International school of nuclear physics

44th Course: From quarks and gluons to hadrons and nuclei

Erice, Italy
September 21, 2023

Markus Q. Huber

Institute of Theoretical Physics
Giessen University

In collaboration with:
Christian S. Fischer
Hèlios Sanchis-Alepuz

Eur.Phys.J.C 80, arXiv:2004.00415 → J=0
Eur.Phys.J.C 80, arXiv:2110.09180 → J=0,2,3,4
vConf21, arXiv:2111.10197 → +higher terms
HADRON2021, arXiv:2201.05163 → +higher terms

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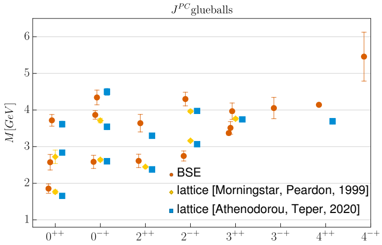

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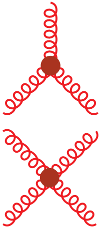

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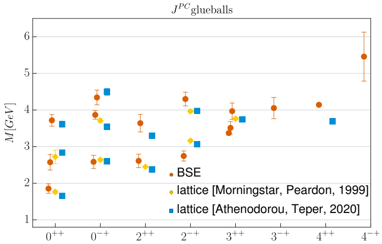

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Glueballs

Non-Abelian nature of QCD $\rightarrow$ self-interaction of force fields.

Mass dynamically created from massless (due to gauge invariance) gluons.

Theory:

Glueballs from gauge inv. operators, e.g., $F_{\mu\nu}F^{\mu\nu}$.

$\rightarrow$ Mixing of operators with equal quantum numbers.

Experiment:

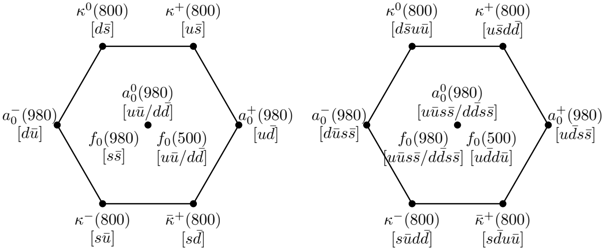
Production in glue-rich environments, e.g., $p\bar{p}$ annihilation (PANDA), pomeron exchange in pp (central exclusive production), radiative J/ψ decays

Reviews on glueballs: [Klempt, Zaitsev, Phys.Rept.454 (2007); Mathieu, Kochelev, Vento, Int.J.Mod.Phys.18 (2009); Crede, Meyer, Prog.Part.Nucl.Phys.63 (2009); Ochs, J.Phys.G40 (2013); Llanes-Estrada, EPJST 230 (2021); Vadacchino, 2305.04869]

Scalar sector

Classification not always easy, e.g., scalar sector $J^{PC} = 0^{++}$:

- $q\bar{q}$ mesons, tetraquarks: (inverted) mass hierarchy? [Jaffe, Phys. Rev. D 15 (1977)]



- Glueballs?

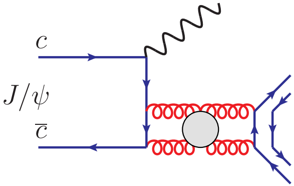
$f_0(500)$
$f_0(980)$
$f_0(1370)$
$f_0(1500)$
$f_0(1710)$

glueball candidates (→ $f_0(1370)$, $f_0(1500)$, $f_0(1710)$)

Functional review:
[Eichmann, Fischer, Santowsky, Wallbott, Few-Body Syst.61 (2020)]

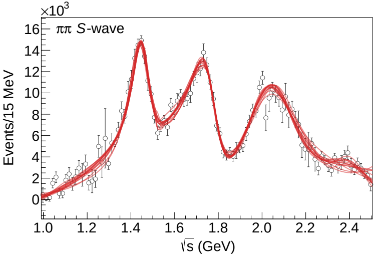
→ Talk by Hoffer

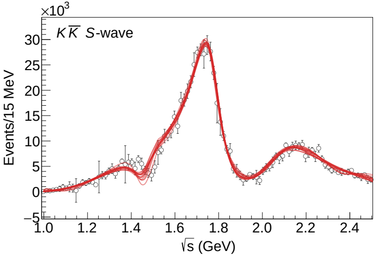
Scalar glueballs from J/ψ decay



Coupled-channel analyses of exp. data (BESIII):

- +add. data, largest overlap with $f_0(1770)$ [Sarantsev, Denisenko, Thoma, Klempt, Phys. Lett. B 816 (2021)]
- largest overlap with $f_0(1710)$ [JPAC Coll., Rodas et al., Eur.Phys.J.C 82 (2022)]



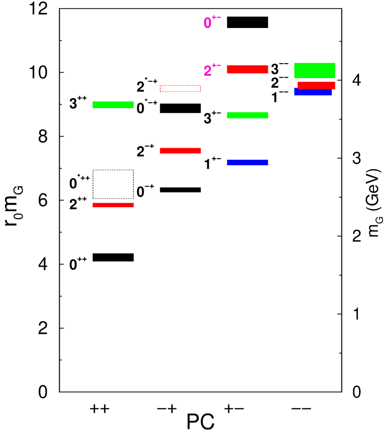


Glueball calculations: Lattice

Lattice methods

Pure gauge theory:
No dynamic quarks.
→ "Pure" glueballs

- [Morningstar, Peardon, Phys. Rev. D60 (1999)]: standard reference
- [Athenodorou, Teper, JHEP11 (2020)]: improved statistics, more states



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“Real QCD”:

- [Gregory et al., JHEP10 (2012)]
- [Brett et al., AIP Conf.Proc. 2249 (2020)]
- [Chen et al., 2111.11929]
- [Vadacchino, Lattice2022, 2305.04869]
- ...

