

# Scattering of glueballs with $J = 0, 2$

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# Overview

Introduction

Glueballonium

TMD of GRG model

Conclusions

# Quantum chromodynamics (QCD)

- Non-Abelian gauge theory [gauge group  $SU(N_C = 3)$ ]
- Theory for strong interactions between **gluons** and **quarks**

$$\mathcal{L}_{QCD} = -\frac{1}{4} \sum_{a=1}^8 F^{a\mu\nu} F_{\mu\nu}^a + \sum_{j=1}^{N_f} \bar{q}_j (i\gamma^\mu D_\mu - m_j) q_j$$

$$D_\mu = \partial_\mu - igA_\mu$$

$$F_{\mu\nu}^a = \partial_\mu A_\nu^a - \partial_\nu A_\mu^a + gf^{abc} A_\mu^b A_\nu^c; a, b, c = 1, \dots, N_C^2 - 1$$

- The **blue** part is called Yang-Mills (YM) Lagrangian

# QCD bound states of $q$ and $g$

Baryon number  $\mathfrak{B}_q = 1/3$ ,  $\mathfrak{B}_{\bar{q}} = -1/3$ ,  $\mathfrak{B}_g = 0$

- **Conventional**

$$\text{Hadrons} \left\{ \begin{array}{ll} \text{Baryons} & [qqq] \quad \mathfrak{B} = 1 \\ \text{Mesons} & [\bar{q}q] \quad \mathfrak{B} = 0 \end{array} \right.$$

- **NON Conventional**

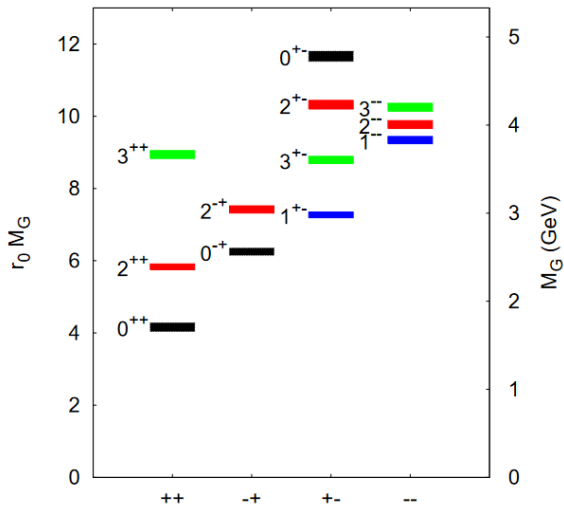
Baryons: pentaquarks ( $\bar{q}qqqq$ ),...  $\mathfrak{B} = 1$

Mesons: tetraquarks ( $\bar{q}\bar{q}qq$ ), glueballs,...  $\mathfrak{B} = 0$

# Glueballs

- Self interaction gluons  $\rightarrow$  bound state (remind talk of M. Huber)
- Several lattice QCD simulations of glueballs
- Scalar glueball stable in pure YM





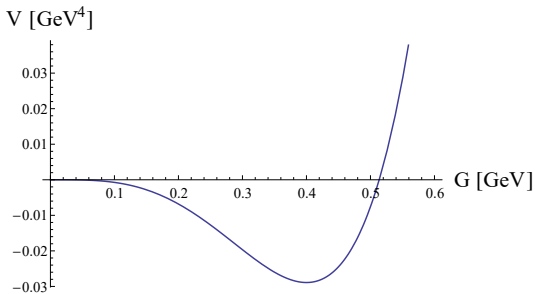
Y. Chen et al, Phys. Rev. D73, 014516 (2006)

# What about the scattering between two scalar glueballs (in YM)?

# Dilaton Lagrangian

$$\mathcal{L}_{dil} = \frac{1}{2}(\partial_\mu G)^2 - V(G) = \frac{1}{2}(\partial_\mu G)^2 - \frac{1}{4} \frac{m_G^2}{\Lambda_G^2} \left( G^4 \ln \left| \frac{G}{\Lambda_G} \right| - \frac{G^4}{4} \right)^{[1]}$$

