International School
of Nuclear Physics, 44th Course
Erice, September 18-24, 2023

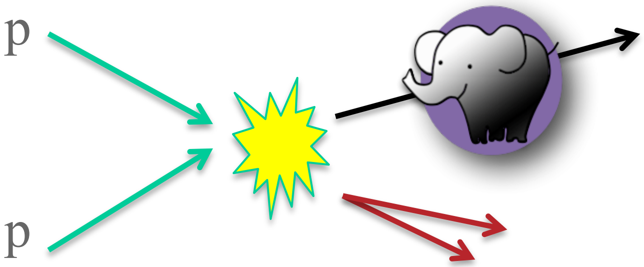
Meson form factors and their impact on the muon g-2

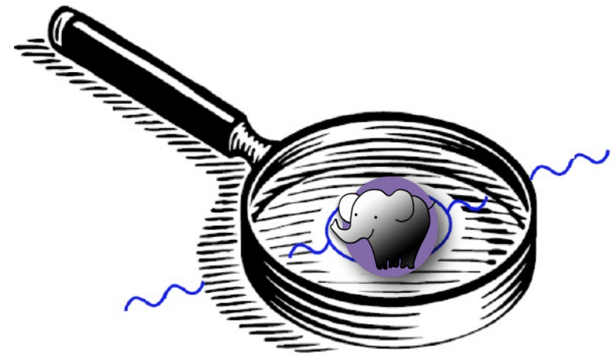
Project TFF
Project XYZ
FOR 5327
Project JRP
Project ALP
Project LBL

Cluster of Excellence PRISMA+
Precision Physics, Fundamental Interactions and Structure of Matter

Achim Denig
JGU Mainz

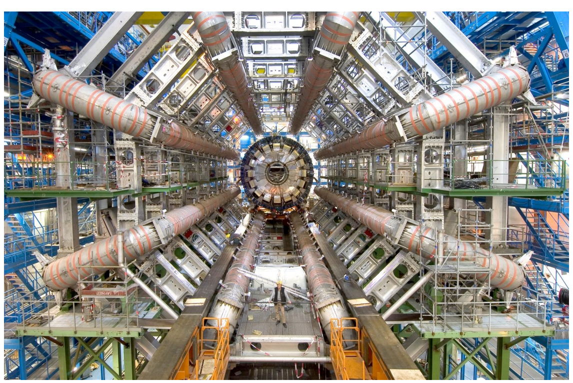
The Search for Physics beyond the Standard Model

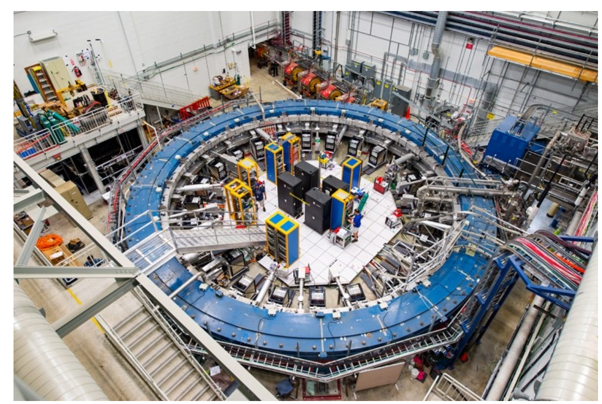




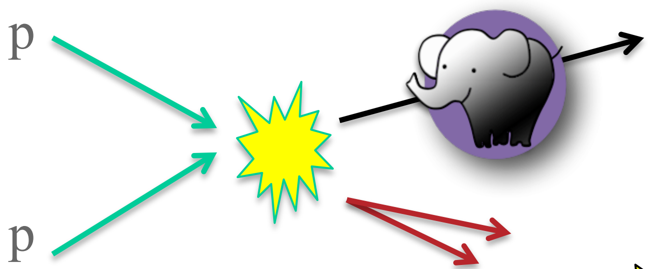
High Energy Frontier
(e.g. LHC/CERN)

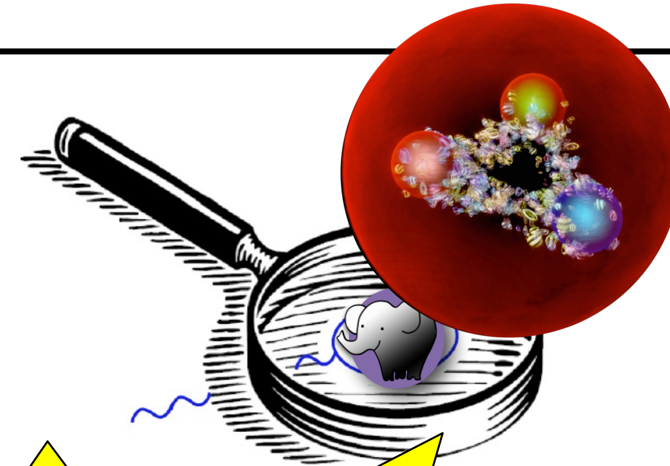
Precision Frontier
(g-2, Flavour Ph., ...)





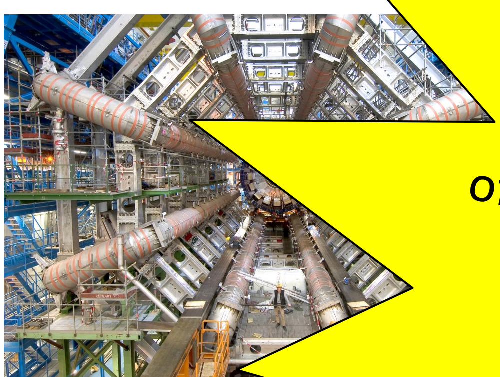
The Search for Physics beyond the Standard Model





High Energy Frontie
(e.g. LHC(CERN))

reci Frontier
our Ph., ...)



Improved understanding of
Hadron Physics
of utmost importance for BSM searches via
precision physics

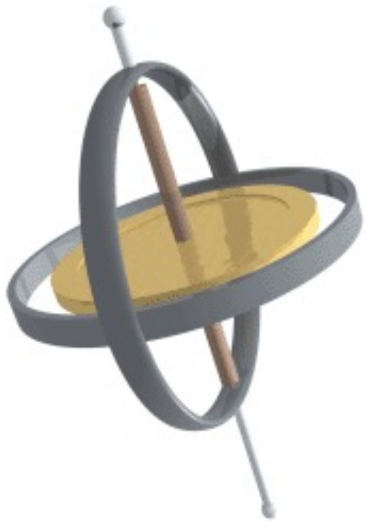
Muon Magnetic Moment: $(g\text{-}2)_\mu$

Confront a high-precision SM prediction with a high-precision measurement

Definition: $\vec{\mu} = \mu_B \cdot g \cdot \vec{S}$

Dirac: $g = 2$

QFT: $a_\mu = (g-2)/2 \approx {}^{\alpha}/_{\pi}$



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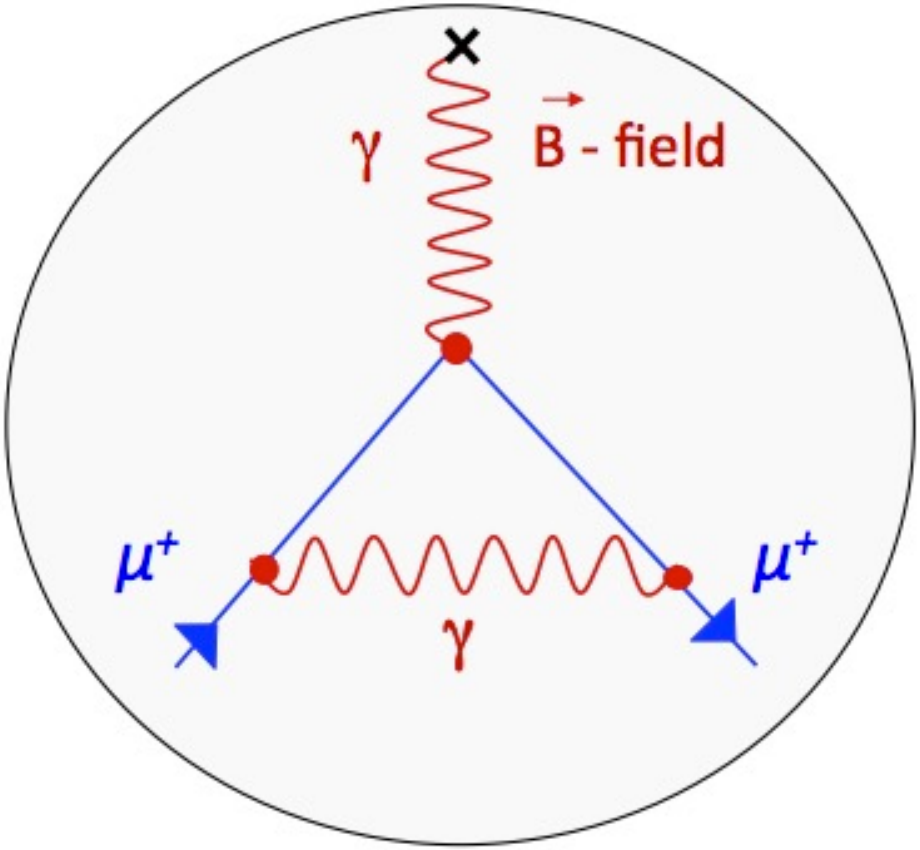

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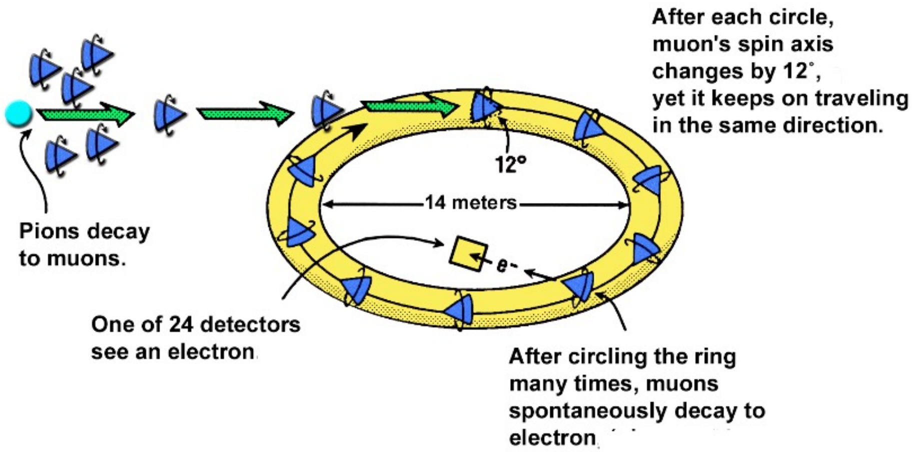
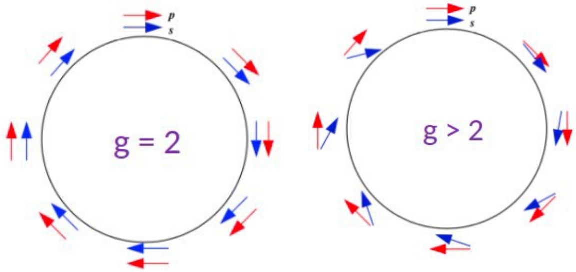
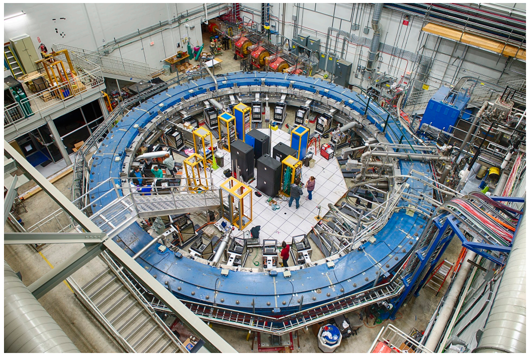




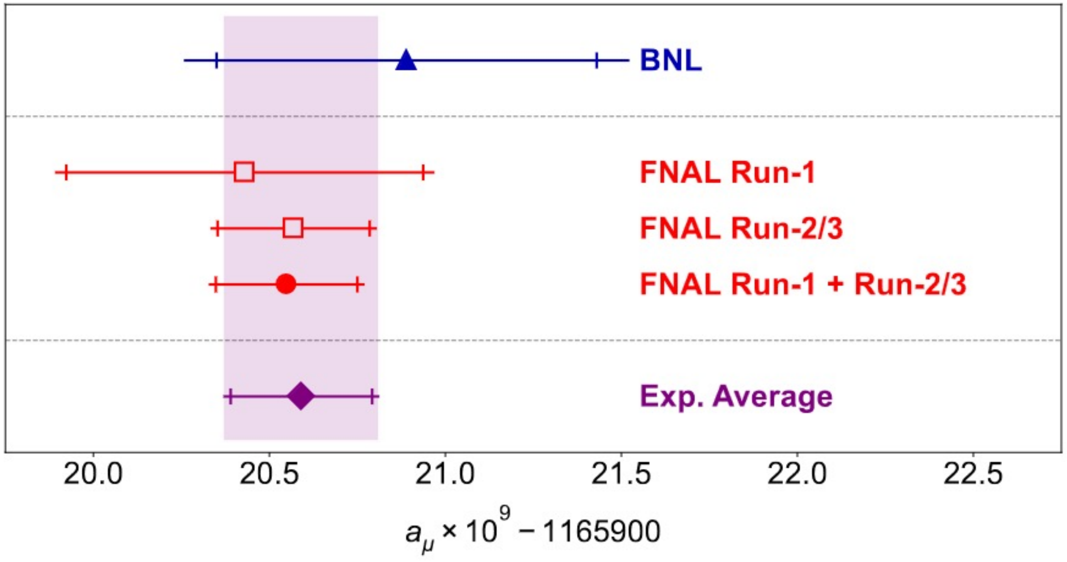
$$\vec{\omega}_a = \vec{\omega}_s - \vec{\omega}_c = -a_\mu \frac{q\vec{B}}{m}$$

