P- and CP-odd effects in heavy ion collisions

Harmen Warringa, Goethe Universität, Frankfurt

Collaborators: Kenji Fukushima, Dmitri Kharzeev and Larry McLerran.

Fukushima, Kharzeev & HJW, arXiv:0912.961
Observation I:

Topological charge fluctuations present in QCD and hence in heavy ion collisions

Topological charge of gauge field: \[ Q = \frac{g^2}{32\pi^2} \int d^4 x \quad F_{\mu\nu}^a \tilde{F}_{\mu\nu}^a = \Delta N_{\text{CS}} \]

Nonzero Q contributes to path-integral, and hence to physical quantities.

The nontrivial vacuum structure of a SU(N) gauge theory

Energy

\[ N_{\text{CS}} = -3 \quad -2 \quad -1 \quad 0 \quad 1 \quad 2 \quad 3 \]

Instanton with Q = 1

Sphaleron with Q = -1

Belavin et. al. ('75), Jackiw & Rebbi ('76), Callan et al. ('76), 't Hooft ('76), Klinkhamer & Manton ('84), ......
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\[ \langle Q^2 \rangle \neq 0 \]
Mass of eta-prime meson. ('t Hooft, Witten, Veneziano)
\[ \frac{2N_f}{f^2 \pi V_4} \langle Q^2 \rangle = m_{\eta'}^2 + m_{\eta}^2 - 2m_K^2 \]

\[ \langle Q \rangle = 0 \]
Neutron electric dipole moment (Baker et.al. ('06))
\[ |\theta| < 10^{-10} \]
No P- and CP-violation in QCD!
But Q can induce P- and CP-odd effects.
Observation II:
Ultra high-energy heavy ion collisions = Ultra strong (EM) magnetic fields

Gold – Gold collision: two currents which carry 79 charges each
Observation II:
Ultra high-energy heavy ion collisions
= Ultra strong (EM) magnetic fields

\[ eB(\tau = 0.2 \text{ fm/c}) \approx 10^3 \sim 10^4 \text{ MeV}^2 \approx 10^{17} \text{ G} \]

See also Minakata and Müller ('96)
Outline

To explain you that

Topological charge + Magnetic Field =

$\langle Q \rangle = 0$

$\langle Q^2 \rangle \neq 0$

Charge separation

This can potentially be observed in experiment by charge correlation study [Voloshin ('04)]
Topological charge induces chirality

This is the P- and CP-odd effect

**Chirality:** difference between number of quarks + antiquarks with right- and left-handed helicity

$$N_5 = \# q_R + \# \bar{q}_R - \# q_L - \# \bar{q}_L$$

**Axial anomaly:** topological charge induces chirality

Steinberger ('49), Schwinger ('51), Alder ('69), Bell and Jackiw ('69)

$$\partial_\mu \langle \bar{\psi} \gamma^\mu \gamma^5 \psi \rangle_A = 2 m \langle \bar{\psi} i \gamma^5 \psi \rangle_A - 2 \frac{g^2}{32 \pi^2} F^{a}_{\mu \nu} \tilde{F}^{\mu \nu a}$$

Change in chirality over time for each flavor

$$\Delta N_5 = -2Q$$
Magnetic field induces polarization

Magnetic field aligns spins, depending on electric charge

No Magnetic Field: No polarization

Magnetic field: Polarization

The momenta of the quarks align along the magnetic field

Quark with R-helicity obtains momentum opposite to one with L-helicity

Hence magnetic field distinguishes between right and left
Topological Charge + Magnetic field = Chirality + Polarization =

\[ Q = -1 \]

\[ \Delta N_5 = 2 \]

Q < -1: Positively charged particles move parallel to magnetic field, negatively charged antiparallel

... = Electromagnetic Current

Chiral Magnetic Effect: Kharzeev, McLerran & HJW ('07)
Topological Charge + Magnetic field = Chirality + Polarization = 

Size of Current: \[ J = \int d^3 x \langle \bar{\psi} \gamma^3 \psi \rangle = -2Q \sum_f |q_f| \]

Valid for full polarization, what about smaller fields?