This form reproduces the trace anomaly in YM



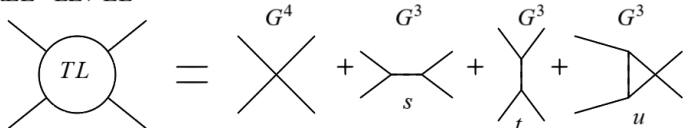
[1] A. A. Migdal and M. A. Shifman, "Dilaton Effective Lagrangian In Gluodynamics," Phys. Lett. B114, 445 (1982)



# Tree-level scattering EPJC 82 (2022)

$$V(G) \approx -\frac{1}{16}\Lambda_G^4 + \frac{1}{2}m_G^2 G^2 + \frac{1}{3!} \left( 5 \frac{m_G^2}{\Lambda_G} \right) G^3 + \frac{1}{4!} \left( 11 \frac{m_G^2}{\Lambda_G^2} \right) G^4 + \dots$$

TREE LEVEL



$$\mathcal{A}(s, t, u) = -11 \frac{m_G^2}{\Lambda_G^2} - \left( 5 \frac{m_G^2}{\Lambda_G} \right)^2 \frac{1}{s - m_G^2} - \left( 5 \frac{m_G^2}{\Lambda_G} \right)^2 \frac{1}{t - m_G^2} - \left( 5 \frac{m_G^2}{\Lambda_G} \right)^2 \frac{1}{u - m_G^2}$$

$$\mathcal{A}(s, t, u) = -11 \frac{m_G^2}{\Lambda_G^2} - \left(5 \frac{m_G^2}{\Lambda_G}\right)^2 \frac{1}{s - m_G^2} - \left(5 \frac{m_G^2}{\Lambda_G}\right)^2 \frac{1}{t - m_G^2} - \left(5 \frac{m_G^2}{\Lambda_G}\right)^2 \frac{1}{u - m_G^2}$$

$$s = (p_1 + p_2)^2,$$

$$t = (p_1 - p_3)^2 = -2k^2(1 - \cos \theta),$$

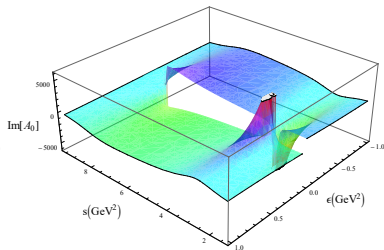
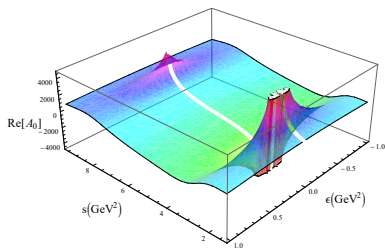
$$u = (p_1 - p_4)^2 = -2k^2(1 + \cos \theta),$$

$$\mathcal{A}(s, t, u) = \mathcal{A}(s, \cos \theta) = \sum_{\ell=0}^{\infty} (2\ell + 1) \mathcal{A}_{\ell}(s) P_{\ell}(\cos \theta)$$

$$\mathcal{A}_0(s) = -11 \frac{m_G^2}{\Lambda_G^2} - 25 \frac{m_G^4}{\Lambda_G^2} \frac{1}{s - m_G^2} + 50 \frac{m_G^4}{\Lambda_G^2} \frac{\log\left(1 + \frac{s - 4m_G^2}{m_G^2}\right)}{s - 4m_G^2}$$

$\mathcal{A}_0(s)$ 

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$$\Delta\delta_0 := \delta_0(s \rightarrow \infty) - \delta_0(4m_G^2) \neq n\pi \rightarrow \text{unitarization}$$

# Unitarization

*Unitarization*

$$U = TL + (TL)\Sigma U$$

Let  $\Sigma(s)$  be a properly subtracted glueball-glueball self-energy loop function, then:

$$\mathcal{A}_0^U(s) = [\mathcal{A}_0^{-1}(s) - \Sigma(s)]^{-1}$$

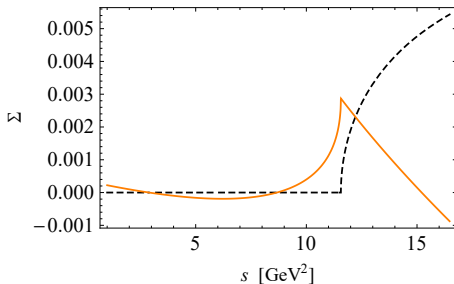
-preserve the pole corresponding to  $s = m_G^2$