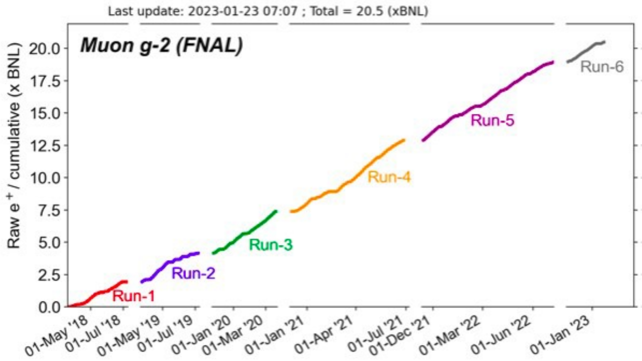
Fermilab $(g\text{-}2)_\mu$ Experiment

- FNAL Run 1 in agreement with BNL measurement
- 08/23: FNAL Run 2 + 3 in agreement with Run 1
- FNAL Run 1-6, further reduction of uncertainty by a factor of 2 !



arxiv:2308.06230



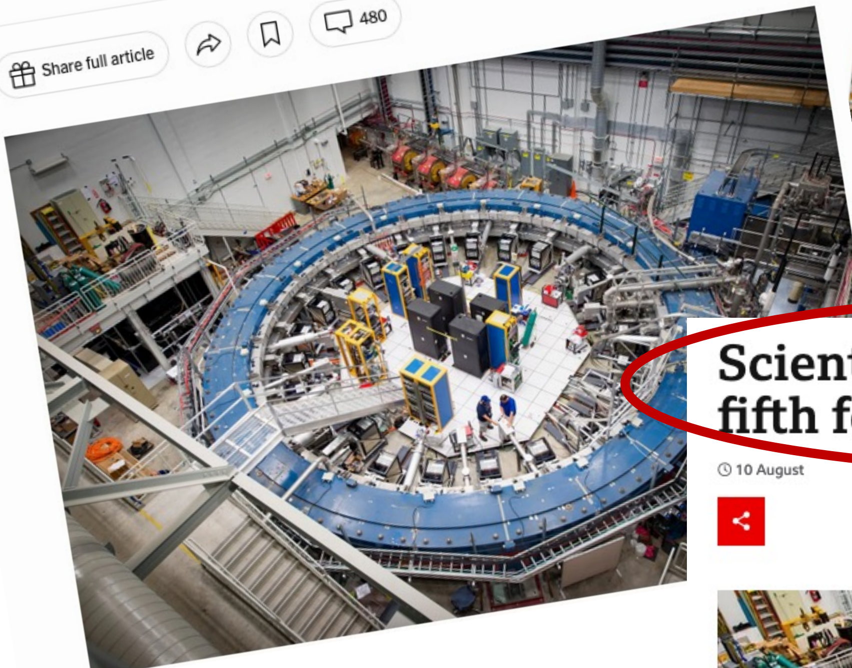


$$a_\mu^{exp} = (11\,659\,205.9 \pm 2.2) \cdot 10^{-10}$$ **statistics dominated error**

Physicists Move One Step Closer to a Theoretical Showdown

The deviance of a tiny particle called the muon might prove that one of the most well-tested theories in physics is incomplete.

Share full article 480



CERNCOURIER Reporting on international high-energy physics

Physics Technology Community In focus Magazine

SEARCHES FOR NEW PHYSICS | NEWS

New Muon g-2 result bolsters earlier measurement

10 August 2023

Scientists at Fermilab close in on fifth force of nature

BBC NEWS

10 August

Standard Model Prediction of $(g-2)_\mu$

EW contributions: A **trimph** of perturbative QFT and computing

$$a_\mu^{SM} = a_\mu^{QED} + a_\mu^{weak} + a_\mu^{had}$$

Absolute contribution dominated by QED

Uncertainty dominated by hadronic contribution

Czarnecki et al.

$(15.4 \pm \mathbf{0.2}) \cdot 10^{-10}$

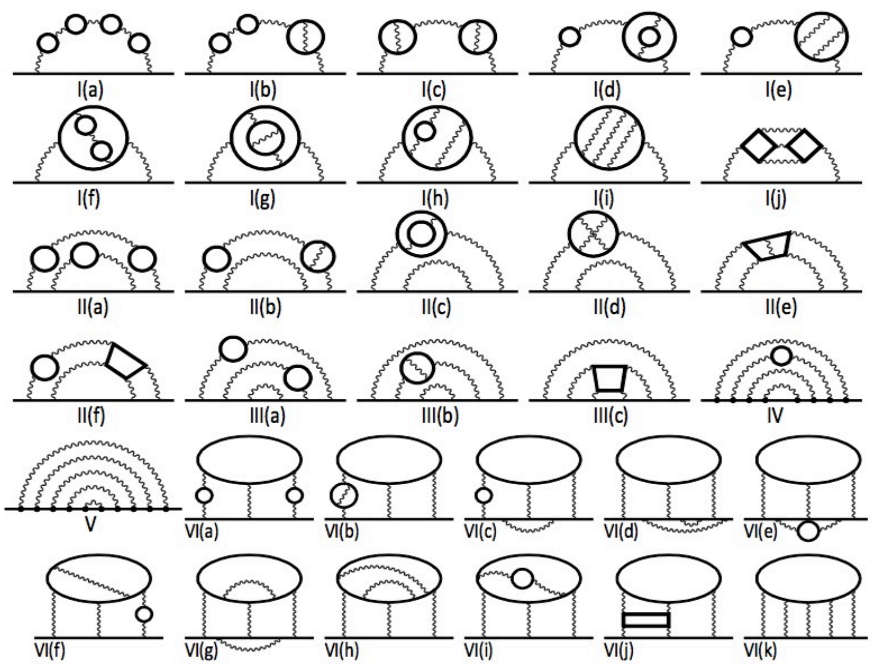
Kinoshita et al. '12

$(11\,658\,471.808 \pm \mathbf{0.015}) \cdot 10^{-10}$

10th

12672

diagrams



Standard Model Prediction of $(g\text{-}2)_\mu$

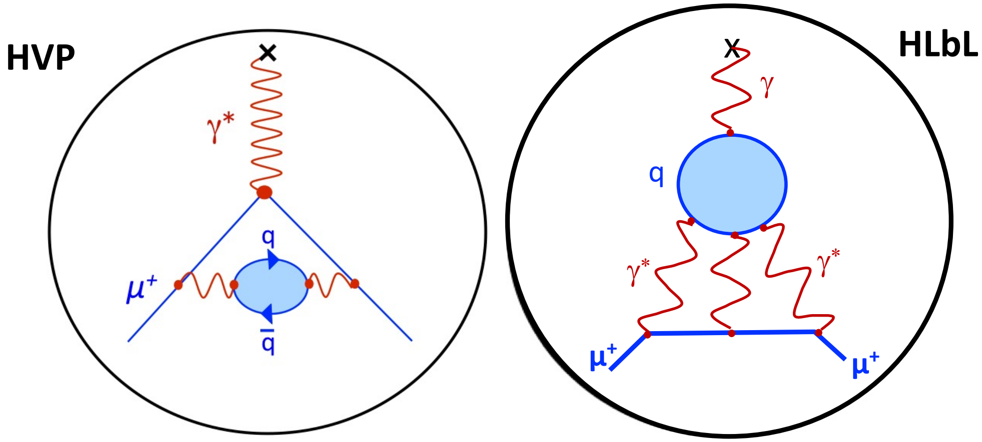
Hadronic contribution **non-perturbative, the limiting** contribution

$$a_\mu^{SM} = a_\mu^{QED} + a_\mu^{weak} + a_\mu^{had}$$

→ **HVP**: Hadronic Vacuum Polarization (≅ **687 ... 694** ± **2.4 ... 4.1**) · 10^{-10}

	BDJ19	DHMZ19	FJ17	KNT19
$a_\mu^{\text{HVP, LO}} \times 10^{10}$	687.1(3.0)	694.0(4.0)	688.1(4.1)	692.8(2.4)

→ **HLbL**: Hadronic Light-by-Light (10.5 ± **2.6**) · 10^{-10} Glasgow „consensus“ value



NLO (-9.8 ± **0.1**) · 10^{-10} ;
NNLO (1.2 ± **0.01**) · 10^{-10}

B. Kubis
D.G.M. Porras

$(g\text{-}2)_\mu$ Theory Initiative (since 2017)


FERMILAB-PUB-20-207-T
INT-PUB-20-021
KEK Preprint 2020-5
MITP/20-028

CERN-TH-2020-075
IFT-UAM/CSIC-20-74
LMU-ASC 18/20
LTH 1234
LU TP 20-20
MAN/HEP/2020/003
PSI-PR-20-06
UWThPh 2020-14
ZU-TH 18/20

The anomalous magnetic moment of the muon in the Standard Model

hep-ph] 8 Jun 2020

196 pages, 103 figures

2006.04822

Goal:

theory consensus value of muon g-2 SM prediction (most relevant hadronic contributions!)

- Working groups on HVP, HLbL, LatticeQCD, ...
- Six collaboration meetings and various workshops on subtopics
- Scrutiny of various theoretical evaluations


Bern 2023


FNAL 2017


Mainz 2018

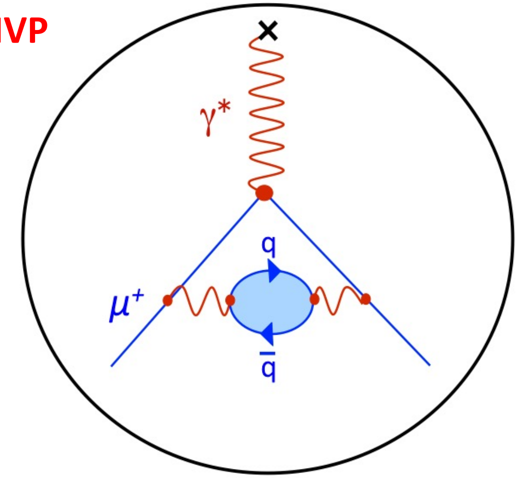

Seattle 2019

KEK (virtual) 2021


Edinburgh 2022

Hadronic Vacuum Polarization (HVP)

HVP



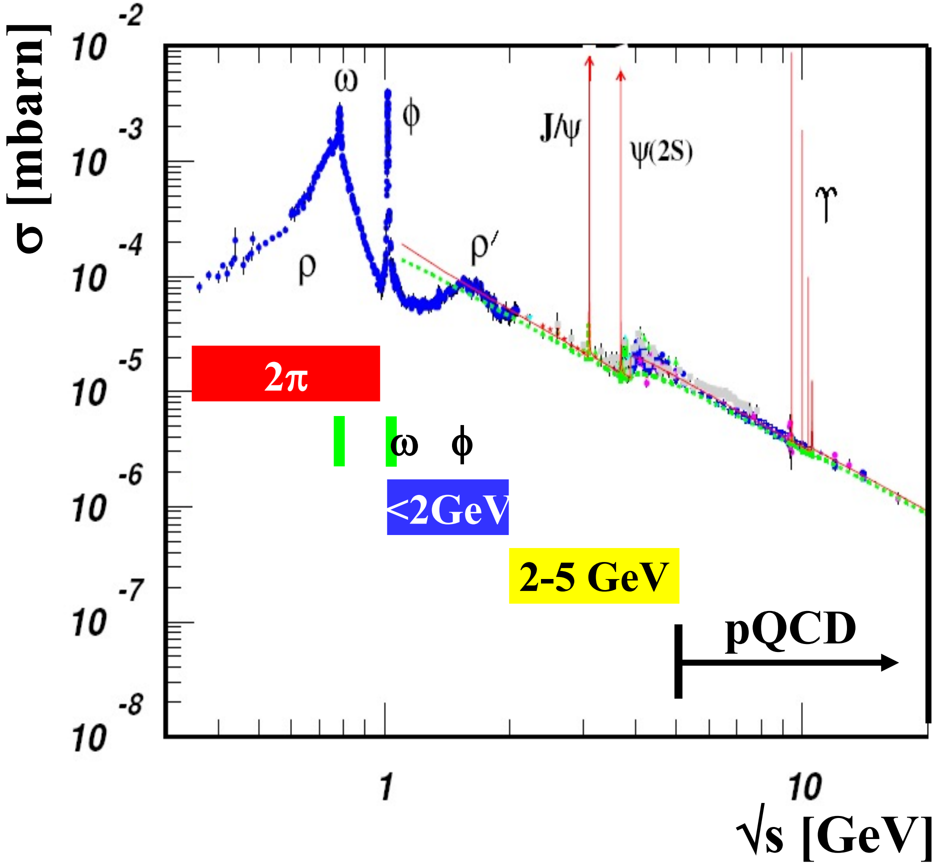
Estimate of (g-2) Theory Initiative based on dispersive approach (including higher orders):

(**693.1 ± 4.0**) · 10^{-10}

was (≅ 687 ... 694 ± 2.4 ... 4.1) · 10^{-10}

Hadronic Vacuum Polarization Contrib. to (g-2)$_\mu$

$$a_\mu^{HVP} = \frac{1}{4\pi^3}\int_{4m_\pi^2}^{\infty} ds\, K(s)\, \boldsymbol{\sigma_{\mathrm{had}}(s)}$$