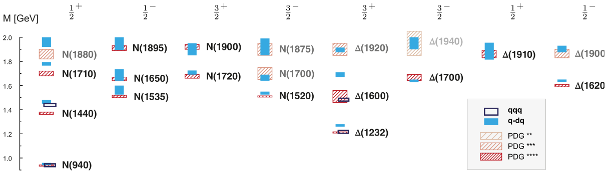
Challenging:

- Much higher statistics required (poor signal-to-noise ratio)
- Continuum extrapolation and inclusion of fermionic operators still to be done
- Mixing with $\bar{q}q$ challenging
- $m_\pi = 360\,\text{MeV}$
- Small unquenching effects found

No quantitative results yet.

Functional spectrum calculations

Functional methods successful in describing many aspects of the hadron spectrum qualitatively and quantitatively! $\rightarrow$ Talk by Eichmann



[Eichmann, Sanchis-Alepuz, Williams, Alkofer, Fischer, Prog.Part.Nucl.Phys. 91 (2016); Eichmann, Few Body Syst. 63 (2022)]

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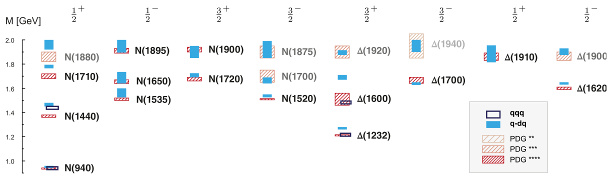

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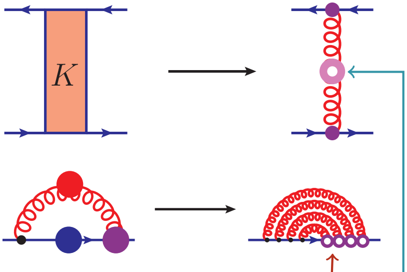


Workhorse for more than 20 years: Rainbow-ladder truncation with an effective interaction, e.g., Maris-Tandy (or similar).

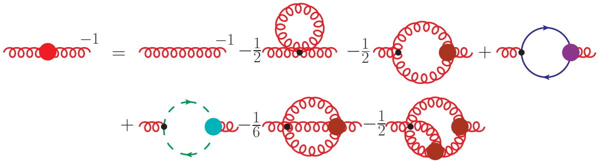
restricted structure of equations ($\Gamma_\mu \to \gamma_\mu$)

IR strength + perturbative UV

Results for mesons beyond rainbow-ladder, e.g., [Williams, Fischer, Heupel, Phys.Rev.D 93 (2016)].

Functional glueball calculations

Glueballs? Rainbow-ladder?



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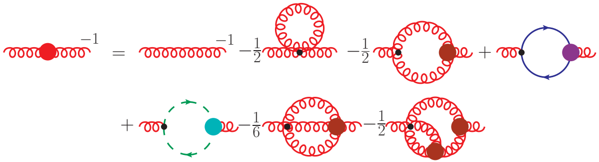

There is no rainbow for gluons!

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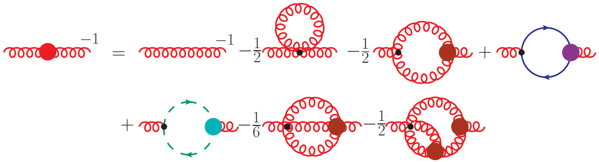

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Model based BSE calculations ($J = 0$):

- [Meyers, Swanson, Phys.Rev.D87 (2013)]
- [Sanchis-Alepuz, Fischer, Kellermann, von Smekal, Phys.Rev.D92, (2015)]
- [Souza et al., Eur.Phys.J.A56 (2020)]
- [Kaptari, Kämpfer, Few Body Syst.61 (2020)]

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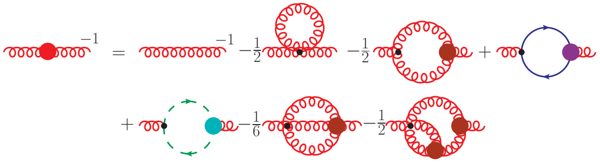

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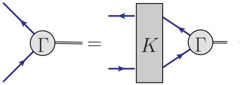
- [Kaptari, Kämpfer, Few Body Syst.61 (2020)]

Alternative: Calculated input [MQH, Phys.Rev.D 101 (2020)]

- $J = 0$: [MQH, Fischer, Sanchis-Alepuz, Eur.Phys.J.C80 (2020)]
- $J = 0, 2, 3, 4$: [MQH, Fischer, Sanchis-Alepuz, Eur.Phys.J.C81 (2021)]

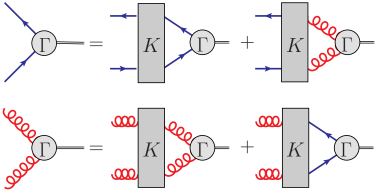
Extreme sensitivity on input!

Bound state equations for QCD



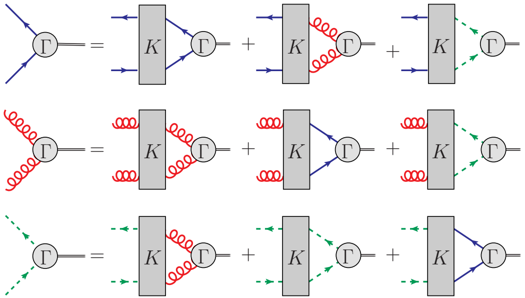
- Require scattering kernel K and propagator.

Bound state equations for QCD



- Require scattering kernels K and propagators.
- Quantum numbers determine which amplitudes Γ couple.

Bound state equations for QCD



- Require scattering kernels K and propagators.
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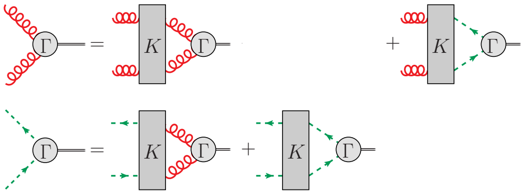
- Ghosts from gauge fixing

One framework

- Natural description of mixing.
- Similar equations for hadrons with more than two constituents

Bound state equations for QCD

Focus on pure glueballs.