- $\mathcal{A}_0^U(s = 3m_G^2) = \infty$ : the branch point  $s = 3m_G^2$  is generated by the single particle pole for  $m_G^2$  along the  $t$  and  $u$  channels.

$$\Sigma(s) = \frac{(s - m_G^2)(s - 3m_G^2)}{\pi} \int_{4m_G^2}^{\infty} \frac{\text{Im}\Sigma(s')}{(s' - s - i\epsilon)(s' - m_G^2)(s' - 3m_G^2)} ds'$$

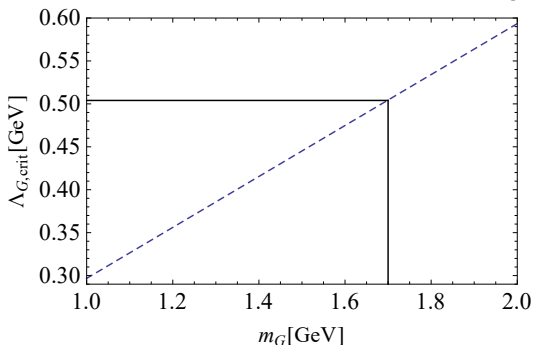
The imaginary part is fixed by (optical theorem)

$$\text{Im}\Sigma(s) = \theta(s - 4m_G^2) \frac{1}{2} \frac{1}{16\pi} \sqrt{1 - \frac{4m_G^2}{s}}$$



$\Lambda_{G,crit}$ 

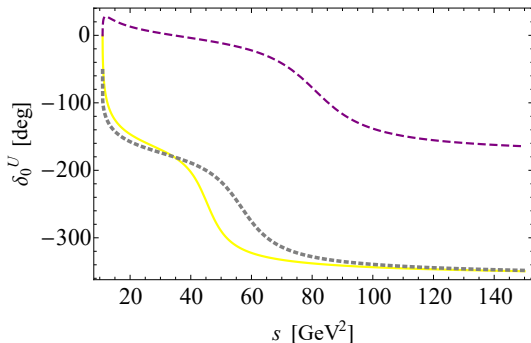
$$\mathcal{A}_0^U(s) = [\mathcal{A}_0^{-1}(s) - \Sigma(s)]^{-1} \xrightarrow{s=s_{th}} \frac{3\Lambda_G^2}{92m_G^2} - \Sigma(4m_G^2) \Big|_{\Lambda_G=\Lambda_{G,crit}} = 0$$



Overall result: attraction  $\longrightarrow$  decreases up to  $\Lambda_{G,crit}$

# Phase-shift

$$\frac{e^{2i\delta_\ell^U(s)} - 1}{2i} = \frac{1}{2} \cdot \frac{k^{2\ell+1}}{8\pi\sqrt{s}} \mathcal{A}_\ell^U(s)$$



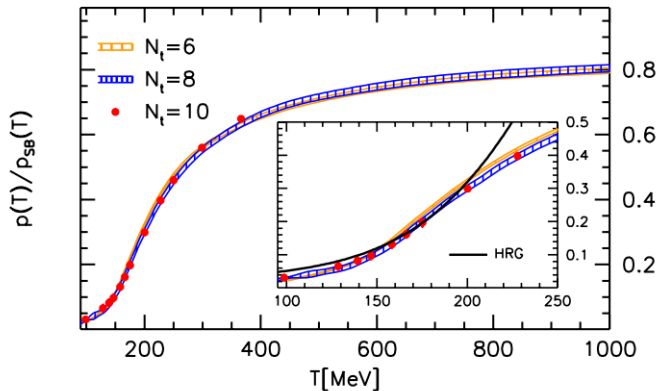
$\Delta\delta_0^U \rightarrow -\pi, -2\pi$  (Levinson theorem fulfilled)

# Can we use these results?



# Hadron Resonance Gas model

- All baryons and mesons ( $m < 2.5$  GeV) from PDG [Borsanyi et al. JHEP11(2010)077]



# Can we think about an analogous method in YM?

NO → Thank you for your attention



YES → GRG model

# Glueball thermodynamics EPJC83 (2023)



# Pressure

$$p_i = -(2J_i + 1)T \int_0^\infty \frac{k^2}{2\pi^2} \ln \left( 1 - e^{-\frac{\sqrt{k^2 + m_i^2}}{T}} \right) dk$$

