n$ transitions from ^{80}Ge at $T = 0$ (upper part), $T = 0.8$ MeV (middle part), and $T = 1.6$ MeV (lower part). E is the transition energy. S is the total strength at given ΔJ and T. Strength distribution for 2^- transitions is calculated for the electron energy 25 MeV.

total transition strength and a new well-pronounced peak appears at the energy 10 MeV. It is worthy to note that, in the formation of the peak, the same single-particle transitions $\pi(1f_{7/2}) \to \nu(1f_{5/2})$ and $\pi(1g_{9/2}) \to \nu(1g_{7/2})$ are involved. However, now these transitions are connected with annihilation of a thermally excited Bogolyubov quasiparticle or, which is the same, with creation of a tilde thermal quasiparticle with negative energy. Consequently, the transition energies are determined by the differences of corresponding thermal quasiparticle energies or, in other words, by the $\varepsilon^{(-)}_{j_p j_n}$ poles of the secular equation (25). These transition energies are less than $\varepsilon^{(+)}_{j_p j_n}$ which play a role at low temperatures. Thus, an increase in temperature leads to a significant redistribution of the strength of allowed $p \to n$ transitions in ^{80}Ge: the peak shifts downward in energy by

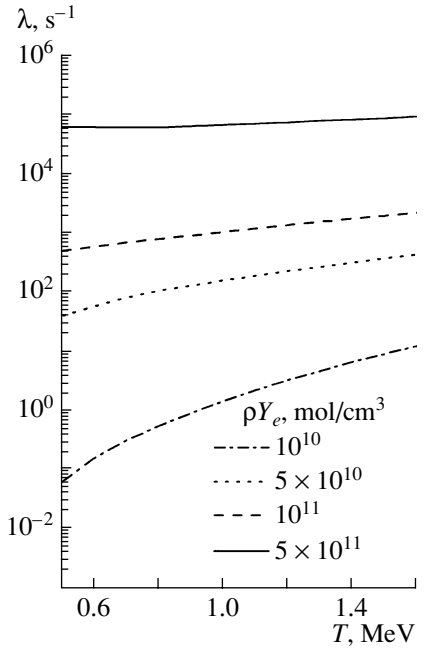

Fig. 3. Electron capture rate λ for ^{80}Ge as a function of temperature T for selected values of density ρY_e (in mol/cm^3). T is in units of 10^9 K.

about 8 MeV and the total transition strength reduces when the temperature reaches the critical one. This, in turn, affects the electron capture rate, as will be demonstrated below.

In Fig. 2, the strength distributions for the first-forbidden 0^-, 1^-, and 2^- $p \to n$ transitions from ^{80}Ge are shown for different values of T. As one can see, the total strengths of forbidden transitions are scarcely affected by the temperature. The reason is that first-forbidden $p \to n$ transitions in ^{80}Ge are mainly dominated by high-energy particle–hole transitions, whose strengths depend only slightly on temperature and pair correlations. Nevertheless, for 1^- and 2^- transitions, an increase in temperature leads to noticeable broadening of the corresponding strength distributions. The broadening appears because of the increase in the number of low-energy single-particle transitions caused by the thermal smearing of the proton and neutron Fermi surfaces. Due to a severe selection rule, the contribution of low-energy transitions to the 0^- strength distribution appears to be negligible.

Since the total strengths of the allowed and first-forbidden $p \to n$ transitions in ^{80}Ge appear to be of the same value, both types should be taken into account in the calculation of the electron capture rate.

The calculated electron capture rate for ^{80}Ge is shown in Fig. 3. The rate increases with increasing temperature and density ρY_e. The lower the density, the higher is the sensitivity of the rate to temperature increase. The reason is that, at low densities (i.e., when the chemical potential of surrounding electrons is relatively small), the capture rates depend crucially on the details of the $p \to n$ strength distribution. At $\rho Y_e = 10^{10}$ mol/cm^3 and in the temperature range covered, the chemical potential μ_e is equal to 10–11 MeV and the electron capture proceeds mainly through low-energy $p \to n$ transitions. A temperature growth increases the fraction of these transitions since the peak of allowed transition strength shifts downward in energy and, moreover, low-energy first-forbidden transitions become possible. As a result, the capture rate increases significantly with increasing temperature. A growth of pressure increases the value of electron chemical potential from $\mu_e \approx 18$–19 MeV at $\rho Y_e = 5 \times 10^{10}$ mol/cm^3 up to $\mu_e \approx 40$–41 MeV at $\rho Y_e = 5 \times 10^{11}$ mol/cm^3. As a consequence, the capture rate becomes less sensitive to temperature change. At $\rho Y_e = 5 \times 10^{11}$ mol/cm^3, the rate depends only on the total transition strength and the position of the energy centroid of the strength distribution.