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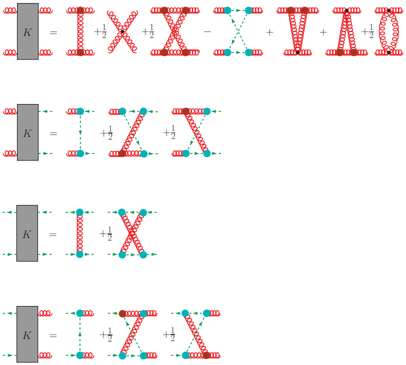
Kernels

Systematic derivation from 3PI effective action: [Berges, Phys. Rev. D 70 (2004); Carrington, Gao, Phys. Rev. D 83 (2011)]

Self-consistent treatment of 3-point functions requires 3-loop expansion.

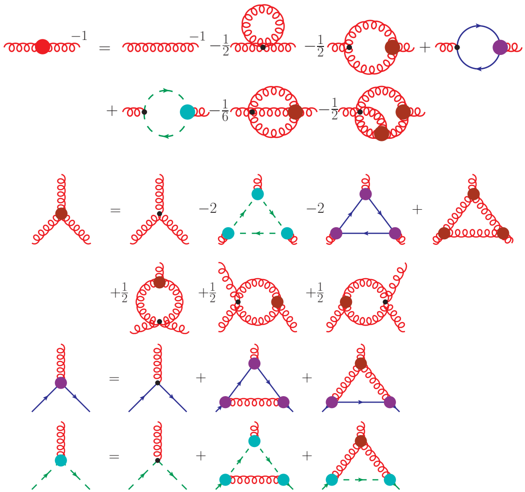


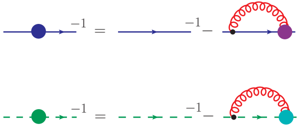
[Fukuda, Prog. Theor. Phys 78 (1987); McKay, Munczek, Phys. Rev. D 40 (1989); Sanchis-Alepuz, Williams, J. Phys: Conf. Ser. 631 (2015); MQH, Fischer, Sanchis-Alepuz, Eur.Phys.J.C80 (2020)]

Correlation functions of quarks and gluons

Equations of motion: 3-loop 3PI effective action

→ [Review: MQH, Phys.Rept. 879 (2020)]





- Conceptual and technical challenges: nonperturbative renormalization, two-loop diagrams, convergence, size of kernels, . . .
- Self-contained: Only parameters are the strong coupling and the quark masses!
- Long way, e.g., ghost-gluon vertex, three-gluon vertex, four-gluon vertex, . . .
- → MQH, Phys.Rev.D 101 (2020)

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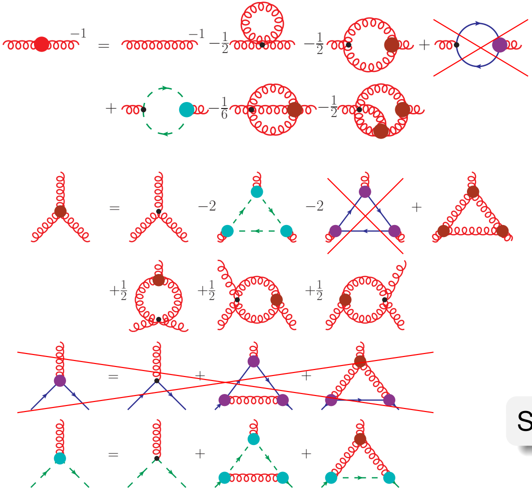

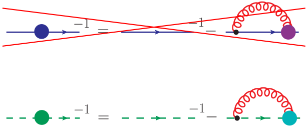

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- Long way, e.g., ghost-gluon vertex, three-gluon vertex, four-gluon vertex, ...
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Start with pure gauge theory.

Landau gauge propagators

Self-contained: Only external input is the coupling! → Ab-initio! [MQH, Phys.Rev.D 101 (2020)]

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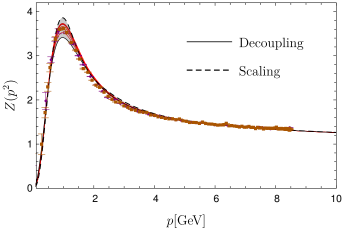
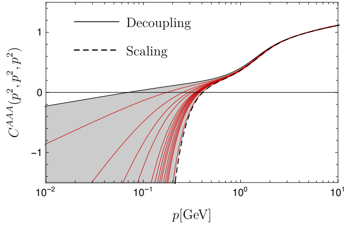
Gluon dressing function:



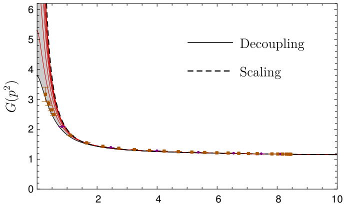
Three-gluon vertex:



Family of solutions [von Smekal, Alkofer, Hauck, PRL79 (1997); Aguilar, Binosi, Papavassiliou, Phys.Rev.D 78 (2008); Boucaud et al., JHEP06 (2008); Fischer, Maas, Pawlowski, Ann.Phys. 324 (2008); Alkofer, MQH, Schwenzer, Phys. Rev. D 81 (2010)]

Nonperturbative completions of Landau gauge [Maas, Phys. Lett. B 689 (2010)]?