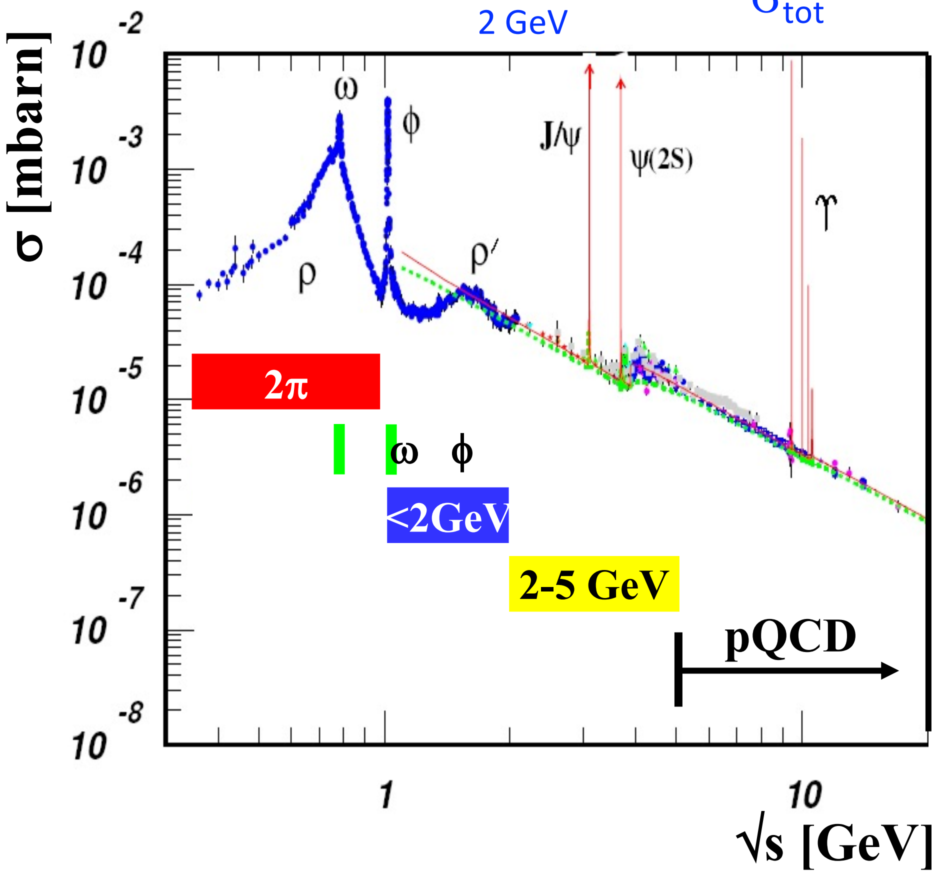


Intrinsic ~ 1 / s^2
low energy contributions
especially important!

Hadronic Vacuum Polarization Contrib. to $(g\text{-}2)_\mu$

exclusive data | inclusive data σ_{tot}

2 GeV



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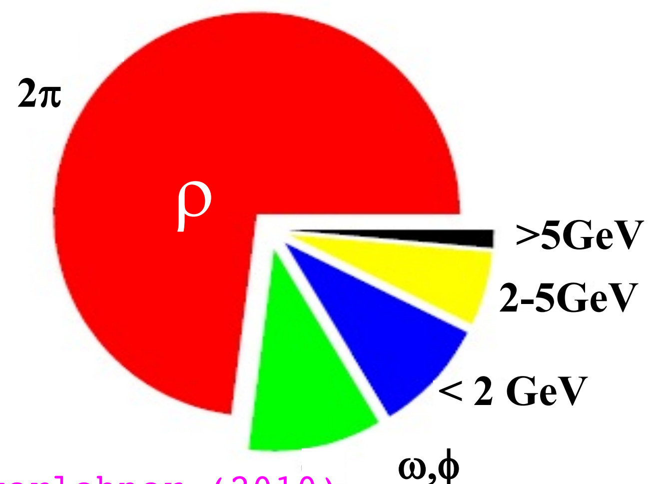
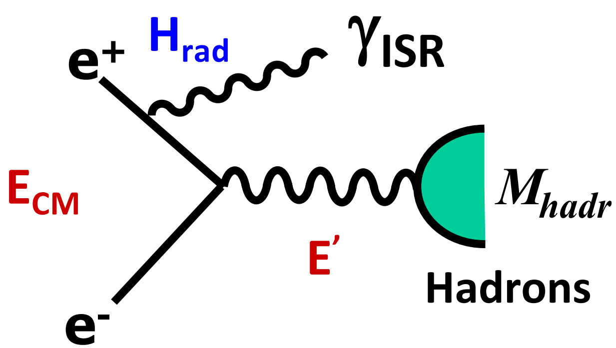
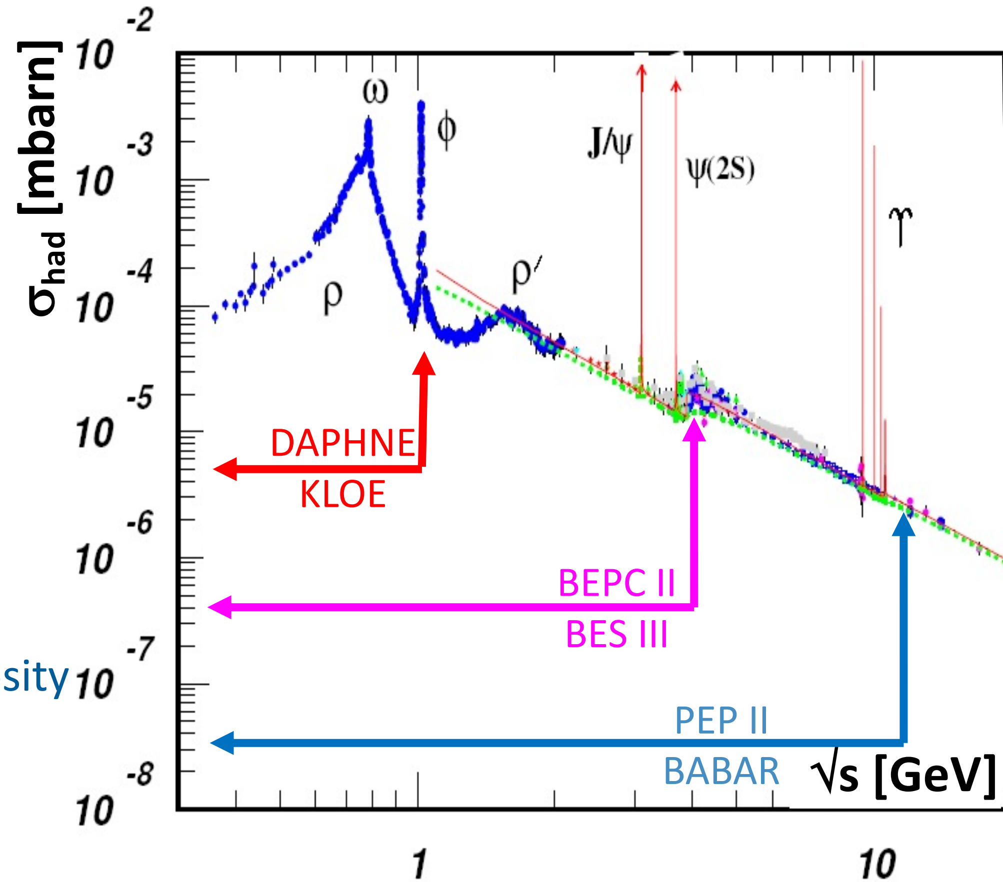


Jegerlehner (2010)

Initial State Radiation (ISR)

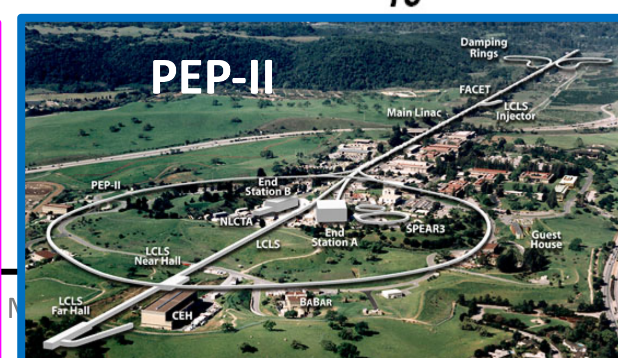
Initial State Radiation (ISR) aka Radiative Return



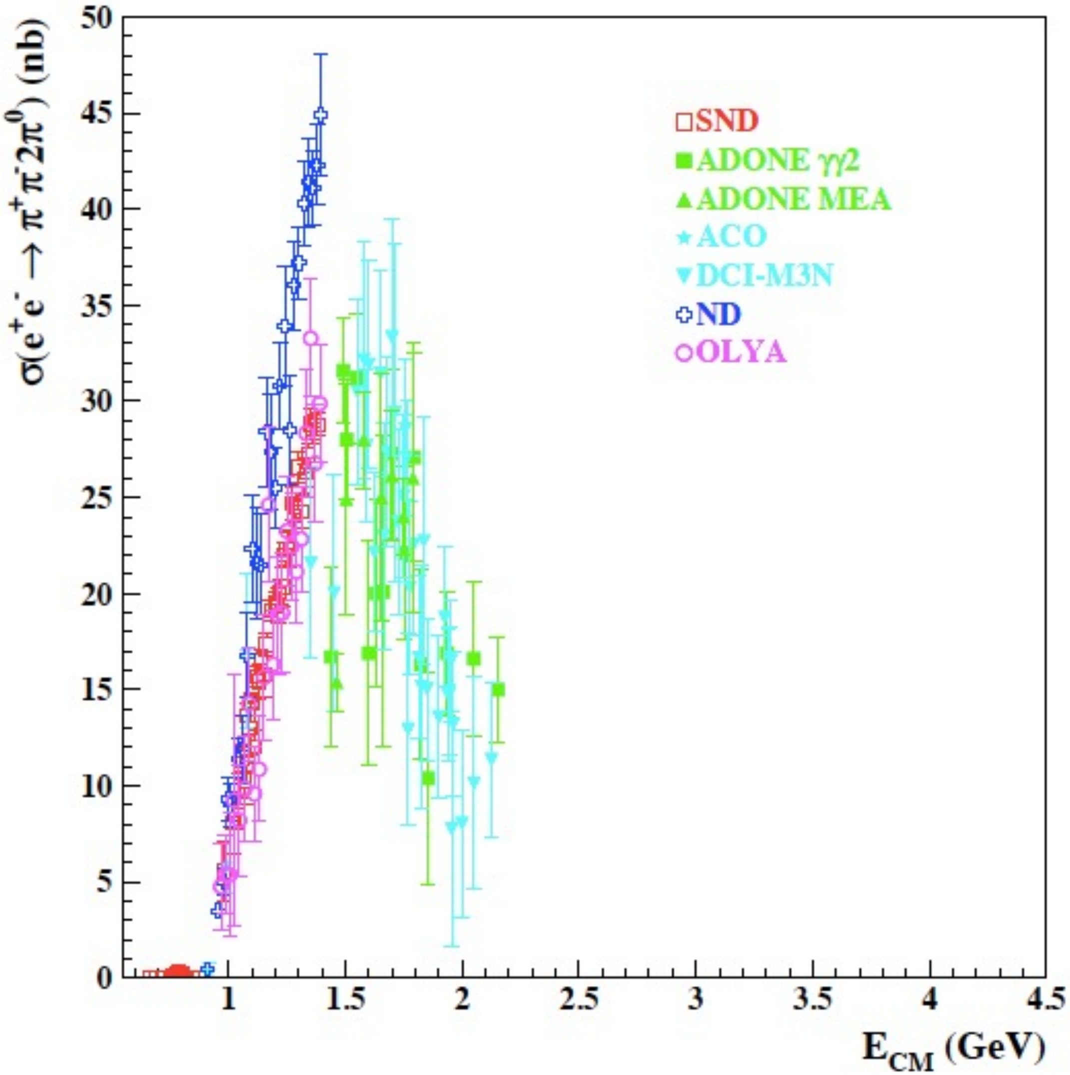


- No systematic variation of E_{beam}
- High statistics thanks to high luminosity
- Precise knowledge of radiative corrections mandatory (H_{rad})

`PHOKHARA event generator`



BABAR: $e^+e^- \to \pi^+\pi^-2\pi^0\gamma_{ISR}$



- Tagged ISR analysis (2017)
  Phys. Rev. D96 (2017) 092009
- Huge improvement over existing data sets suffering from normalization issues
- Confirmed by BES III analysis (preliminary)