Lattice calculated  $p$  from Borsanyi et al. JHEP07(2012)056

VS

GRG model with masses from:

**Chen et al.** Phys.Rev.D 69 (2004) 076003

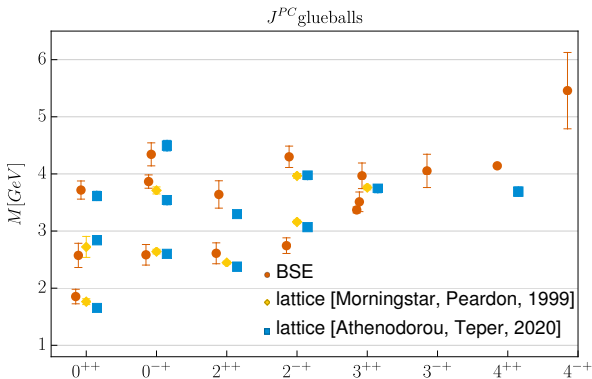
**Athenodorou, Teper** JHEP 11 (2020) 172

**Meyer** hep-lat/0508002 [hep-lat]

$n J^{PC}$	M[MeV]			$n J^{PC}$	M[MeV]		
	Chen et al.	Meyer	A & T		Chen et al.	Meyer	A & T
$1 0^{++}$	1710(50)(80)	1475(30)(65)	1653(26)	$11^{--}$	3830(40)(190)	3240(330)(150)	4030(70)
$2 0^{++}$		2755(30)(120)	2842(40)	$12^{--}$	4010(45)(200)	3660(130)(170)	3920(90)
$3 0^{++}$		3370(100)(150)		$2 2^{--}$		3740(200)(170)	
$4 0^{++}$		3990(210)(180)		$1 3^{--}$	4200(45)(200)	4330(260)(200)	
$1 2^{++}$	2390(30)(120)	2150(30)(100)	2376(32)	$1 0^{+-}$	4780(60)(230)		
$2 2^{++}$		2880(100)(130)	3300(50)	$1 1^{+-}$	2980(30)(140)	2670(65)(120)	2944(42)
$1 3^{++}$	3670(50)(180)	3385(90)(150)	3740(70)	$2 1^{+-}$			3800(60)
$1 4^{++}$		3640(90)(160)	3690(80)	$1 2^{+-}$	4230(50)(200)		4240(80)
$1 6^{++}$		4360(260)(200)		$1 3^{+-}$	3600(40)(170)	3270(90)(150)	3530(80)
$1 0^{-+}$	2560(35)(120)	2250(60)(100)	2561(40)	$2 3^{+-}$		3630(140)(160)	
$2 0^{-+}$		3370(150)(150)	3540(80)	$1 4^{+-}$			4380(80)
$1 2^{-+}$	3040(40)(150)	2780(50)(130)	3070(60)	$1 5^{+-}$		4110(170)(190)	
$2 2^{-+}$		3480(140)(160)	3970(70)				
$1 5^{-+}$		3942(160)(180)					
$1 1^{-+}$			4120(80)				
$2 1^{-+}$			4160(80)				
$3 1^{-+}$			4200(90)				

LQCD papers	Number of glueballs	Lattice Parameter	$T_c$
Chen et al.	12	$r_0^{-1} = 410(20)$ MeV	$317 \pm 23$ MeV
Meyer	22	$\sqrt{\sigma} = 440(20)$ MeV	$277 \pm 13$ MeV
Athenodorou and Teper	20	$r_0^{-1} = 418(5)$ MeV	$323 \pm 18$ MeV