Ghost dressing function:



Stability of the solution

- Agreement with lattice results. ✓

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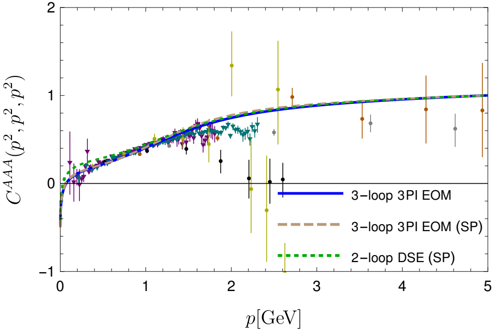
- Concurrence between functional methods:

3PI vs. 2-loop DSE:



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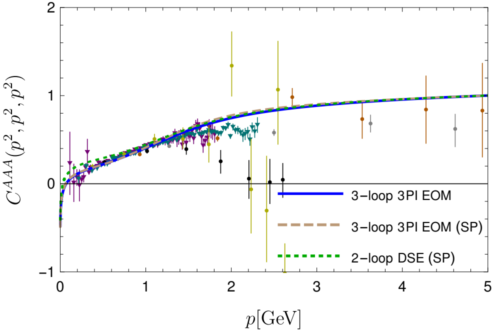
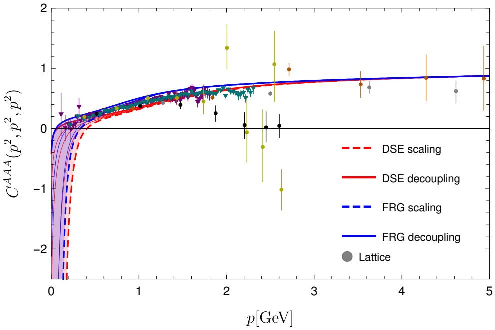


DSE vs. FRG:



[Cucchieri, Maas, Mendes, Phys.Rev.D77 (2008); Sternbeck et al., Proc.Sci. LATTICE2016 (2017); Cyrol et al., Phys.Rev.D 94 (2016); MQH, Phys.Ref.D101 (2020)]

Stability of the solution: Extensions

- Three-gluon vertex: Tree-level dressing dominant, others subleading [Eichmann, Williams, Alkofer, Vujinovic, Phys.Rev.D89 (2014); Pinto-Gómez et al., 2208.01020] ✓

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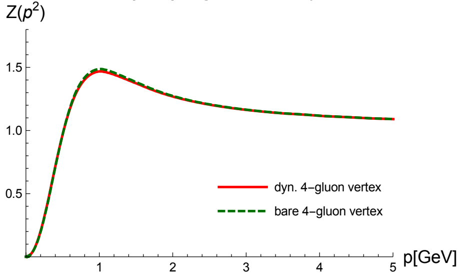

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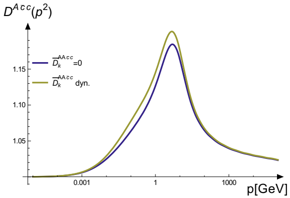
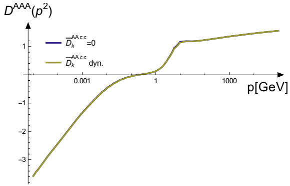
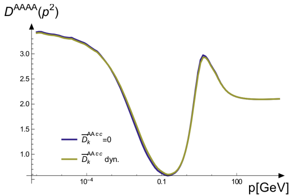
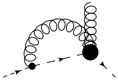
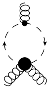
- Two-ghost-two-gluon vertex [MQH, Eur. Phys.J.C77 (2017)]: ✓
  (FRG: [Corell, SciPost Phys. 5 (2018)])

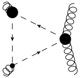




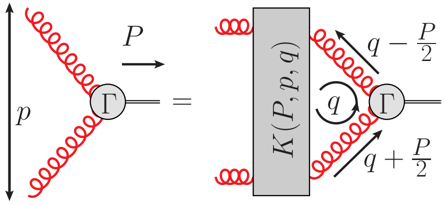








Correlation functions for complex momenta



(pseudoscalar glueball)

$$\boldsymbol{\lambda}(\boldsymbol{P})\Gamma(P) = \mathcal{K} \cdot \Gamma(P)$$

$\rightarrow$ Eigenvalue problem for $\Gamma(P)$:

1. Solve for $\lambda(P)$.
2. Find P with $\lambda(P) = 1$.
   $\Rightarrow M^2 = -P^2$

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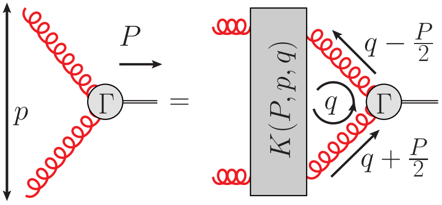

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However:

Propagators are probed at $\left(q \pm \frac{P}{2}\right)^2 = \frac{P^2}{4} + q^2 \pm \sqrt{P^2\, q^2}\cos\theta = -\frac{M^2}{4} + q^2 \pm i\, M\sqrt{q^2}\,\cos\theta$

$\rightarrow$ Complex for $P^2 < 0$!

Time-like quantities ($P^2 < 0$) $\rightarrow$ Correlation functions for complex arguments.

Extrapolation of $\lambda(P^2)$

Extrapolation method

- Extrapolation to time-like P^2 using Schlessinger's continued fraction method (proven superior to default Padé approximants) [Schlessinger, Phys.Rev.167 (1968)]
- Average over extrapolations using subsets of points for error estimate

$$f(x) = \frac{f(x_1)}{1 + \frac{a_1(x-x_1)}{1+\frac{a_2(x-x_2)}{1+\frac{a_3(x-x_3)}{\dots}}}}$$

Coefficients a_i can determined such that $f(x)$ exact at x_i.