4. CONCLUSIONS

We presented the approach to study thermal effects on weak interaction rates in hot nuclei based on the ideas and methods of the thermo field dynamics. The equations allowing one to calculate strength distributions of charge-exchange transitions in hot nuclei were derived within the thermal pnQRPA. Strength distributions of allowed and first-forbidden transitions from the ^{80}Ge were calculated and analyzed. For allowed transitions, it was found that the temperature increase leads to a considerable downward shift of the peak in the strength distribution and strongly reduces the total transition strength in the vicinity of the critical temperature. Both effects are caused by the diminishing of pair correlations with temperature increase. At the same time, the total transition strength of the first-forbidden transitions in ^{80}Ge appears to be scarcely affected by the temperature changes. The temperature rise increases the fraction of low-energy 1^- and 2^- first-forbidden transitions, which leads to noticeable broadening of the corresponding strength distributions.

The temperature dependence of the electron capture rate on ^{80}Ge was calculated. It was shown how the changes in the $p \to n$ strength distributions affect the capture rates at different densities.

It is of interest to go beyond the thermal pnQRPA within the present approach and take into account coupling with complex thermal (e.g., two-phonon) configurations. For charge-exchange transitions in cold nuclei, this problem was resolved within the quasiparticle-phonon nuclear model [20, 21]

and other approaches [30]. It was found that the coupling with complex configuration strongly affects the QRPA strength distribution. As discussed above, the details of the strength distribution are of particular importance for electron capture rates at low densities.

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APPENDIX

Charge-exchange transition probabilities (transition strengths) from the thermal vacuum to thermal one-phonon states are given by the following expressions:

$$\Phi^{(-)}_{\lambda i} = \left(\sum_{j_p j_n} d^{(-)}_{\lambda}(j_p j_n)\Omega_1(j_p j_n;\lambda i)\right)^2, \quad \text{(A.1)}$$

$$\widetilde{\Phi}^{(-)}_{\lambda i} = \left(\sum_{j_p j_n} d^{(-)}_{\lambda}(j_p j_n)\widetilde{\Omega}_1(j_p j_n;\lambda i)\right)^2,$$

$$\Phi^{(+)}_{\lambda i} = \left(\sum_{j_p j_n} (-1)^{j_n-j_p+\lambda} d^{(+)}_{\lambda}(j_n j_p)\Omega_2(j_p j_n;\lambda i)\right)^2,$$

$$\widetilde{\Phi}^{(+)}_{\lambda i} = \left(\sum_{j_p j_n} (-1)^{j_n-j_p+\lambda} d^{(+)}_{\lambda}(j_n j_p)\widetilde{\Omega}_2(j_p j_n;\lambda i)\right)^2,$$

where $d^{(\mp)}_{\lambda}(j_{p(n)} j_{n(p)})$ is a reduced single-particle matrix element of the transition operator

$$d^{(\mp)}_{\lambda}(j_{p(n)} j_{n(p)}) = \langle j_{n(p)}||D^{(\mp)}_{\lambda}||j_{p(n)}\rangle, \quad \text{(A.2)}$$

and functions $\Omega_{1,2}(j_p j_n;\lambda i)$ are linear combinations of the thermal phonon amplitudes (26):