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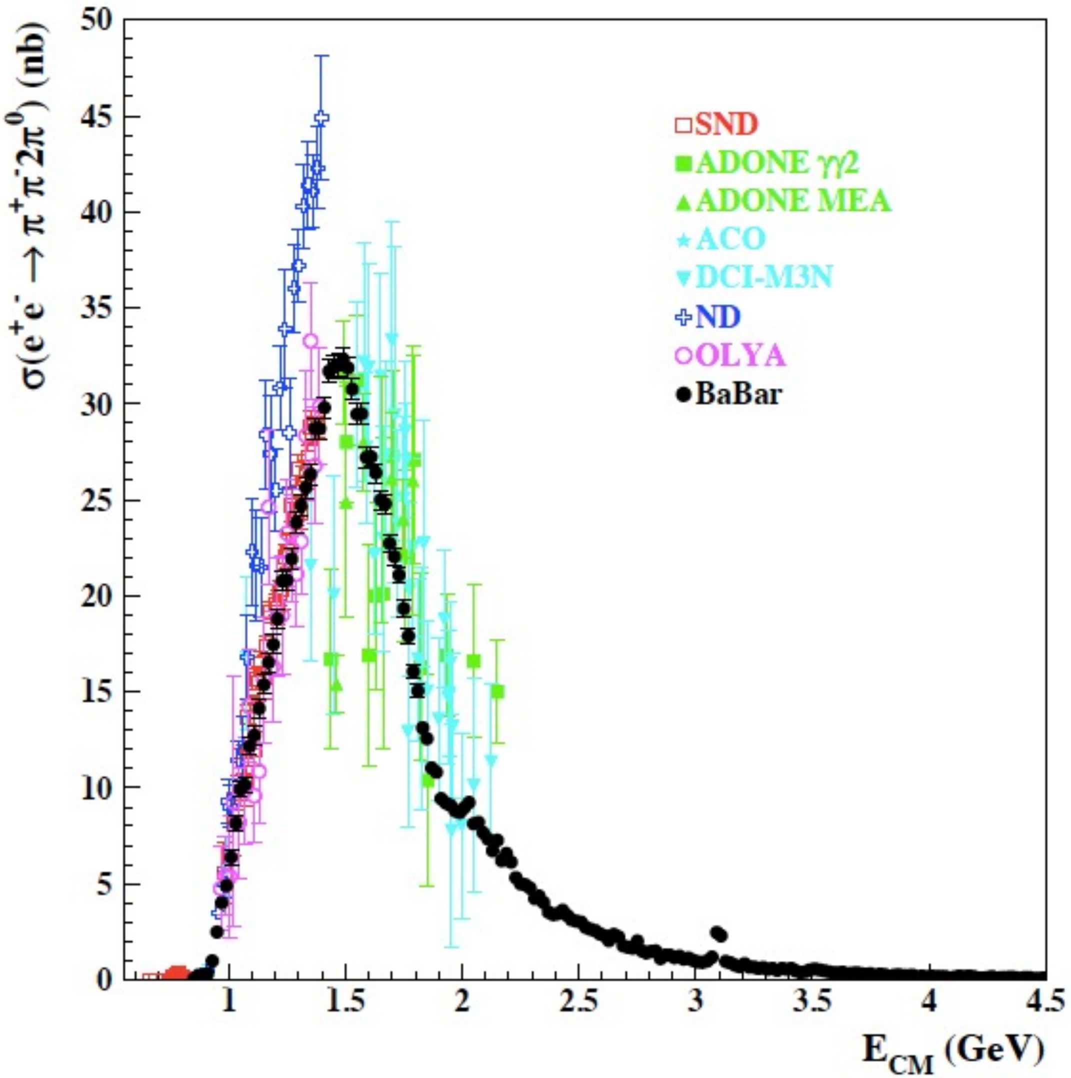

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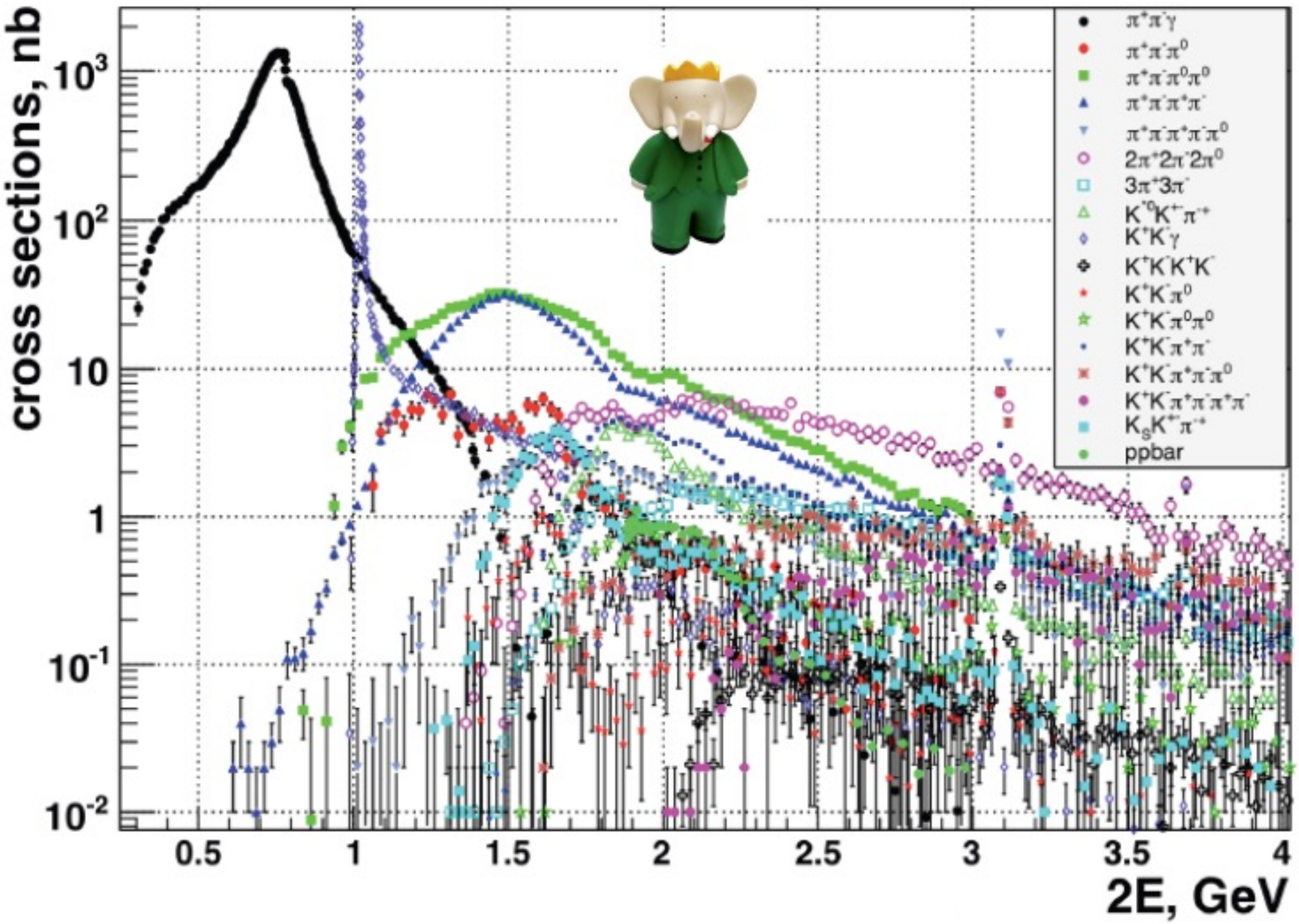
BaBar ISR Results



Precision:

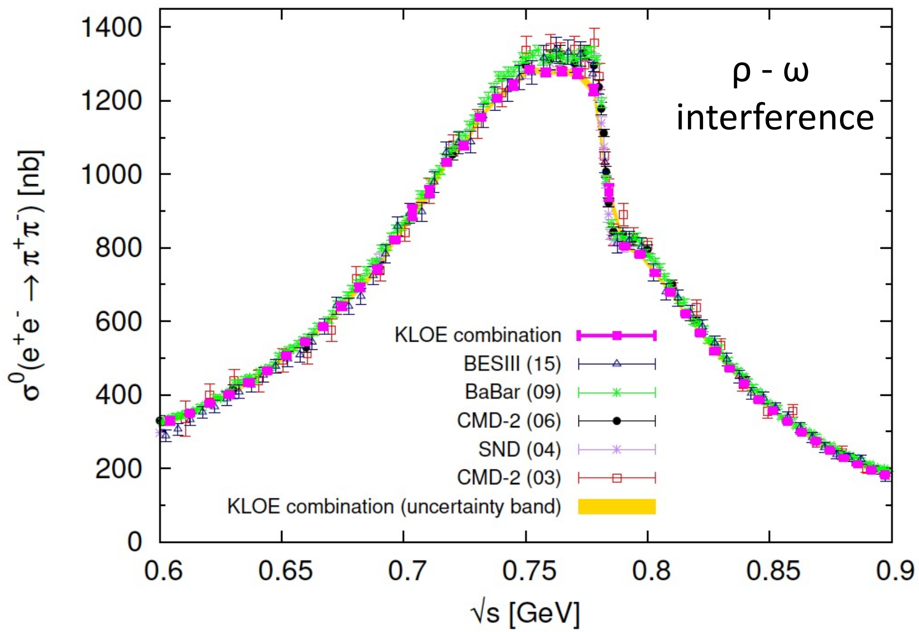
2π: < 1%

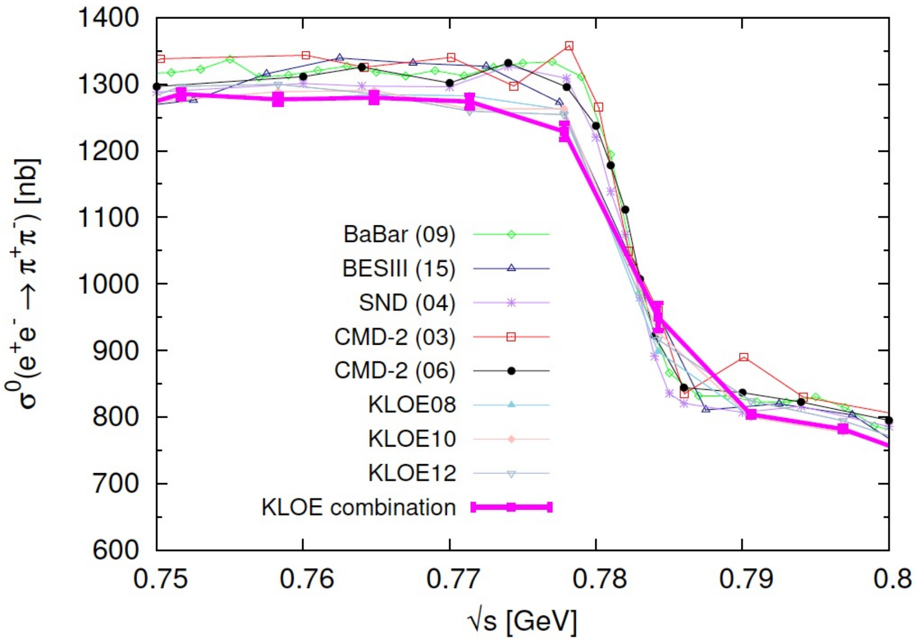
3π: ~10%

4π: ~ 3%

$\geq 5\pi$: 10% and higher

Most relevant Channel: $e^+e^- \rightarrow \pi^+\pi^-$



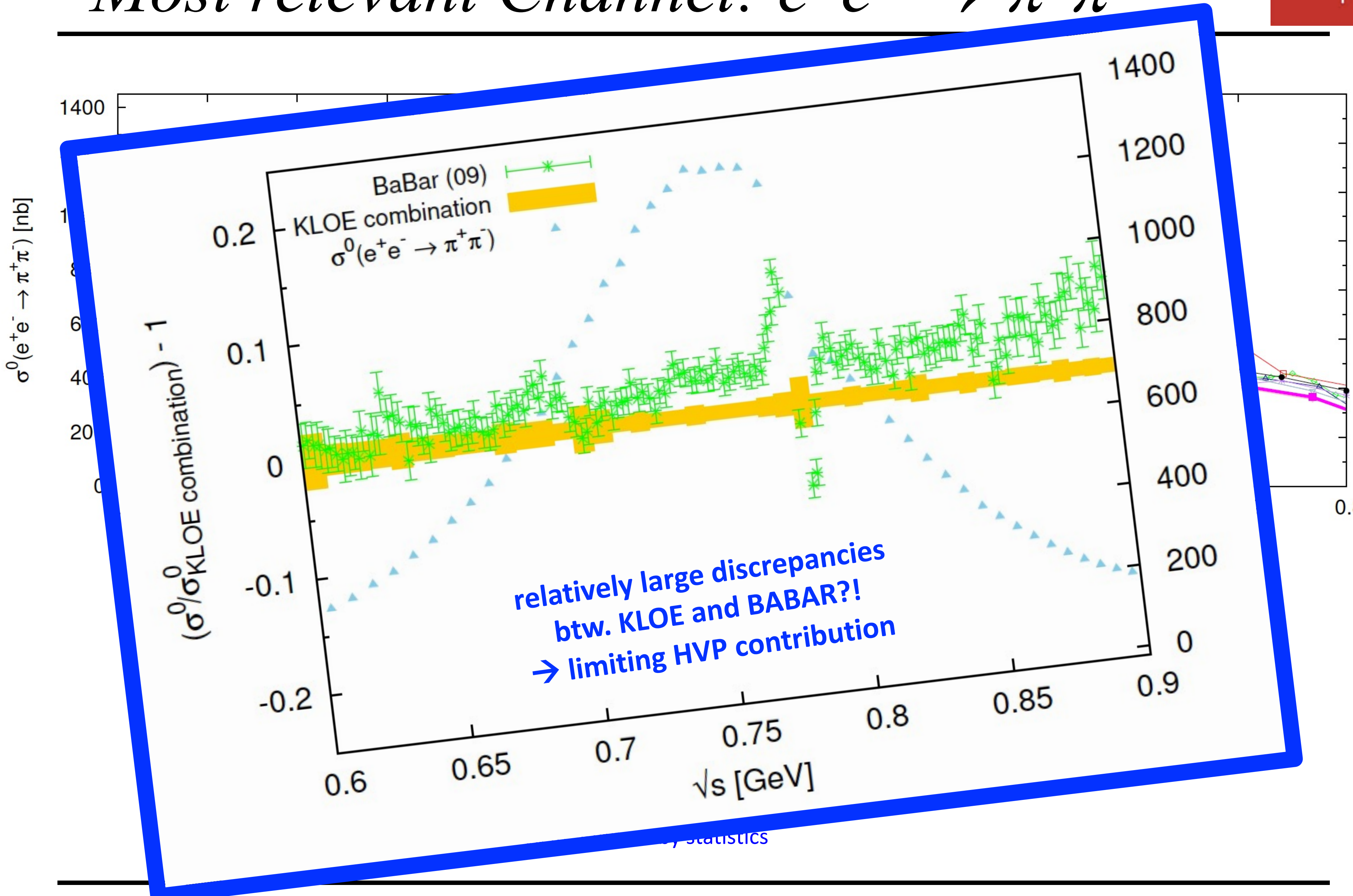


Systematic Uncertainties on ρ(770) peak

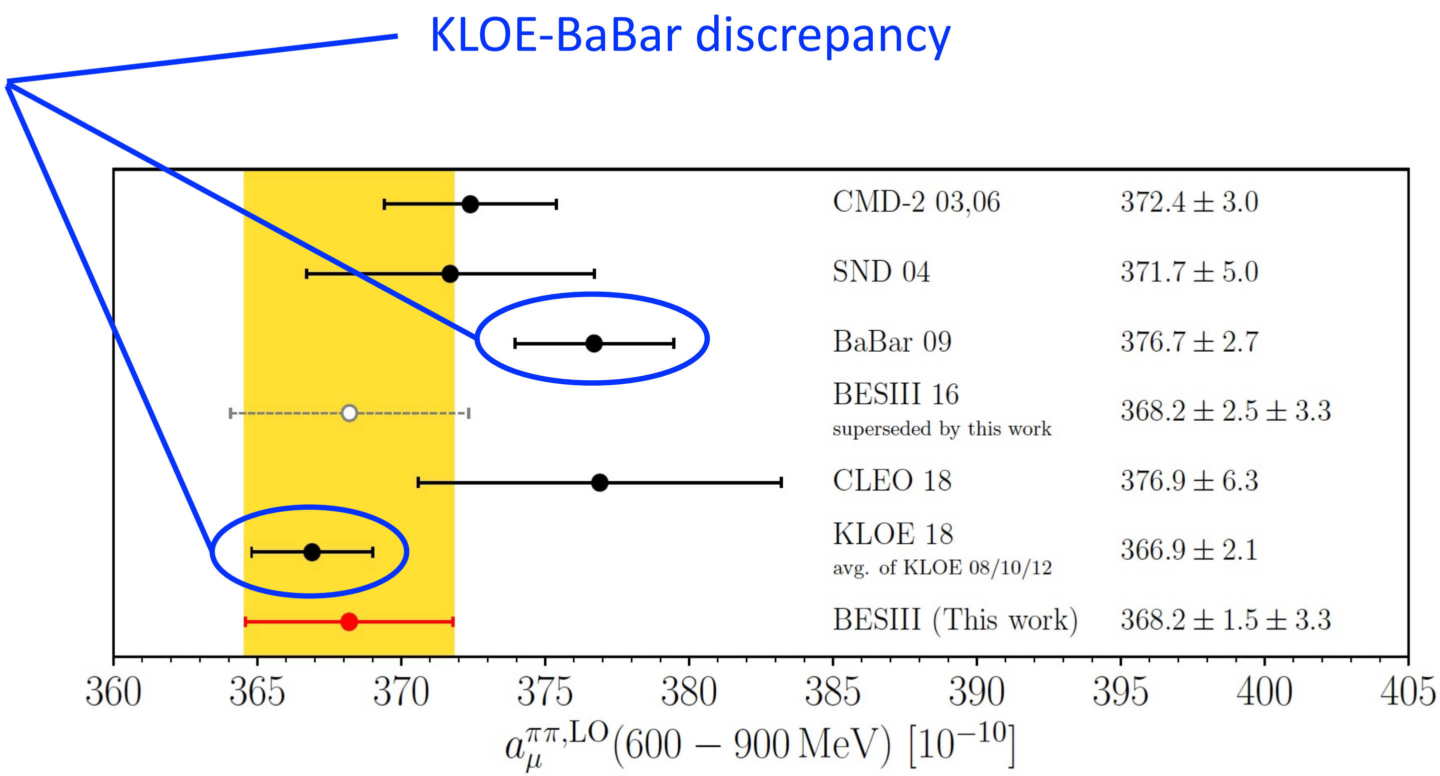
- ISR BABAR 0.5%
- ISR KLOE 0.6% (average of 3 analyses)
- ISR BESIII 0.9%
- Energy Scan CMD2 0.8%*

* limited in addition by statistics

Most relevant Channel: $e^+e^- \rightarrow \pi^+\pi^-$



Knowledge of the 2π Contribution

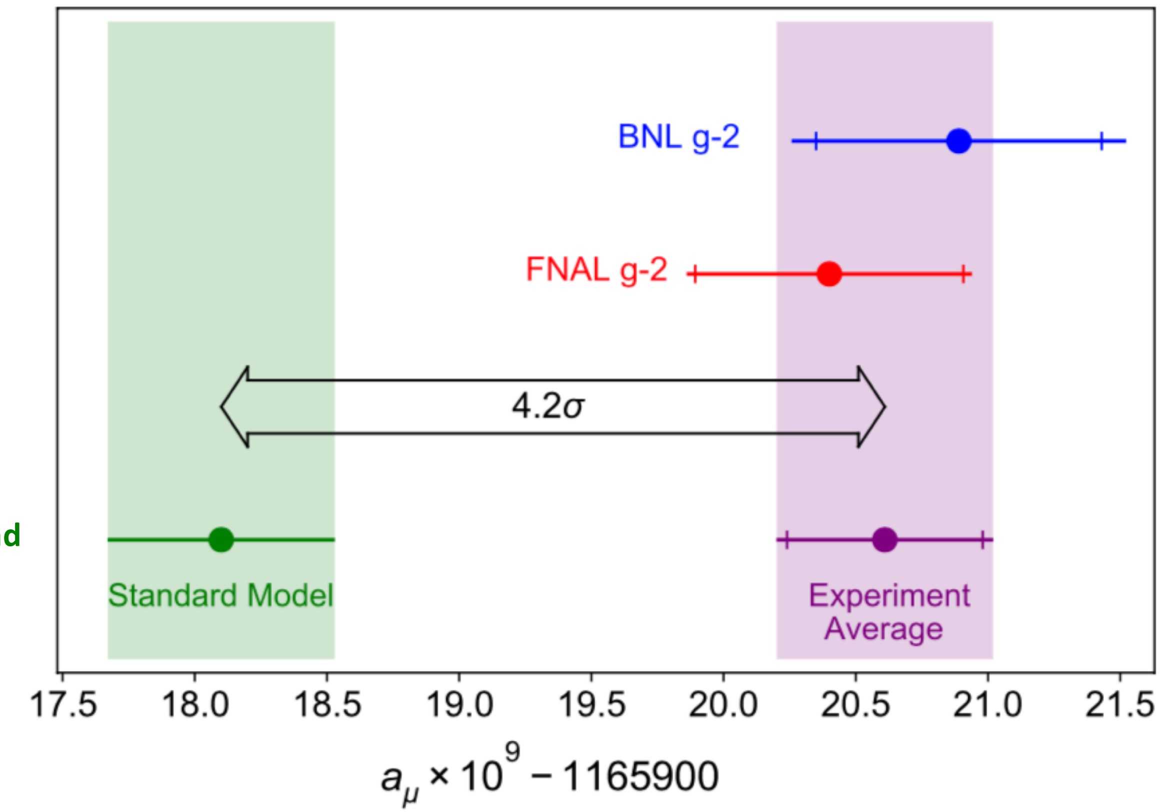


<u>Situation 2021</u>: Knowledge of 2π contribution to HVP largely limits accuracy of SM prediction to the muon g-2

→ New data needed to clarify situation ….

*Situation **2021**: Muon Magnetic Moment: $(g\text{-}2)_\mu$*

$$a_\mu^{SM} = a_\mu^{QED} + a_\mu^{weak} + a_\mu^{had}$$



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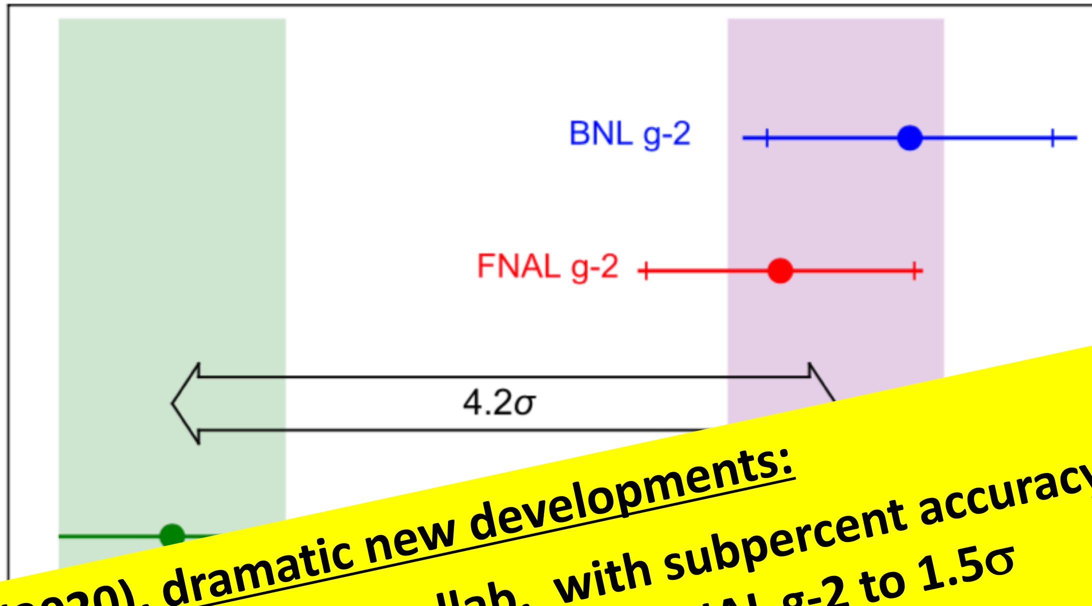


<u>Since release of whitepaper (2020), dramatic new developments:</u>

2021 Lattice QCD calculation of HVP by BMW collab. with subpercent accuracy
→ reduces discrepancy between SM prediction and FNAL g-2 to 1.5σ

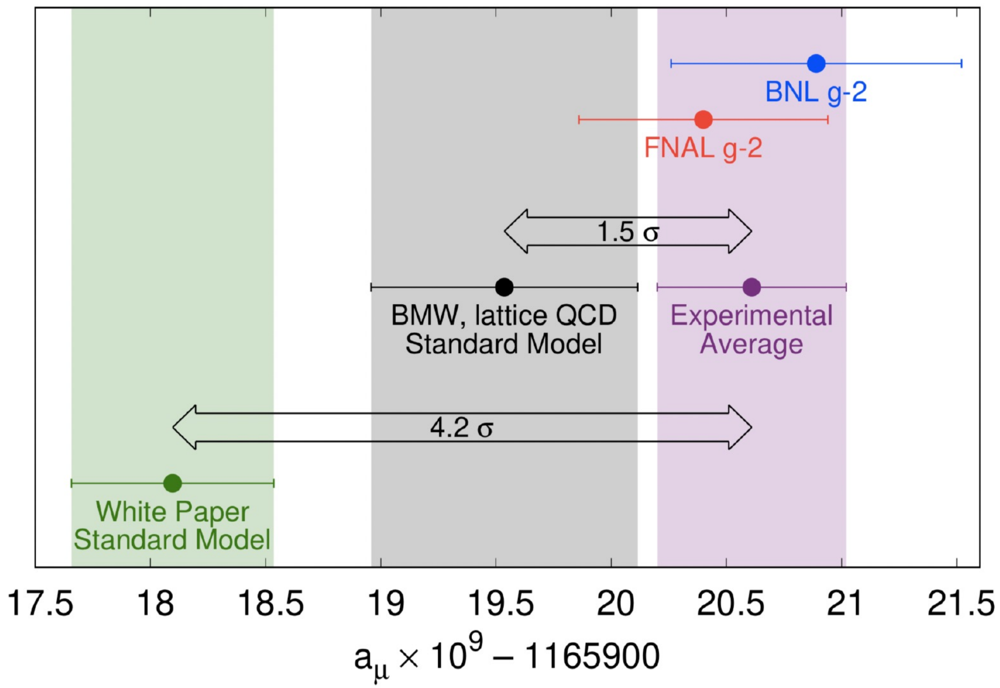
2022 Lattice QCD window fever

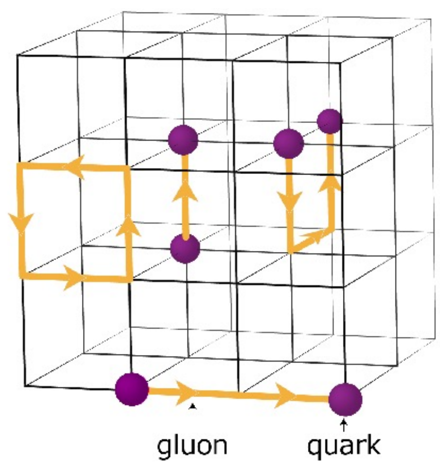
2023 CMD-3 measurement of pion FF: $e^+e^- \to \pi^+\pi^-$