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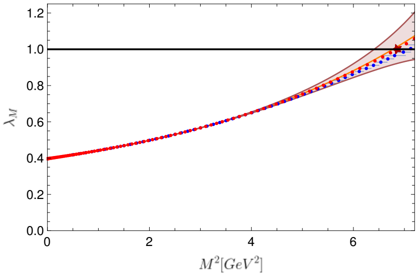
- Average over extrapolations using subsets of points for error estimate

$$f(x) = \frac{f(x_1)}{1 + \frac{a_1(x-x_1)}{1+\frac{a_2(x-x_2)}{1+\frac{a_3(x-x_3)}{\cdots}}}}$$

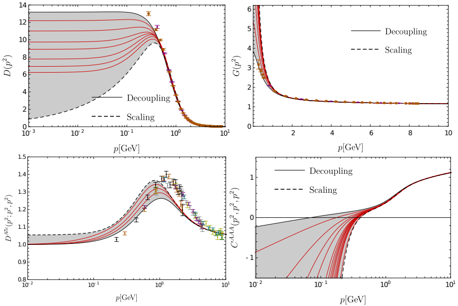
Coefficients a_i can determined such that $f(x)$ exact at x_i.

Test extrapolation for solvable system:
Heavy meson [MQH, Sanchis-Alepuz, Fischer, Eur.Phys.J.C 80 (2020)]



Glueball results J=0

Gauge-variant correlation functions:



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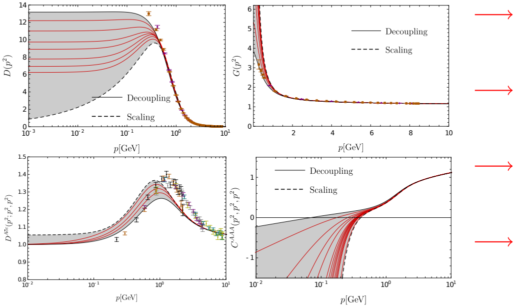
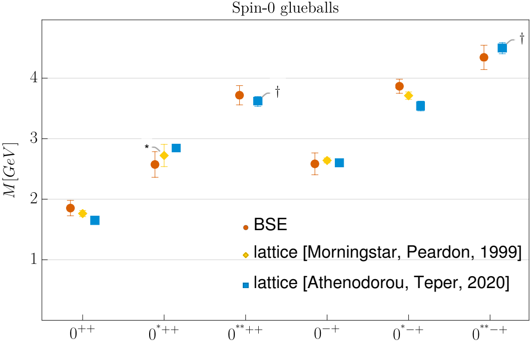


Unique physical spectrum:



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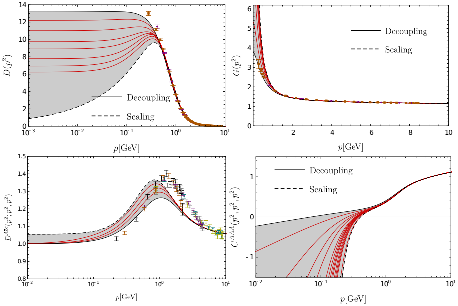

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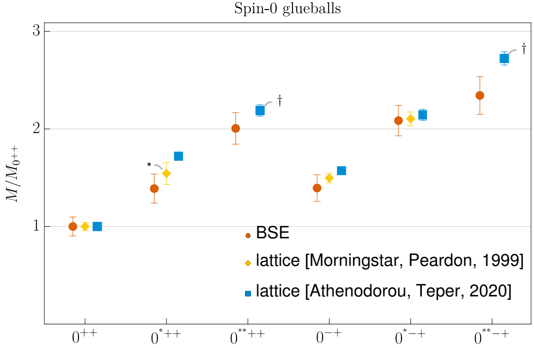


Spectrum independent! → Family of solutions yields the same physics.

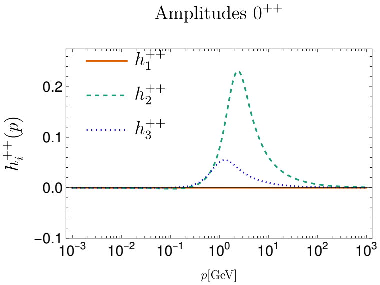
All results for $r_0 = 1/418(5)$ MeV.

[MQH, Fischer, Sanchis-Alepuz, Eur.Phys.J.C80 (2020)]

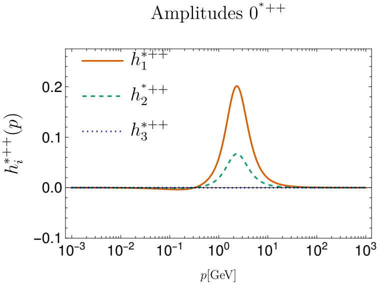
Amplitudes

Information about significance of single parts.

Ground state scalar glueball:



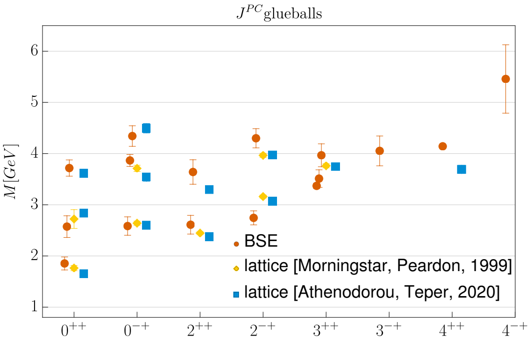
Excited scalar glueball:

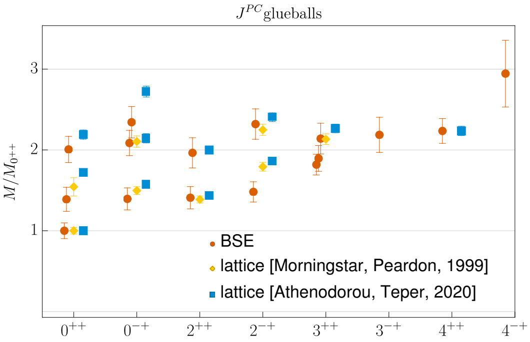


→ Amplitudes have different behavior for ground state and excited state. Useful guide for future developments.

→ Meson/glueball amplitudes: Information about mixing.