Chiral Magnetic Effect: Kharzeev, McLerran & HJW ('08)
Nonzero Chirality: Nonzero chiral chemical potential $\mu_5$

$$H \rightarrow H - \mu_5 \int d^3 x \bar{\psi} \gamma^0 \gamma^5 \psi$$

Compute induced current in magnetic field
Magnitude of the induced current

Alekseev, Cheianov, Fröhlich ('98), Fukushima, Kharzeev and HJW ('08)

1. Energy conservation
   \[ j = \frac{N_c \sum_f q_f^2}{2 \pi^2} \mu_5 B \]
   Nielsen and Ninomiya ('83)

2. Density in Lowest Landau Level
   \[ j = \frac{N_c \sum_f q_f^2}{2 \pi^2} \mu_5 B \]
   See also Metlitsky and Zhitnitsky ('06)

3. Chern-Simons term
   \[ j = \frac{N_c \sum_f q_f^2}{2 \pi^2} \mu_5 B \]

4. Thermodynamic potential
   \[ j = \frac{N_c \sum_f q_f^2}{2 \pi^2} \mu_5 B \]

5. Linear response
   \[ j = \frac{N_c \sum_f q_f^2}{2 \pi^2} \mu_5 B \]

6. Propagator in magnetic field
   \[ j = \frac{N_c \sum_f q_f^2}{2 \pi^2} \mu_5 B \]

Result follows from EM axial anomaly. Therefore exact and independent of coupling strength. Anomaly induced currents: c.f. Goldstone and Wilczek ('81)
Magnitude of the induced current

\[ j = \frac{N_c \sum_f q_f^2}{2 \pi^2} \mu_5 B \]

But what is \( \mu_5 \)?

\[ n_5 = \frac{\partial \Omega}{\partial \mu_5} \]

\[ N_5 = -2Q \]

Computed at high T (lo. pert. QCD)

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Current as a function of magnetic field

\[ \frac{j}{eN} \left| \frac{J}{q N_5^0} \right|_5 \]

\( T = 0 \)

\( T = 2n_5^{1/3} \)

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Strong fields:

\[ J = -2Q \sum_f |q_f| \]

\( \rightarrow \)

\[ J = -\frac{3}{\pi^2} \frac{Q}{T^2 + \mu^2 / \pi^2} B \sum_f q_f^2 \]

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\( Q = -1 \)

\( \Delta N_s = 2 \)
Chiral Magnetic Effect in time-dep. field

Kharzeev and HJW ('09)

\[ \vec{j} = \sigma_E \vec{E} \quad \sigma_E = \text{electrical conductivity} \]

\[ \vec{j} = \sigma_\chi \vec{B} \quad \sigma_\chi = \text{chiral magnetic conductivity} \]

Compute induced current using linear response

\[ \langle j^\mu(x) \rangle = \int d^4x' \, \Pi_R^{\mu\nu}(x-x') \, A_\mu(x') + o(A_\mu^2) \]

\[ \sigma_\chi(\omega) = \lim_{p' \to 0} \frac{1}{2i \, p^i} \epsilon^{ijk} \, \tilde{\Pi}_R^{jk}(\omega, p) \]

Leading order \( \tilde{\Pi}_R^{jk} \)

\[ \mu_5, T \]

Off diagonal, antisymmetric part of photon polarization tensor. Nonzero with \( \mu_5 \)
CM conductivity: weak vs. strong coupling

Displayed: normalized conductivity as a function of frequency.

\[ \sigma_0 = \frac{N_c \sum_f q_f^2}{2\pi^2} \mu_s \]

Weak coupling (1 loop pert. QCD)

Strong coupling (holographic model of QCD)

Kharzeev and HJW ('09)

Ho-Ung Yee ('09)

Real part: in-phase response, imaginary part: 90 degrees out of phase response
Chiral Magnetic Effect in time-dep. field

\[ j(t) = \int_0^\infty \frac{d\omega}{\pi} \left[ \sigma'_x(\omega) \cos(\omega t) + \sigma''_x(\omega) \sin(\omega t) \right] \tilde{B}(\omega) \]

Current: const. chirality + time dep. mag. field

\[ B(t) = \frac{B_0}{\left[1 + (t/\tau)^2\right]^{3/2}} \]

Red: current in slowly changing fields, adiabatic appr. = ok

Blue and green curves, faster changing mag field, but still induced current.