2022: Lattice-QCD calculation of HVP

First time ab-initio precision calculation of HVP contribution by means of Lattice QCD (BMW 2021)

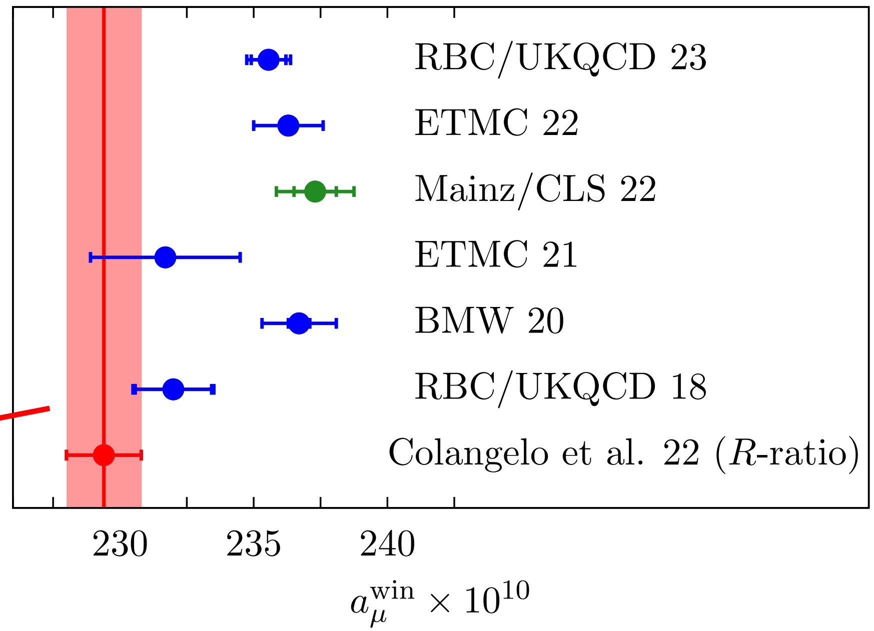
Similar accuracy as dispersive data-based calculation





2022: Lattice-QCD Window Fever

Lattice QCD calculations (spacelike!) with largest systematic effects at small and long distances → **intermediate window (~1/3 of total HVP contribution)!**



Dispersive data-driven value

Courtesy Hartmut Wittig

Excellent agreement among all Lattice QCD calculations and disagreement with data-driven approach confirmed ! However, effect is quite (too?) large in intermediate window?!

2023 Shock: CMD-3@Novosibirsk $e^+e^- \rightarrow \pi^+\pi^-$

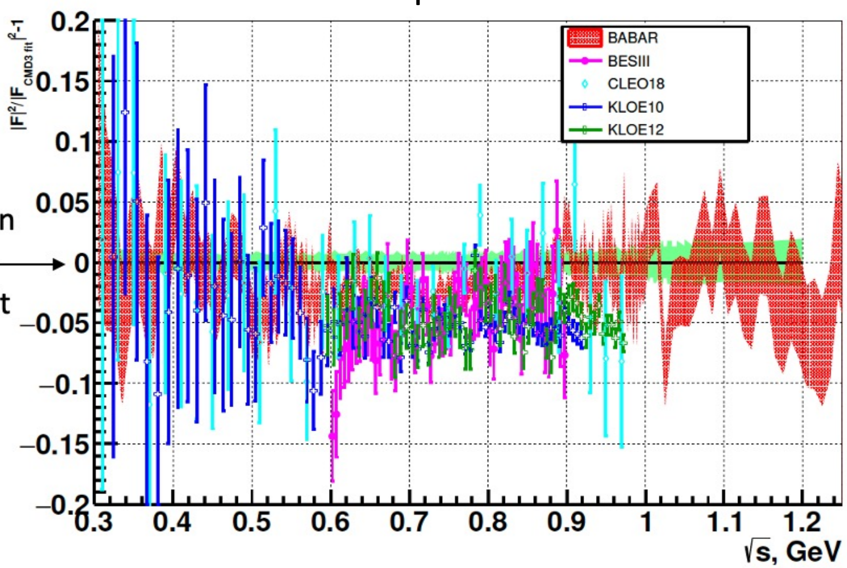
arxiv:2302.08834

- New result from CMD-3 collaboration @ VEPP-2000 collider in Novosbirsk
- Energy scan method, no ISR!
- Energy range from threshold up to 1.2 GeV
- Highest statistics data sample up to now, systematic uncertainty 0.7% on ρ peak

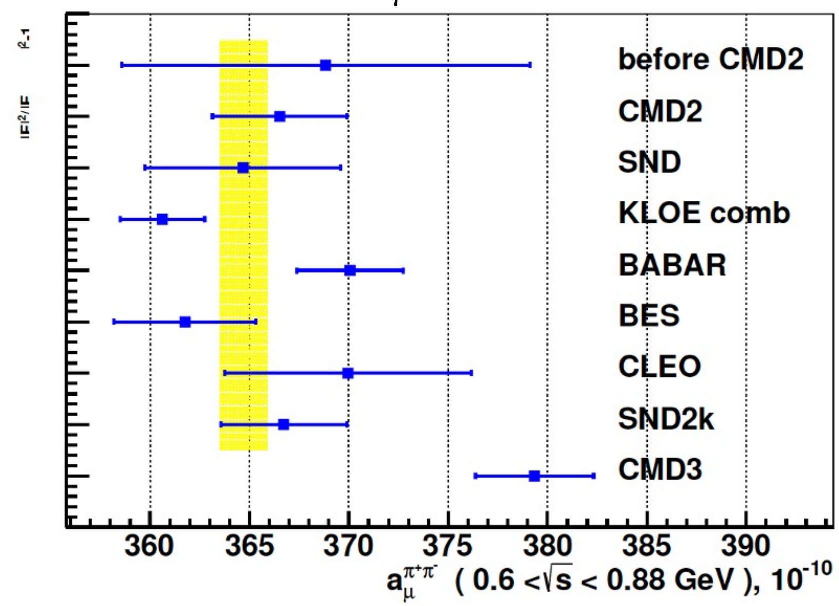
→ **Significant deviation from previous ISR and energy scan experiments ! Why?**

Differential spectrum

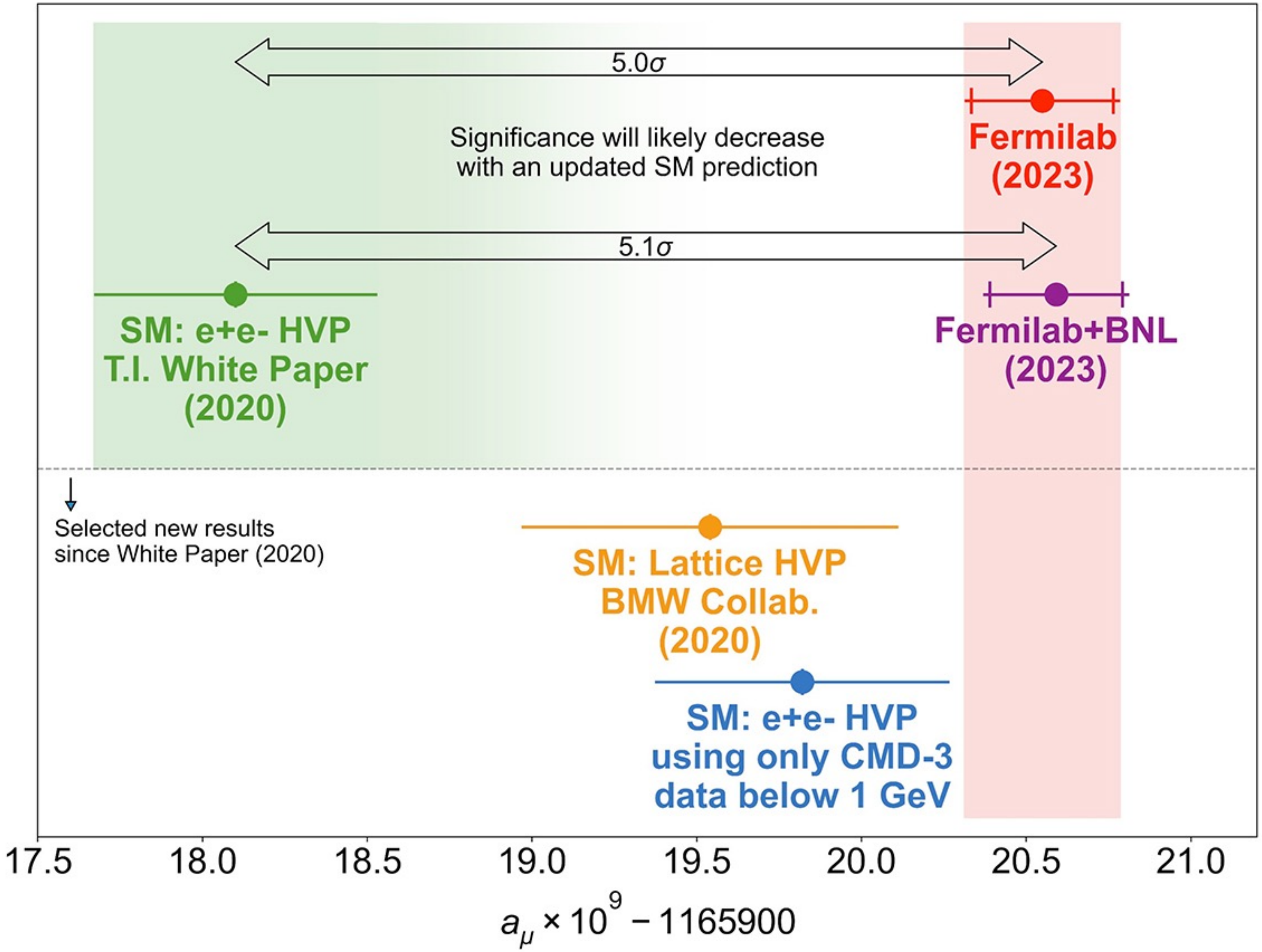
Deviation from CMD3 fit →



(g-2) Integral $a_\mu^{\pi\pi}$ (600 − 880 MeV)

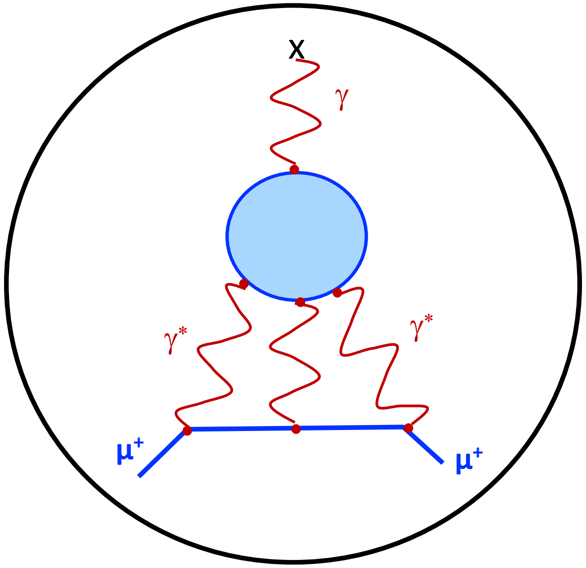


*Situation **2023**: Muon Magnetic Moment: $(g\text{-}2)_\mu$*



*Situation **2023**: Muon Magnetic Moment: $(g\text{-}2)_\mu$*

Hadronic Light-by-Light Contribution (HLbL)

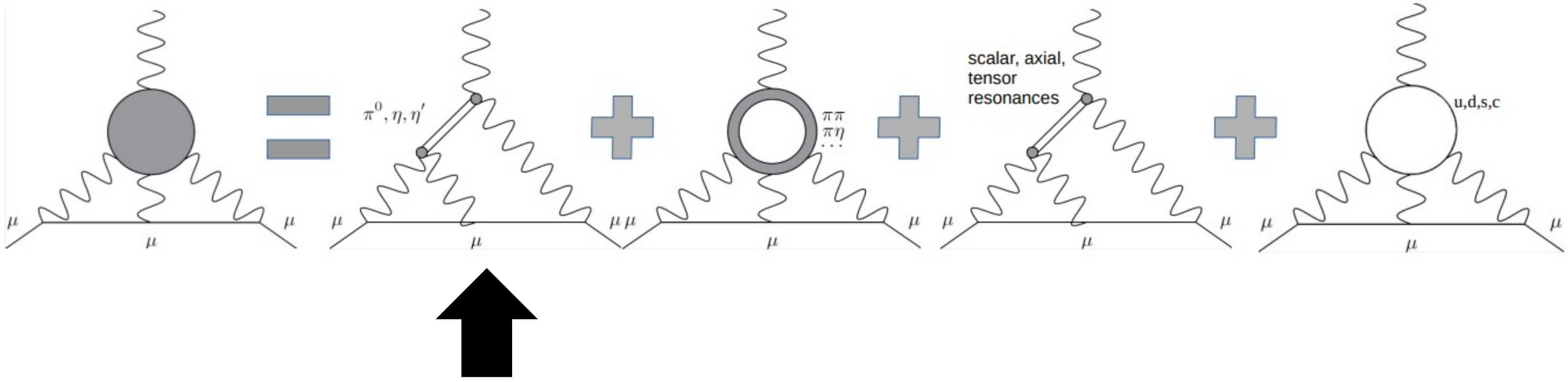


Estimate of (g-2) Theory Initiative:

$(\mathbf{9.2 \pm 1.8}) \cdot 10^{-10}$

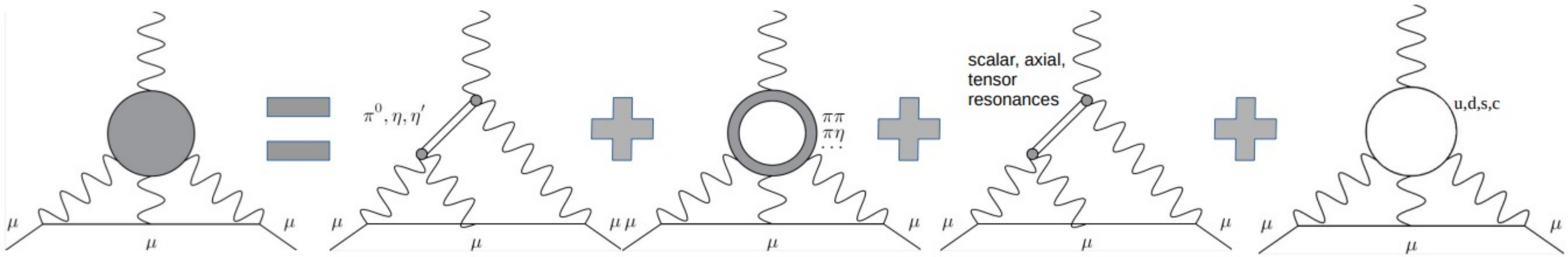
was $(10.5 \pm \mathbf{2.6}) \cdot 10^{-10}$

HLbL and Impact of BESIII Data



leading contribution

HLbL and Impact of BESIII Data

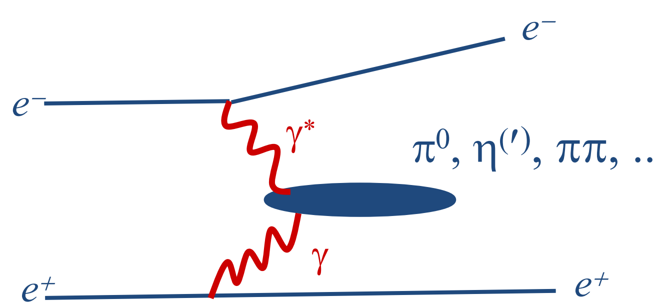


leading contribution

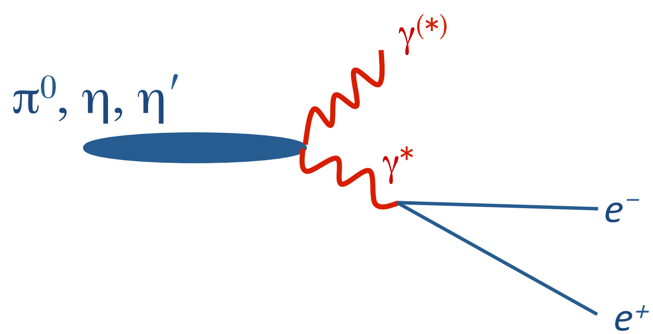
NEW

Data-driven approach!

Exp. Input:
Transition
Form Factors $F(Q^2)$
below ~ 2 GeV²



BESIII
e^+e^- collider
$\sqrt{s}$ 2 ... 5 GeV



MAMI MAINZER MIKROTRON

Hadronic Light-by-Light $(g\text{-}2)_\mu$

Leading contributions are pole contribution from π^{0}, η, η'

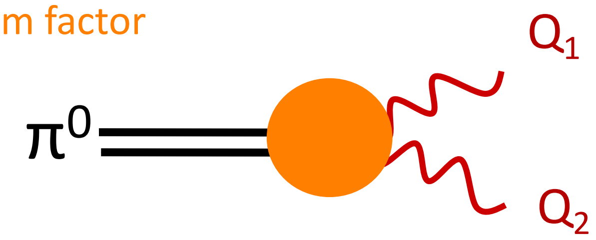
$$a_\mu^{\mathrm{HLbL};\pi^{0(1)}} = \int_0^\infty dQ_1 \int_0^\infty dQ_2 \int_{-1}^{1} d\tau \; w_1(Q_1,Q_2,\tau)\,\mathcal{F}_{\pi^0\gamma^*\gamma^*}(-Q_1^2,-(Q_1+Q_2)^2)\,\mathcal{F}_{\pi^0\gamma^*\gamma^*}(-Q_2^2,0)$$

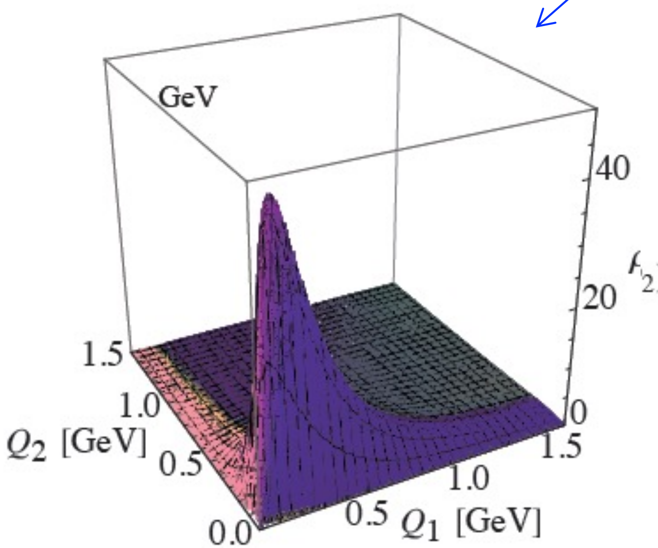
weighting function

transition form factor

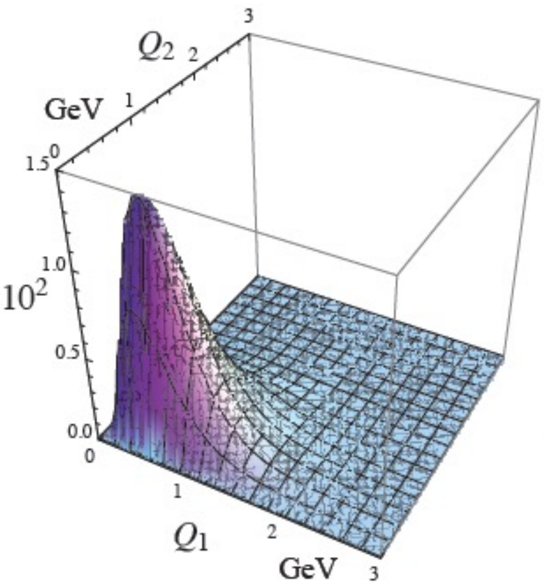
3D integral representation

[Nyffeler 2016]





Pseudoscalar Mesons



Axial Vector Mesons

→ Need doubly virtual form factors of π^{0}, η, η' at low Q^2

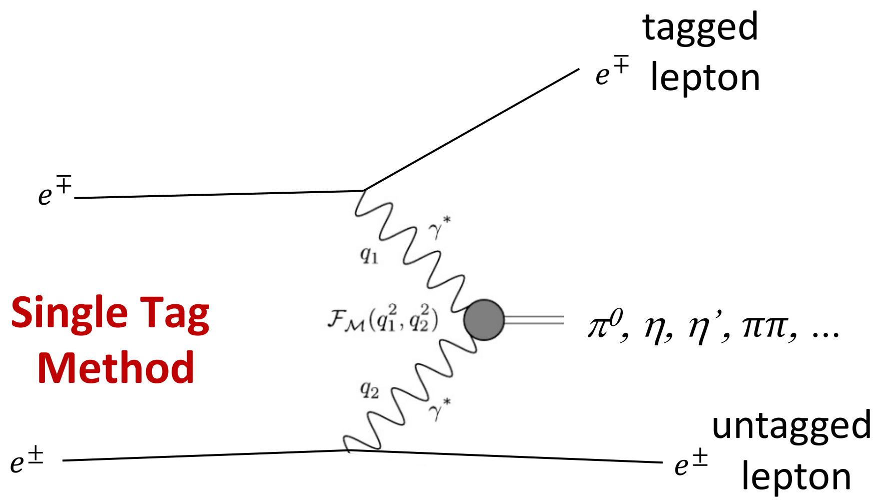
Two-Photon Physics Programme at BESIII

Selection criteria

- 1 electron (positron) detected
- 1 positron (electron) along beam axis
- Meson fully reconstructed

→ **cut on angle of missing momentum**

Single Tag Method



Momentum transfer

- tagged: $Q^2 = -q_1^2 = -(p - p')^2$

  → Highly virtual photon

- untagged: $q^2 = -q_2^2 \sim 0$ GeV^2

  → Quasi-real photon

EKHARA event generator

$$Q^2 = 4 \cdot E \cdot E' \cdot \sin^2(\theta/2)$$

Two-Photon Physics Programme at BESIII

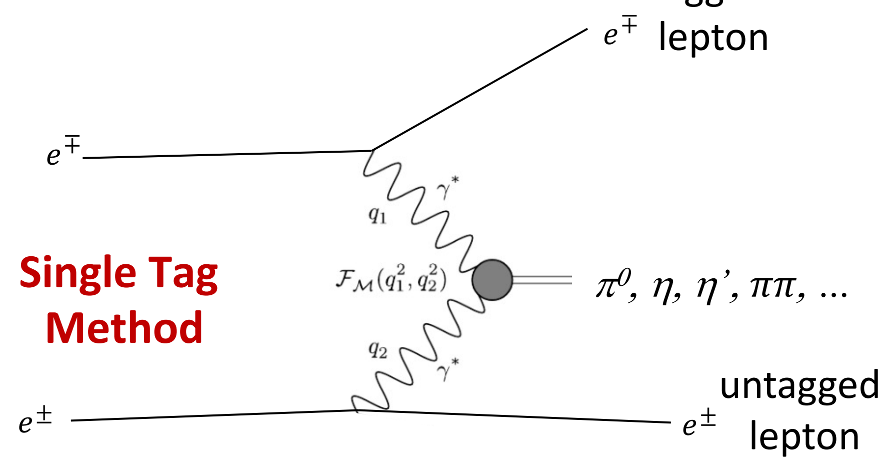
JG|U

Selection criteria

- 1 electron (positron) detected
- 1 positron (electron) along beam axis
- Meson fully reconstructed

→ **cut on angle of missing momentum**

Single Tag Method



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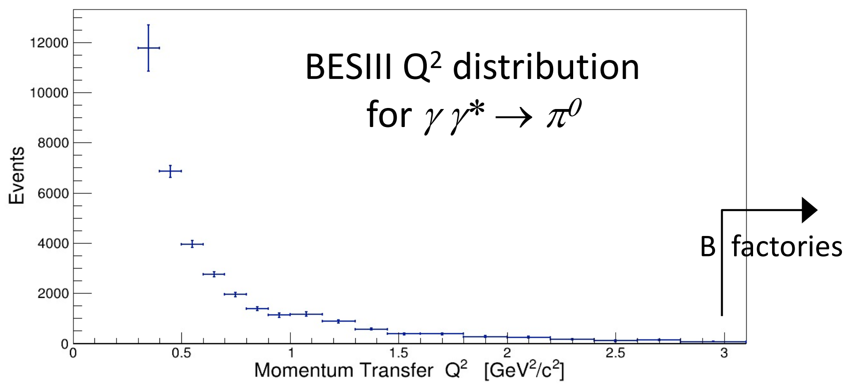

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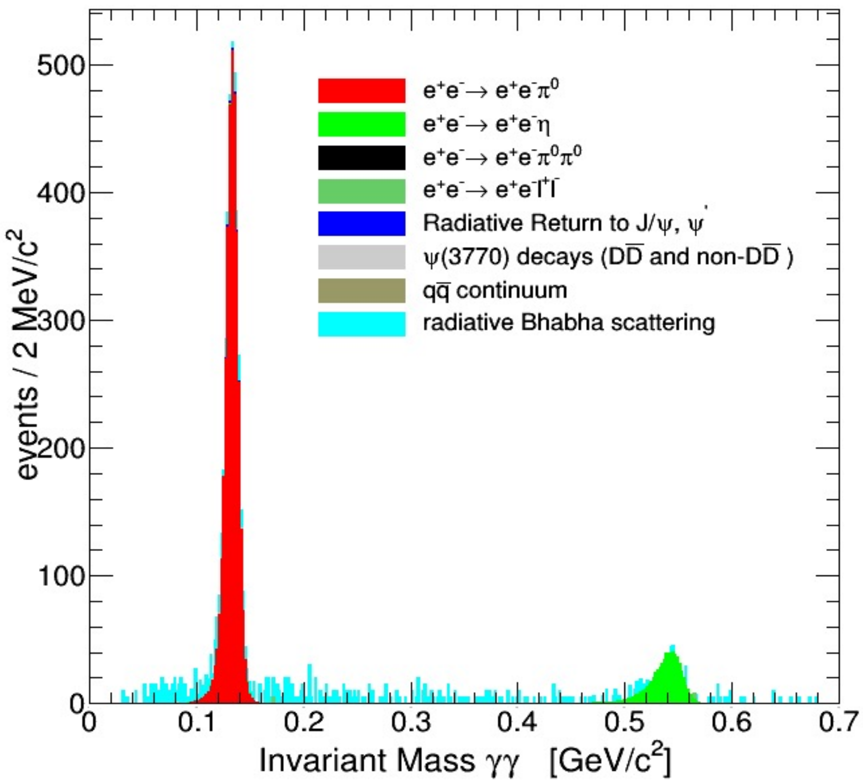
BES III Analysis: $e^+e^- \rightarrow e^+e^- \pi^0$