# Masses also from functional methods

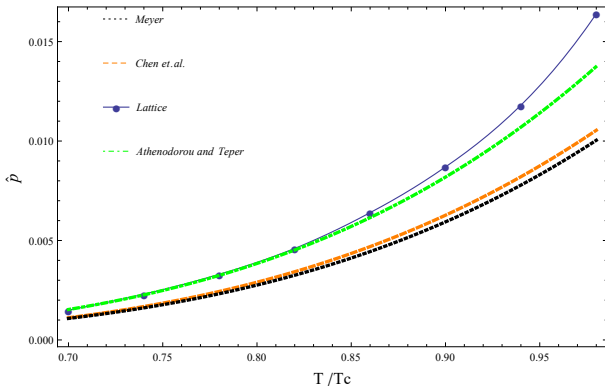


**M. Q. Huber, C. Fischer, H. Sanchis-Alepuz, EPJC 81 (2021) 12, 1083**

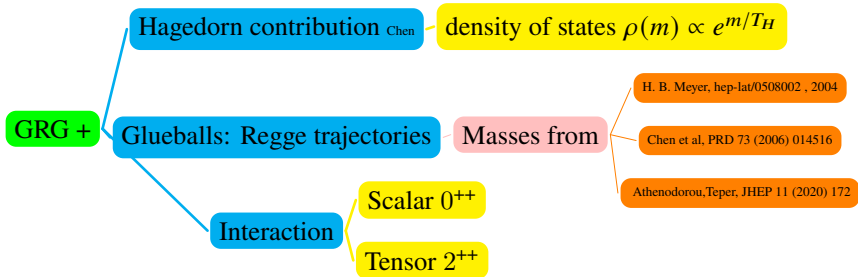
Glueball masses on an absolute scale set by  $r_0^{-1} = 418(5)$  MeV ( $T_c = 323 \pm 18$  MeV)

# Lattice comparison

[Borsnanyi et al.]HEP07(2012)056

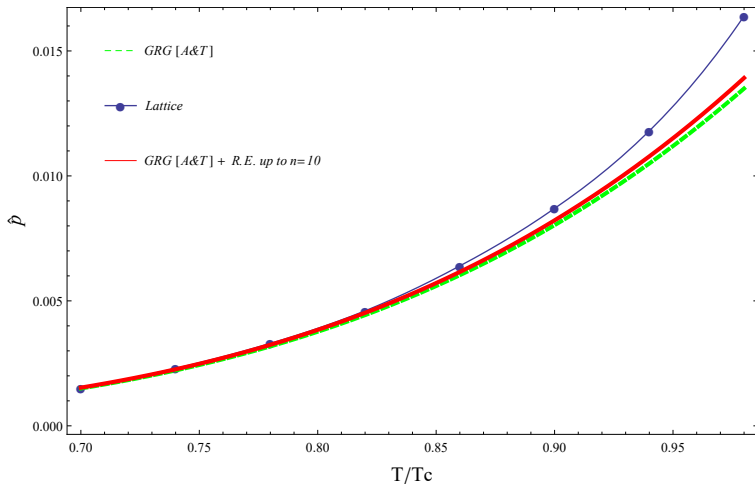


with Chen et al.: Hagedorn term  $\propto e^{m/T_H}$  needed (Meyer PRD 80 (2009))  
with A and T: better agreement without Hagedorn term  $\rightarrow$  hint of accuracy.



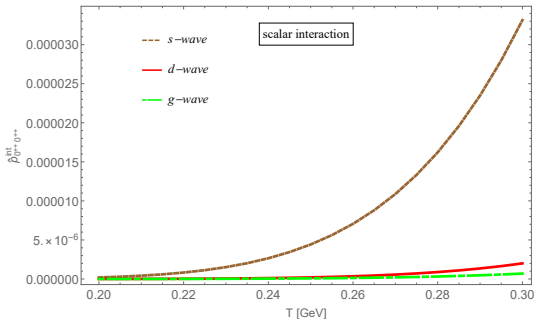


# Additional glueballs



# Scalar glueball

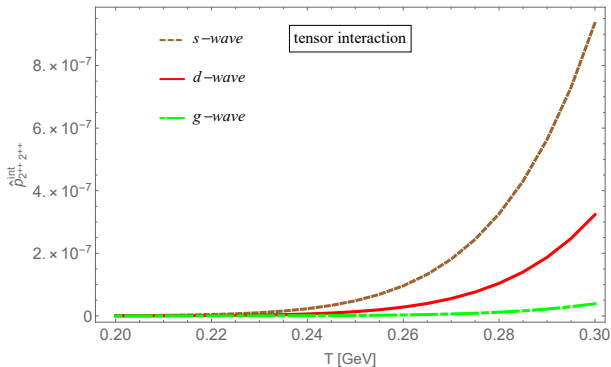
$$\hat{p}_{0^{++}0^{++}}^{\text{int}} = -\frac{1}{T^3} \sum_{l=0}^{\infty} \int_{2m_{0^{++}}}^{\infty} dx \frac{2l+1}{\pi} \frac{d\delta_l^{0^{++}0^{++}}(x)}{dx} \int \frac{d^3k}{(2\pi)^3} \ln\left(1 - e^{-\frac{\sqrt{k^2+x^2}}{T}}\right) + \hat{p}_B$$