Even stronger response in strongly coupled regime.

Conclusion: also sizable current in fast changing magnetic field
Chiral Magnetic Effect: other methods

Lattice QCD:
Buividovich, Chernodub, Luschevskaya and Polikarpov ('09)
Abramczyk, Blum, Petropoulos and Zhou ('09)

AdS/CFT:
H.U. Yee ('09)
Rebhan, Schmitt and Stricker ('09)

Instanton:
S. Nam ('09)
Induced current in color-flux tube

By $B_z$, $F_z$ Flux tube generates chirality dynamically

Perpendicular magnetic field to color flux tube

Flux tubes naturally arise in glasma

Krasnitz et al. ('02), Lappi & McLerran, ('06)
P- and CP-odd effects in Heavy Ion Collisions

Topological charge $Q$ fluctuates anywhere in the QGP

Measure: variances = nonzero

Medium causes screening

Variance of charge difference between upper and lower side reaction plane:

$$\langle \Delta_{\pm}^2 \rangle = 2 \int_{t_i}^{t_f} dt \int_V d^3 x \Gamma \left[ \xi_+^2(x_\perp) + \xi_-^2(x_\perp) \right] \left( \sum_f \frac{3 q_f^2 e B}{\pi^2 T^2} \right)$$

Time & Volume integral
Overlap region
Rate of creation
Topological charge
Screening Functions
Square of Change
Charge difference

Estimate magnitude relative asymmetry for large impact parameter $10^{-4}$ with 1-2 orders of magnitude uncertainty.
Experimental observables

Correlations in azimuthal angle of charged particles

\[
a_{++} = \left\langle \frac{1}{N_+ N_+} \sum_{i, j=1}^{N_+, N_+} \cos (\phi_i + \phi_j - 2 \Psi_{RP}) \right\rangle
\]

\[
= \left\langle \frac{1}{N_+^2} \left[ \sum_{i=1}^{N_+} \cos (\phi_i - \Psi_{RP}) \right]^2 \right\rangle
\]

\[
- \left\langle \frac{1}{N_+^2} \left[ \sum_{i=1}^{N_+} \sin (\phi_i - \Psi_{RP}) \right]^2 \right\rangle
\]

Charge fluctuations in x-direction

Minus fluctuations in y direction

Average is over many similar minimum bias events

Take symmetric interval around zero rapidity

Analysis (and problems) similar to elliptic flow.
See also talks by Jean-Yves Ollitrault and Raimond Snellings

STAR detector
Full azimuthal coverage
Charge correlations at RHIC

Au-Au and Cu-Cu @ 200 GeV

min. bias, $|\eta|<1.0, \ 0.15<p_t<2\text{GeV/c}$

Strong charge correlations observed at RHIC
is it due to P- and CP-odd effects or something else?


See also B. Müller, Physics 2, 104 (2009)

Data cannot be explained by

HIJING
HIJING+v2,
MeVSIM,
UrQMD
STAR data due to P- and CP-odd effects?

**Deconfinement necessary** to separate quarks
**Chiral Symmetry restoration necessary** to induce chirality

Hence no Chiral Magnetic Effect at low energies. Test energy scan. Also test at LHC

**Magnetic field** the correlators proportional to $Z^2$.

Test: compare collisions with same A and different Z, isobars
Argon-40 ($Z=18$), vs. Calcium-40 ($Z=20$), 23% increase in signal

**More quantitative phenomenology** really necessary
More data also possible: individual charged particle correlations

**Think of other explanations**
Cluster model of F. Wang ('09), .... ???
Conclusions: P- and CP-odd effects in heavy ion collisions

\[ \langle Q^2 \rangle \neq 0 \]

\[ \langle N_5^2 \rangle \neq 0 \]

\[ \langle J_z^2 \rangle > \langle J_{x,y}^2 \rangle \]

\[ \langle \cos(\phi_i^\pm + \phi_j^\pm, \mp - 2 \Psi_{RP}) \rangle \neq 0 \]

\[ \langle \Delta_{\pm}^2 \rangle > 0, \quad \langle \Delta + \Delta_{-} \rangle < 0 \]
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