Event Selection:

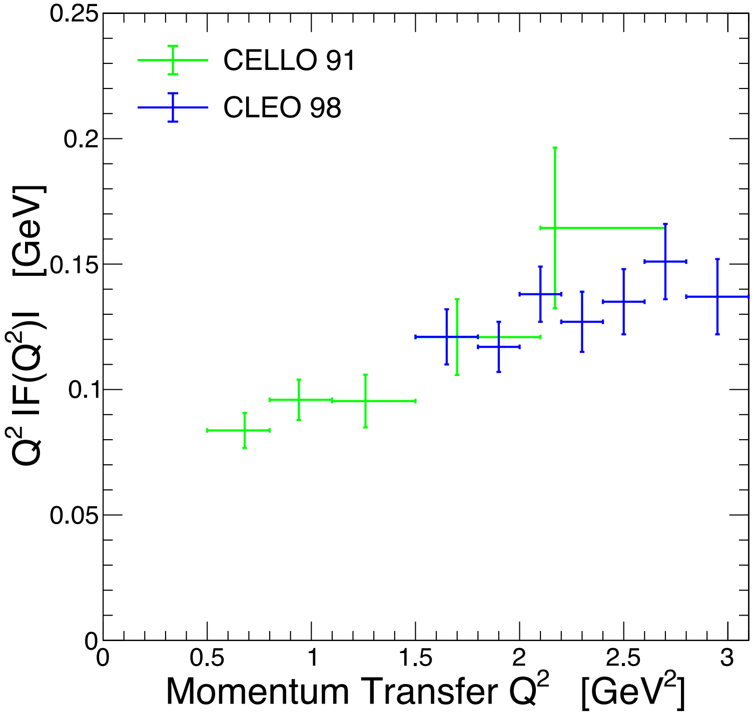
- exactly one lepton candidate
- at least two, max four photons
- Helicity angle $\cos \Theta_H > 0.8$
- Kinematic cuts to reject ISR background

→ **cut on angle of missing momentum**

Strategy: Count π^0 yield in bins of Q^2 → $d\sigma/dQ^2$ → Form factor $F(Q^2)$

BES III Analysis: $\gamma\gamma^* \rightarrow \pi^0$

PPNP107 (2019) 20



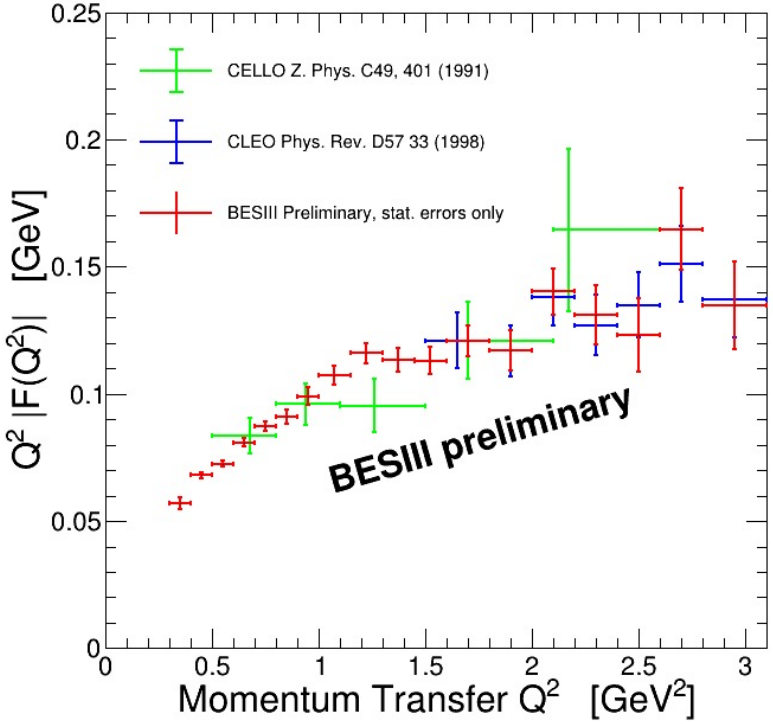
- $\sqrt{s}_{\mathrm{BESIII}} = 3.77$ GeV, L= 2.9/fb
- Unprecedented accuracy of BESIII
- Relevant Q^2 range for HLbL

→ **Very good agreement with recent dispersive analysis and of Lattice QCD calculation**

- Q^2 range below 0.3 GeV^2 accessible at BESIII with data from lower c.m. energy

BES III Analysis: $\gamma\gamma^* \to \pi^0$

PPNP107 (2019) 20



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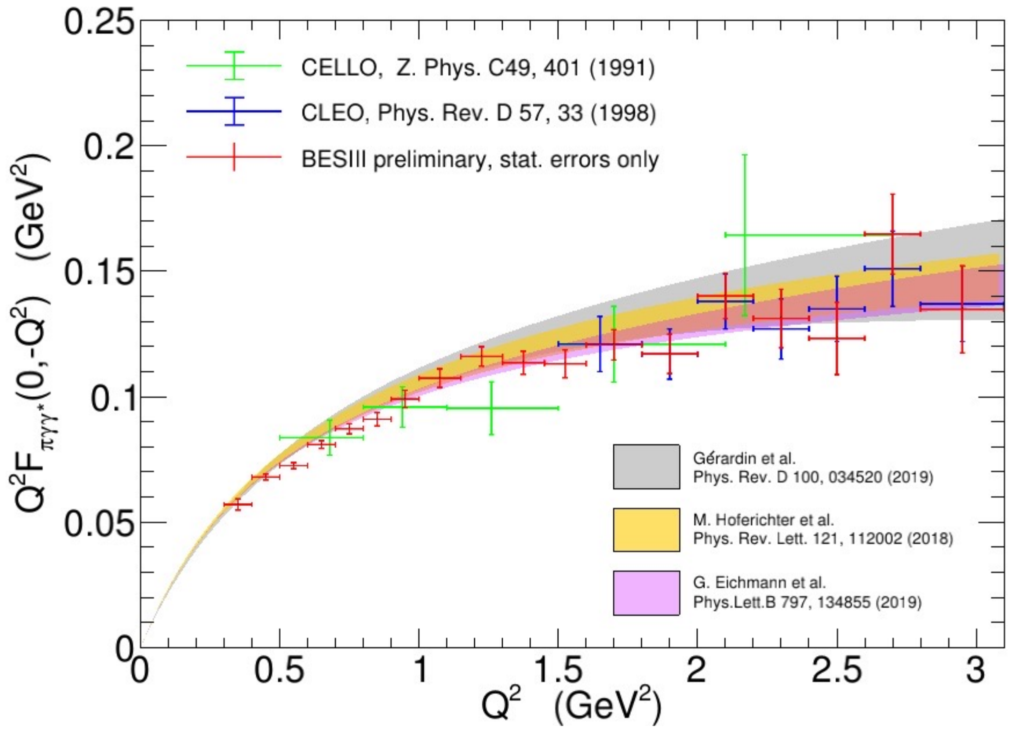

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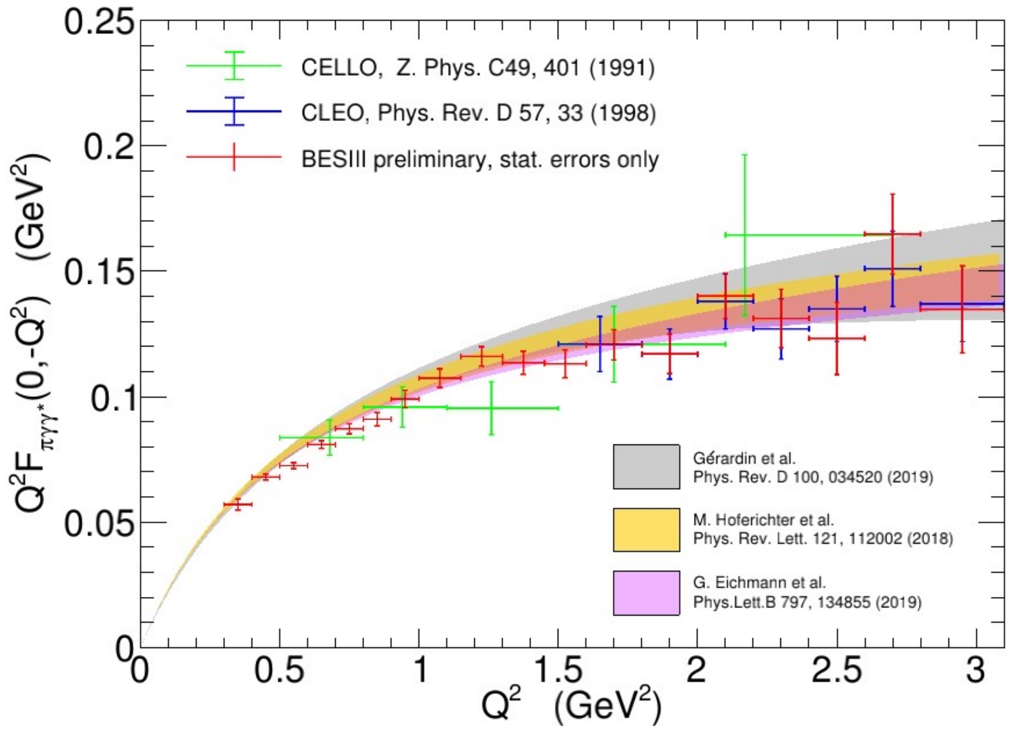

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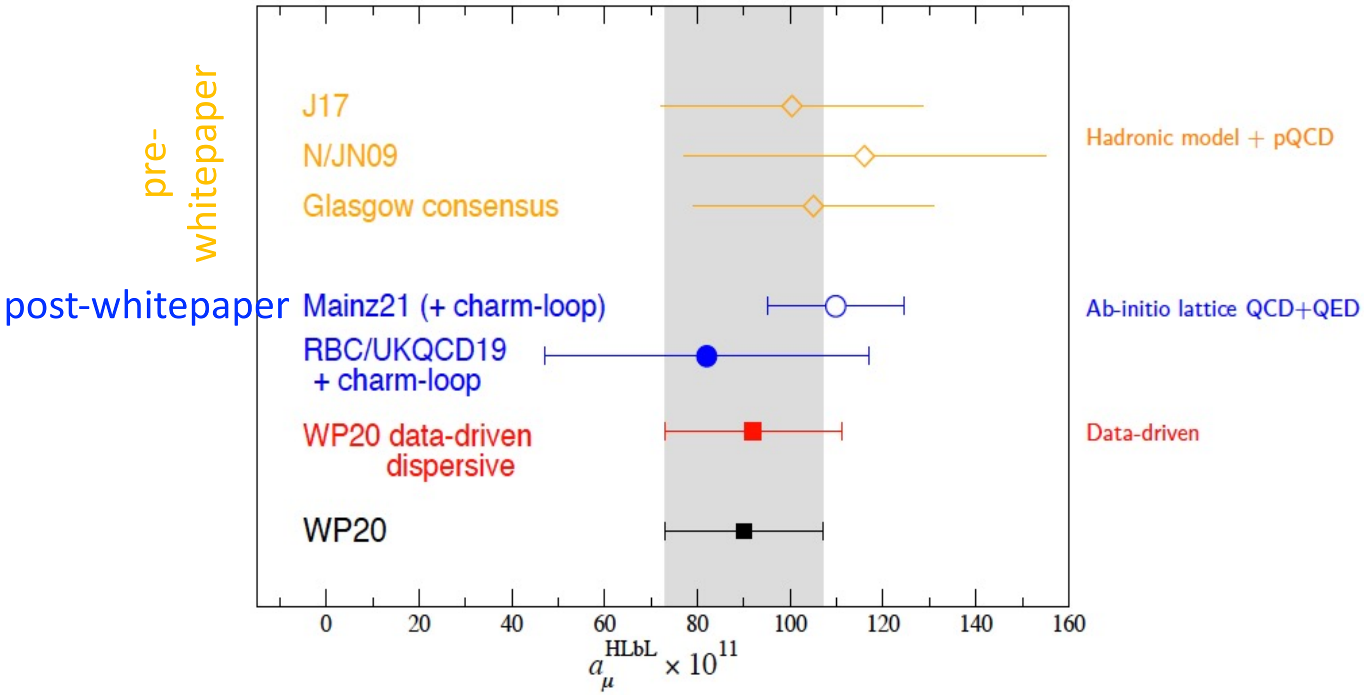
similar results for η and η' TFFs;
first measurement ever ππ;
many other channels

Hadronic Light-by-Light (HLbL)



- Differently from the HVP contribution, in the case of HLbL excellent agreement between the data-driven estimate and Lattice QCD
- With upcoming new form factor data ample room for improvement
- Community goal: relative uncertainty <10%

Hadronic Light-by-Light (HLbL)

Conclusions

Conclusions

- Interpretation of FNAL muon g-2 experiment calls for a detailed understanding of hadronic effects:
  - Hadronic Vacuum Polarization (HVP) contribution
  - Hadronic Light-by-Light (HLbL) contribution
- Following the standard approach to determine HVP contribution via dispersion relation shows a discrepancy of 5.1 σ between (g-2) SM theory and experiment:

New Physics ?
Too early to say - Two new g-2 puzzles:

- Lattice QCD calculation (BMW) of HVP suggests significantly lower discrepancy
- Recent CMD-3 result for timelike pion form factor also suggest lower discrepancy → reason for deviation to all previous experiments unknown
  → Quest for new measurements of form factors !

Thank you !

g-2 is not an experiment
[not a number] –
It is a way of life ...

John Adams, former Director General CERN