$$\Omega_1(j_p j_n;\lambda i) = u_{j_p} v_{j_n}\big(x_{j_p} x_{j_n}\psi^{\lambda i}_{j_p j_n} + y_{j_p} y_{j_n}\widetilde{\phi}^{\lambda i}_{j_p j_n}\big) + v_{j_p} u_{j_n}\big(y_{j_p} y_{j_n}\widetilde{\psi}^{\lambda i}_{j_p j_n} + x_{j_p} x_{j_n}\phi^{\lambda i}_{j_p j_n}\big) + u_{j_p} u_{j_n}\big(x_{j_p} y_{j_n}\eta^{\lambda i}_{j_p j_n} + y_{j_p} x_{j_n}\widetilde{\xi}^{\lambda i}_{j_p j_n}\big) - v_{j_p} v_{j_n}\big(y_{j_p} x_{j_n}\widetilde{\eta}^{\lambda i}_{j_p j_n} + x_{j_p} y_{j_n}\xi^{\lambda i}_{j_p j_n}\big), \quad \text{(A.3)}$$

$$\Omega_2(j_p j_n;\lambda i) = v_{j_p} u_{j_n}\big(x_{j_p} x_{j_n}\psi^{\lambda i}_{j_p j_n} + y_{j_p} y_{j_n}\widetilde{\phi}^{\lambda i}_{j_p j_n}\big) + u_{j_p} v_{j_n}\big(y_{j_p} y_{j_n}\widetilde{\psi}^{\lambda i}_{j_p j_n} + x_{j_p} x_{j_n}\phi^{\lambda i}_{j_p j_n}\big) - v_{j_p} v_{j_n}\big(x_{j_p} y_{j_n}\eta^{\lambda i}_{j_p j_n} + y_{j_p} x_{j_n}\widetilde{\xi}^{\lambda i}_{j_p j_n}\big) + u_{j_p} u_{j_n}\big(y_{j_p} x_{j_n}\widetilde{\eta}^{\lambda i}_{j_p j_n} + x_{j_p} y_{j_n}\xi^{\lambda i}_{j_p j_n}\big).$$

The function $\widetilde{\Omega}_{1,2}(j_p j_n;\lambda i)$ results from $\Omega_{1,2}(j_p j_n;\lambda i)$ by changing nontilde phonon amplitudes by their tilde partners and vice versa.

For $D^{(-)}_{\lambda}$ and $D^{(+)}_{\lambda}$ differing only by the isospin operator (this is true, in particular, for the Fermi and Gamow–Teller transitions), $d^{(-)}_{\lambda}(j_p j_n) = (-1)^{j_n-j_p+\lambda} d^{(+)}_{\lambda}(j_n j_p)$ and transition strengths (A.1) obey the following relation

$$\widetilde{\Phi}^{(\mp)}_{\lambda i} = \exp(-\omega_{\lambda i}/T)\Phi^{(\pm)}_{\lambda i} \quad (\omega_{\lambda i} > 0). \quad \text{(A.4)}$$

The similar relation was found in [17] for charge-neutral transitions of electric type in hot nuclei.

The validity of the Ikeda sum rule (ISR) for the Fermi and Gamow–Teller transitions in hot nuclei can be proved within the present approach. Defining the total $n \to p$ and $p \to n$ transition strengths

$$S^{(\mp)}_{\lambda} = \sum_i (\Phi^{(\mp)}_{\lambda i} + \widetilde{\Phi}^{(\mp)}_{\lambda i}) \quad (\lambda = 0^+, 1^+) \quad \text{(A.5)}$$

and taking into account the closure relation for phonon amplitudes, one gets

$$\mathrm{ISR} = S^{(-)}_{\lambda} - S^{(+)}_{\lambda} = \sum_{j_p j_n} [d^{(-)}_{\lambda}(j_p j_n)]^2 \times \Big([u^2_{j_n} y^2_{j_n} + v^2_{j_n} x^2_{j_n}] - [u^2_{j_p} y^2_{j_p} + v^2_{j_p} x^2_{j_p}]\Big). \quad \text{(A.6)}$$

Making use of the relations

$$\sum_{j_{p(n)}} |\langle j_p||t^{(-)}||j_n\rangle|^2 = (2j_{n(p)}+1), \quad \text{(A.7)}$$
$$\sum_{j_{p(n)}} |\langle j_p||\sigma t^{(-)}||j_n\rangle|^2 = 3(2j_{n(p)}+1),$$

and taking into account the particle number conservation within the thermal BCS (14), one finds that $\mathrm{ISR} = N - Z$ for the Fermi transitions and $\mathrm{ISR} = 3(N - Z)$ for the Gamow–Teller transitions.

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