Samanta, Giacosa PRD, 102 (2020)

# Tensor glueball

$$\hat{p}_{2^{++}2^{++}}^{\text{int}} = -\frac{1}{T^3} \sum_{J=0}^4 \sum_{l=0}^{\infty} \int_{2m_2^{++}}^{\infty} dx (2J+1) \frac{2l+1}{\pi} \frac{d\delta_l^{2^{++}2^{++},J}(x)}{dx} \int \frac{d^3k}{(2\pi)^3} \ln \left( 1 - e^{-\frac{\sqrt{k^2+x^2}}{T}} \right)$$



- Emergence of a 2 scalar glueball bound state, that we named "glueballonium". If existent, could be seen in experiments -G mix, unstable- (PANDA, BesIII, BelleII, LHCb, TOTEM) and in lattice (Yamanaka et al. PRD102 (2020)).
- Free glueball gas with lightest states: sufficient for TMD description of LQCD results for pure YM.
- The critical temperature in YM seems to be near  $T_C \approx 323$  MeV.
- Effect of heavier glueballs and of interactions are very small.
- GRG works well with the masses of Athenodorou & Teper → those masses are favoured.

# Thank you for your attention

## Phase shift

$$A_\ell(s) = \frac{1}{2} \int_{-1}^1 d \cos \theta A(s, \cos \theta) P_\ell(\cos \theta) .$$

where  $P_\ell(\cos \theta)$  are the Legendre polynomials.

$$\hat{A}_\ell(s) = \frac{1}{k^{2l}} A_\ell(s) .$$

Scattering lengths:

$$a_\ell = \frac{\hat{A}_\ell(4m_G^2)}{32\pi m_G} .$$

hence

$$\delta_\ell(s) = \frac{1}{2} \arg \left[ 1 + 2i\rho_\ell(s)\hat{A}_\ell(s) \right] ,$$

$$\frac{1}{2} \cdot \frac{k^{2\ell+1}}{8\pi\sqrt{s}} = \rho_\ell(s)$$

# YM field

$$\mathcal{L}_{\text{YM}} = -\frac{1}{4} G_{\mu\nu}^a G^{a,\mu\nu}, \quad G_{\mu\nu}^a = \partial_\mu A_\nu^a - \partial_\nu A_\mu^a + g_0 f^{abc} A_\mu^b A_\nu^c$$

gluonic quantum fluctuation: dilatation symmetry

$$g_0 \xrightarrow[\text{anomaly}]{\text{trace}} g(\mu)$$

$$\Rightarrow \partial_\mu J_{\text{dil}}^\mu = (T_\mu^\mu)_{\text{YM}} = \frac{\beta(g)}{2g} G_{\mu\nu}^a G^{a,\mu\nu} \neq 0, \quad \beta(g) = \partial g / \partial \ln \mu$$

At one loop ( $\beta(g) = -bg^3$  &  $b = 11N_c/(48\pi^2)$ ):

$$g^2(\mu) = \frac{1}{2b \ln(\mu/\Lambda_{\text{YM}})}, \quad \Lambda_{\text{YM}} \approx 250 \text{ MeV}$$

Nonvanishing expectation value of the trace anomaly:

$$\langle T_\mu^\mu \rangle_{\text{YM}} = -\frac{11N_c}{24} \left\langle \frac{\alpha_s}{\pi} G_{\mu\nu}^a G^{a,\mu\nu} \right\rangle = -\frac{11N_c}{24} C^4$$

# Glueball field

Gluons  $\xrightarrow{\text{confinement}}$  not the asymptotic states of the theory