Glueball results

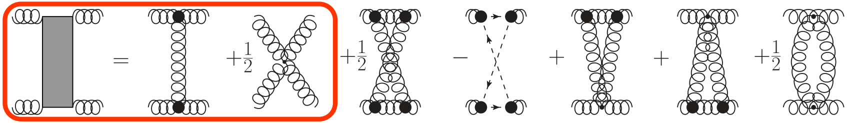




[MQH, Fischer, Sanchis-Alepuz, Eur.Phys.J.C81 (2021)]

- Agreement with lattice results
- New states: 0^{**++}, 0^{**-+}, 3^{-+}, 4^{-+}

Higher order diagrams



One-loop diagrams only:

[MQH, Fischer, Sanchis-Alepuz, Eur.Phys.J.C80 (2020); MQH, Fischer, Sanchis-Alepuz, Eur.Phys.J.C81 (2021)]

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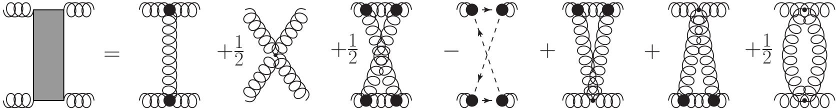


One-loop diagrams only:

[MQH, Fischer, Sanchis-Alepuz, Eur.Phys.J.C80 (2020); MQH, Fischer, Sanchis-Alepuz, Eur.Phys.J.C81 (2021)]

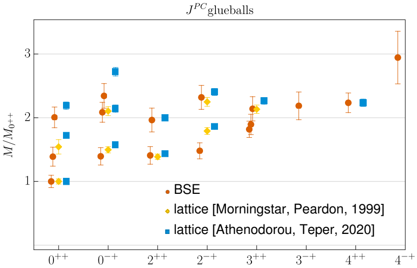
Two-loop diagrams: subleading effects

- 0^{-+}: none
  [MQH, Fischer, Sanchis-Alepuz, EPJ Web Conf. 258 (2022)]
- 0^{++}: $< 2\%$
  [MQH, Fischer, Sanchis-Alepuz, HADRON2021, arXiv:2201.05163]
- 2^{++}: none

Summary and outlook

- Alternative to models in bound state equations: Direct calculation of input.
- Large system of equations may be necessary.

Pure glueball spectrum from first principles.



Tests

- Input:
  - Agreement with other methods: lattice + continuum
  - Extensions
- BSEs: Higher orders negligible

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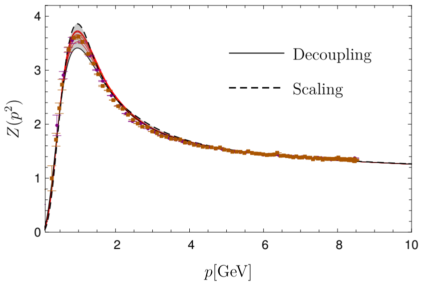

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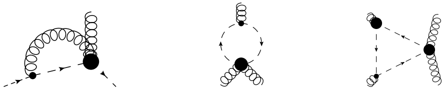

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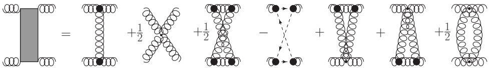

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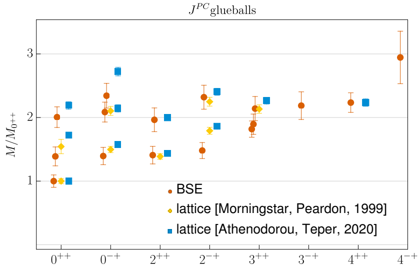

Tests

- Input:
  - Agreement with other methods: lattice + continuum
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- BSEs: Higher orders negligible

Future:

- +quarks $\to$ QCD
- three-body bound state eq. $\to C = -1$

Thank you for your attention.

$J = 1$ glueballs

Landau-Yang theorem

Two-photon states cannot couple to $J^P = \mathbf{1}^{\pm}$ or $(2n+1)^-$

[Landau, Dokl.Akad.Nauk SSSR 60 (1948); Yang, Phys. Rev. 77 (1950)].

($\rightarrow$ Exclusion of $J = 1$ for Higgs because of $h \rightarrow \gamma\gamma$.)

Applicable to glueballs?

$\rightarrow$ Not in this framework, since gluons are not on-shell.

$\rightarrow$ Presence of $J = 1$ states is a dynamical question.

$J = 1$ not found here.

Glueballs as bound states

Hadron masses from correlation functions of color singlet operators.

Glueballs as bound states

Hadron masses from correlation functions of color singlet operators.

Example: For $J^{\mathrm{PC}} = 0^{++}$ glueball take $O(x) = F_{\mu\nu}(x)F^{\mu\nu}(x)$:

$$\boldsymbol{D}(\boldsymbol{x}-\boldsymbol{y}) = \langle O(x)O(y)\rangle$$

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Lattice: Mass exponential Euclidean time decay:

$$\lim_{t\to\infty} \langle O(x) O(0) \rangle \sim e^{-t\,M}$$

Glueballs as bound states

Hadron masses from correlation functions of color singlet operators.

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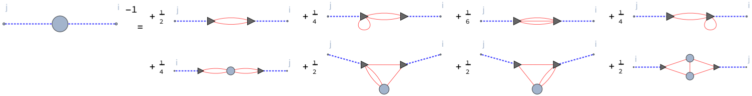
$$\boldsymbol{D}(\boldsymbol{x} - \boldsymbol{y}) = \langle O(x)O(y)\rangle$$

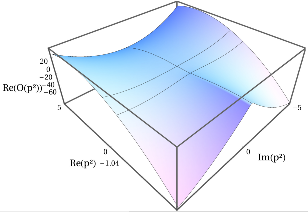
Functional approach: Complicated object in a diagrammatic language, 2-, 3- and 4-gluon contributions [MQH, Cyrol, Pawlowski, Comput.Phys.Commun. 248 (2020)]



+ 3-loop diagrams

Leading order:

[Windisch, MQH, Alkofer, Phys.Rev.D87 (2013)]



Glueballs as bound states

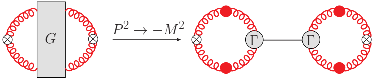
Hadron masses from correlation functions of color singlet operators.