Scalar field  $G$  describing scalar glueball & trace anomaly at the composite level

$$\mathcal{L}_{\text{dil}} = \frac{1}{2} (\partial_\mu G)^2 - V(G)$$

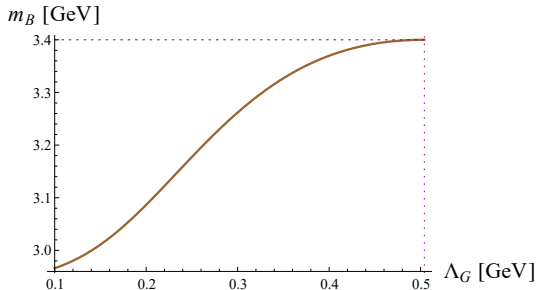
$$\partial_\mu J_{\text{dil}}^\mu = 4V - G \partial_G V \propto G^4 \text{ only if } V(G) = \frac{1}{4} \frac{m_G^2}{\Lambda_G^2} \left( G^4 \ln \left| \frac{G}{\Lambda_G} \right| - \frac{G^4}{4} \right)$$

$\Lambda_G \simeq 0.4 \text{ GeV}$ :  $\left[ \partial_\mu J_{\text{dil}}^\mu = -\frac{1}{4} \frac{m_G^2}{\Lambda_G^2} G^4 \right] \langle T_\mu^\mu \rangle_{YM}$ : Dilaton field saturates the trace of the dilatation current.



## Mass of the glueballonium as function of $\Lambda_G$

For  $\Lambda_G = 0.4$  GeV: bound state (glueballonium) with a mass of 3.37 GeV.  
For  $m_B = 2m_G = 3.4$  GeV one has the critical value  $\Lambda_{G,crit} \sim 0.5$  GeV.



$$T_c = 1.26(7) \cdot \Lambda_{MS} = 1.26(7) \cdot 0.614(2) \cdot r_0^{-1}, \quad (1)$$

$$T_c = 0.629(3) \cdot \sqrt{\sigma}. \quad (2)$$

LQCD papers	Number of glueballs	Lattice Parameter	$T_c$ (using Eqs. (1)-(2))
Chen et.al	12	$r_0^{-1} = 410(20) \text{ MeV}$	$317 \pm 23 \text{ MeV}$
Meyer	22	$\sqrt{\sigma} = 440(20) \text{ MeV}$	$277 \pm 13 \text{ MeV}$
Athenodorou and Teper	20	$r_0^{-1} = 418(5) \text{ MeV}$	$323 \pm 18 \text{ MeV}$

## Additional glueballs

$$\chi^2(a, b_{0^{++}}, b_{2^{++}}, b_{0^{-+}}, b_{2^{-+}}, b_{1^{+-}}) = \sum_{J^{PC}} \left( \frac{M(n, J^{PC}) - M^{\text{lat}}(n, J^{PC})}{\delta M^{\text{lat}}(n, J^{PC})} \right)^2$$

Glueball spectrum compared to the fit				Parameters [GeV <sup>2</sup> ]
$n J^{PC}$	$m$ [GeV] (A and T)	Fit [GeV]	$\chi_i^2$	
<b>1 0<sup>++</sup></b>	1.653(26)	1.647(25)	0.04	$b_{0^{++}} = -2.78 \pm 0.21$
<b>2 0<sup>++</sup></b>	2.842(40)	2.865(30)	0.3	
<b>1 2<sup>++</sup></b>	2.376(32)	2.367(30)	0.08	$b_{2^{++}} = -10.87 \pm 0.57$
<b>2 2<sup>++</sup></b>	3.30(5)	3.33(3)	0.38	
<b>1 0<sup>-+</sup></b>	2.561(40)	2.572(38)	0.08	$b_{0^{-+}} = 1.12 \pm 0.27$
<b>2 0<sup>-+</sup></b>	3.54(8)	3.48(4)	0.57	
<b>1 2<sup>-+</sup></b>	3.07(6)	3.11(5)	0.52	$b_{2^{-+}} = -6.79 \pm 0.66$
<b>2 2<sup>-+</sup></b>	3.97(7)	3.90(4)	1.10	
<b>1 1<sup>+-}</sup></b>	2.944(42)	2.955(37)	0.07	$b_{1^{+-}} = -2.25 \pm 0.45$
<b>2 1<sup>+-}</sup></b>	3.80(6)	3.77(3)	0.23	
			$\chi_{\text{tot}}^2 = 3.38$	$a = 5.49 \pm 0.17$

# Trace anomaly