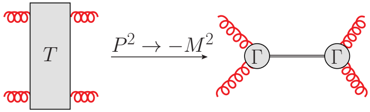
Example: For $J^{PC} = 0^{++}$ glueball take $O(x) = F_{\mu\nu}(x)F^{\mu\nu}(x)$:

$$\boldsymbol{D(x-y)} = \langle O(x)O(y)\rangle$$

Put total momentum on-shell and consider individual 2-, 3- and 4-gluon contributions. $\rightarrow$ Each can have a pole at the glueball mass.

A^4-part of $D(x-y)$, total momentum on-shell:

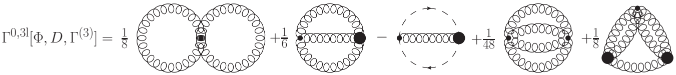


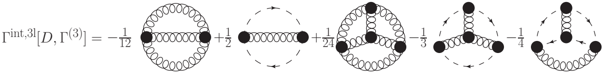


Kernel construction

From 3PI effective action truncated to three-loops: [Berges, Phys. Rev. D 70 (2004); Carrington, Gao, Phys. Rev. D 83 (2011)]

$$\Gamma^{3l}[\Phi, D, \Gamma^{(3)}] = \Gamma^{0,3l}[\Phi, D, \Gamma^{(3)}] + \Gamma^{int,3l}[\Phi, D, \Gamma^{(3)}]$$





Kernels constructed by cutting two legs:
gluon/gluon,ghost/gluon, gluon/ghost, ghost/ghost
[Fukuda, Prog. Theor. Phys 78 (1987); McKay, Munczek, Phys. Rev. D 40 (1989); Sanchis-Alepuz, Williams, J. Phys: Conf. Ser. 631 (2015); MQH, Fischer, Sanchis-Alepuz, Eur.Phys.J.C80 (2020)]

Charge parity

Transformation of gluon field under charge conjugation:

$$A^a_\mu \to -\eta(a) A^a_\mu$$

where

$$\eta(a) = \begin{cases} +1 & a = 1,3,4,6,8 \\ -1 & a = 2,5,7 \end{cases}$$

Color neutral operator with two gluon fields:

$$A^a_\mu A^a_\nu \to \eta(a)^2 A^a_\mu A^a_\nu = A^a_\mu A^a_\nu.$$

$\Rightarrow C = +1$

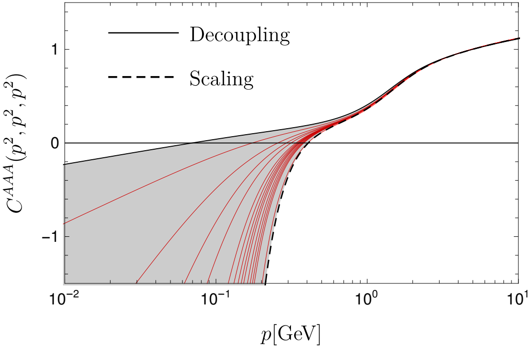
Negative charge parity, e.g.:

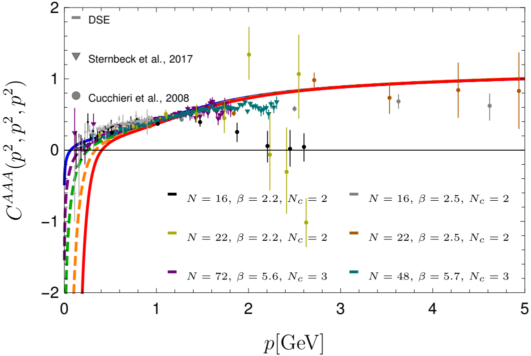
$$\begin{aligned} d^{abc} A^a_\mu A^b_\nu A^c_\rho \to & - d^{abc} \eta(a)\eta(b)\eta(c) A^a_\mu A^b_\nu A^c_\rho = \\ & - d^{abc} A^a_\mu A^b_\nu A^c_\rho. \end{aligned}$$

Only nonvanishing elements of the symmetric structure constant d^{abc}: zero or two indices equal to 2, 5 or 7.

Three-gluon vertex

[Cucchieri, Maas, Mendes, Phys. Rev. D 77 (2008); Sternbeck et al., 1702.00612; MQH, Phys. Rev. D 101 (2020)]

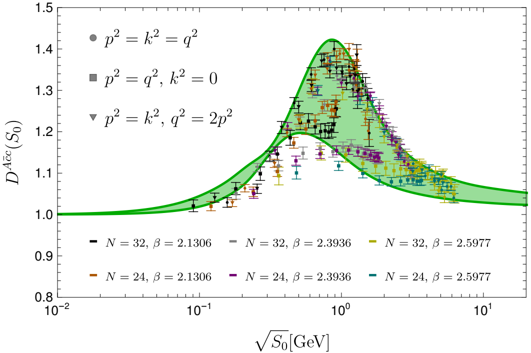




- Simple kinematic dependence of three-gluon vertex (only singlet variable of S_3)
- Large cancellations between diagrams

Ghost-gluon vertex

Ghost-gluon vertex:



[Maas, SciPost Phys. 8 (2019);

MQH, Phys. Rev. D 101 (2020)]

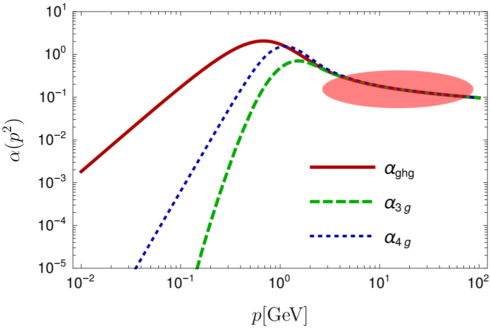
- Nontrivial kinematic dependence of ghost-gluon vertex
- Qualitative agreement with lattice results, though some quantitative differences (position of peak!).

Gauge invariance

[MQH, Phys. Rev. D 101 (2020)]

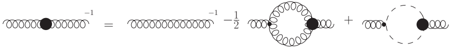
Couplings can be extracted from each vertex.

- Slavnov-Taylor identities (gauge invariance): Agreement perturbatively (UV) necessary.
  [Cyrol et al., Phys.Rev.D 94 (2016)]
- Difficult to realize: Small deviations → Couplings cross and do not agree.
- Here: Vertex couplings agree down to GeV regime (IR can be different).



Landau gauge propagators in the complex plane

Simpler truncation:



[Fischer, MQH, Phys.Rev.D 102 (2020)]

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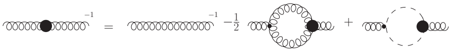
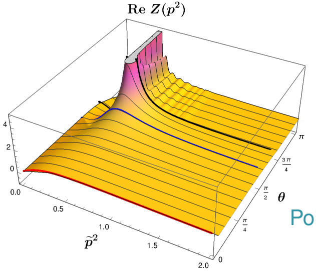
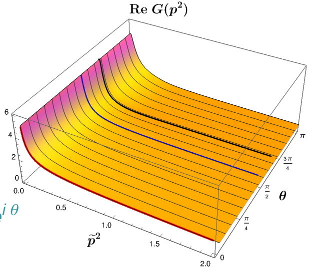


[Fischer, MQH, Phys.Rev.D 102 (2020)]

Ray technique for self-consistent solution of a DSE:



Polar coordinates: $p^2 = \tilde{p}^2 e^{i\,\theta}$



- Current truncation leads to a pole-like structure in the gluon propagator.
- Analyticity up to 'pole' confirmed by various tests (Cauchy-Riemann, Schlessinger, reconstruction)

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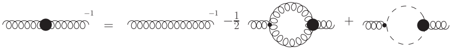

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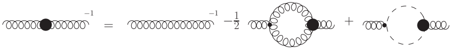

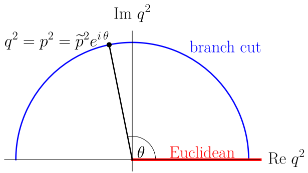

$\rightarrow$ Opening at $q^2 = p^2$.

Landau gauge propagators in the complex plane

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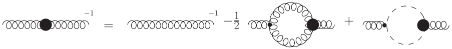
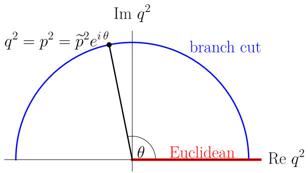
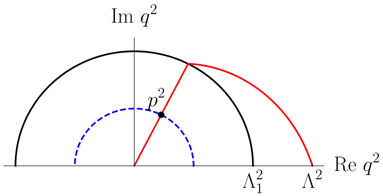




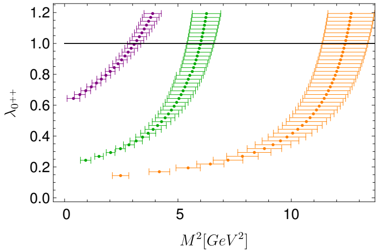


$\rightarrow$ Opening at $q^2 = p^2$.

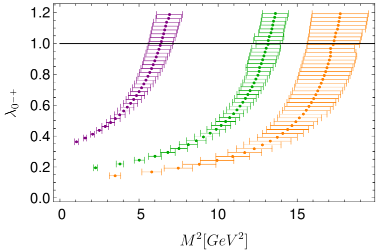
Appearance of branch cuts for complex momenta forbids integration directly to cutoff.

Extrapolation for glueball eigenvalue curves

0^{++}:



0^{-+}:



Several curves: ground state and excited states.

Glueball studies

- Reviews on glueballs: [Klempt, Zaitsev, Phys.Rept.454 (2007); Mathieu, Kochelev, Vento, Int.J.Mod.Phys.18 (2009); Crede, Meyer, Prog.Part.Nucl.Phys.63 (2009); Ochs, J.Phys.G40 (2013); Llanes-Estrada, EPJST 230 (2021); Vadacchino, 2305.04869]
- Lattice: [Morningstar, Peardon, Phys. Rev. D60 (1999); Athenodorou, Teper, JHEP11 (2020); Gregory et al., JHEP10 (2012); Brett et al., AIP Conf.Proc. 2249 (2020); Chen et al., 2111.11929; . . .]
- Hamiltonian many body methods: [Szczepaniak, Swanson, Ji, Cotanch, PRL 76 (1996); Szczepaniak, Swanson, Phys. Lett. B 577 (2003); . . .]
- Chiral Lagrangians: [Janowski, Parganlija, Giacosa, Rischke, Phys. Rev. D 84 (2011); Eshraim, Janowski, Giacosa, Rischke, Phys. Rev. D 87 (2013); . . .]
- Holographic QCD: [Brower, Mathur, Tan, Nucl. Phys. B 587 (2000); Colangelo, De Fazio, Jugeau, Nicotri, Phys. Lett. B 652 (2007); Brünner, Parganlija, Rebhan, Phys. Rev. D 93 (2016); Hechenberger, Leutgeb, Rebhan, Phys. Rev. D 107 (2023); . . .]
- Gribov-Zwanziger framework: [Dudal, Guimaraes, Sorella, Phys. Lett. B 732 (2014)]
- Functional studies: [Meyers, Swanson, Phys.Rev.D87 (2013); Sanchis-Alepuz, Fischer, Kellermann, von Smekal, Phys.Rev.D92, (2015); Souza et al., Eur.Phys.J.A56 (2020); Kaptari, Kämpfer, Few Body Syst.61 (2020); MQH, Phys.Rev.D 101 (2020); MQH, Fischer, Sanchis-Alepuz, Eur.Phys.J.C80 (2020); Pawlowski et al., 2212.01113]