Perfect-fluid hydrodynamics for RHIC – successes and problems

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HIRSCHEGG 2010: Strongly Interacting Matter under Extreme Conditions
Jan. 17-23, 2010
Google Maps: http://maps.google.com/?ll=40.874649,-72.870598&spn=0.047118,0.079823&z=14
Experimental data (soft hadronic sector)

1) transverse-momentum spectra, $p_T$ distributions

Experimental data

2) elliptic flow coefficient $v_2$

Animation by Jeffery Mitchell (Brookhaven National Laboratory)

PHENIX,
Experimental data

3) correlations of identical particles (Hanbury-Brown, Twiss)

two identical pions, $\pi^+\pi^+$, $\pi^-\pi^-$

source emitting particles

three projections of the correlation functions

Experimental data

3) source sizes (HBT radii)

- "Fourier transform"
- HBT radii
  - $R_{\text{side}}$ - spatial transverse extension
  - $R_{\text{out}}$ - spatial transverse extension + emission time
  - $R_{\text{long}}$ - longitudinal extension

HBT radii depend on $k_T$

![Graph showing HBT radii as a function of $k_T$](STAR, Phys.Rev.C71,044906(2005))
“Standard Model/Scheme” of heavy-ion collisions

main ingredients of the 2+1 models:

- **initial conditions**, short thermalization time, $\tau_i \leq 1$ fm
  - Glauber model, e.g., initial entropy/energy density is proportional to the linear combination of the wounded-nucleon density and binary-collision density,
    \[
    \sigma_i(x_\perp) \text{ or } \varepsilon_i(x_\perp) \propto \rho_{sr}(x_\perp) = \frac{1 - \kappa}{2} \bar{w}(x_\perp) + \kappa \bar{n}(x_\perp)
    \]

- Color Glass Condensate

- initial transverse flow, usually set equal to zero (?)

**HYDRODYNAMIC STAGE**

- $v_2$ data suggest that matter behaves like a perfect fluid main tool:
  - perfect-fluid hydrodynamics (Shuryak, Heinz, ...)

- hadronization included in the equation of state

**freeze-out, Cooper-Frye formula**

- freeze-out hypersurface, thermal description of hadron production

- transition hypersurface, change to a hadronic cascade
Motivation for our research
an attempt to obtain a uniform description of soft observables
(resolution of the HBT puzzle)


2+1 Cracow hydrodynamic model

- **initial conditions**, short thermalization time, $\tau_i \leq 1$ fm
  - Glauber and Gaussian initial conditions (including fluctuations)
  - option: initial transverse flow obtained from free-streaming (Yu. Sinyukov)

- **HYDRODYNAMIC STAGE**
  - perfect-fluid hydrodynamics — implemented in the code LHYQUID (M. Chojnacki)
  - hadronization included in the modern equation of state

- freeze-out, Cooper-Frye formula
  - freeze-out hypersurface, thermal description of hadron production — THERMINATOR (A. Kisiel et al.), complete set of resonances, single-freeze-out scenario assumed, two-particle method, with or without Coulomb, used to calculate the HBT radii: $R_{\text{side}}, R_{\text{out}}, R_{\text{long}}, R_{\text{out}}/R_{\text{side}}$
Initial conditions

- most of the approaches use the Glauber model or Color Glass Condensate,
- we assume the Gaussian profile (Gaussian approximation to Glauber)

\[
\frac{dN}{dx\,dy} \sim \exp \left( -\frac{x^2}{2a^2} - \frac{y^2}{2b^2} \right)
\]

the widths \(a\) and \(b\) determined from GLISSANDO

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\[1\] W. Broniowski, M. Rybczyński, P. Bożek arXiv:0710.5731[nucl-th]
Initial conditions

Free-streaming

- thermalization requires some time ($\tau \approx 0.25 - 1.0$ fm)
- two scenarios
  - early thermalization
  - free-streaming

- model for early stage dynamics
  - free streaming of particles, no interactions
  - sudden thermalization – Landau’s matching conditions.

- our results indicate that the two scenarios are equivalent
Thermodynamics

phase diagrams

- phase diagram for water

- phase diagram for QCD

![Thermodynamics phase diagrams](image-url)
Thermodynamics

modeling of the QCD EOS

- hadron gas model for low temperatures

  pliki inputowe z SHARE: Statistical hadronization with resonances

- lattice QCD simulations for large temperatures


- cross-over phase transition

  thermodynamic variables change suddenly at $T_c$ but smoothly, the sound velocity does not drop to zero

![Graph of $c_s^2$ vs. $T/T_c$](image-url)
Hydrodynamics

- energy-momentum conservation law

\[ \partial_\mu T^{\mu\nu} = 0 \]

- energy-momentum of the perfect fluid

\[ T^{\mu\nu} = (\epsilon + P) u^\mu u^\nu - Pg^{\mu\nu} \]

\( \epsilon \) - energy density, \( P \) - pressure, \( u^\mu \) - fluid four-velocity

- mid-rapidity \( (|y| \leq 1) \) for RHIC

\( \mu_B \approx 0 \), temperature is the only independent parameter

- boost-invariance

equations solved at \( z = 0 \), solutions for \( z \neq 0 \) obtained by Lorentz boosts

Hydrodynamics results from LHYQUID

reLativistic HYdrodynamics of QUark-gluon fluID

<table>
<thead>
<tr>
<th>Temperature ( T / T_c )</th>
<th>Velocity ( v / c )</th>
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<tr>
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<tr>
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</tbody>
</table>

Wojciech Florkowski (IFJ PAN & UJK Kielce)  Hydro description of heavy-ion collisions
Hydrodynamics

freeze-out hypersurfaces

- freeze-out temperature $T_f = 145$ MeV

**central collisions**

- because of the strong transverse flow, hadron do not reenter the medium
- space-like and time-like emission is similar

**peripheral collisions**

Wojciech Florkowski (IFJ PAN & UJK Kielce)
primordial particles are emitted according to the Cooper-Frye formula

\[ \frac{dN}{dy \, d^2 p_T} = \int d\Sigma^\mu_\mu f_{eq} (p \cdot u), \]

\( d\Sigma^\mu_\mu \) - element of the freeze-out hypersurface – obtained from hydro

\( u^\mu \) - four-velocity of the fluid

all resonances included

elliptic flow coefficient \( v_2 \)

\[ \frac{dN}{dy \, d^2 p_T} = \frac{dN}{dy \, 2\pi p_T \, dp_T} \left( 1 + 2v_2(p_T) \cos(2\phi_p) + \ldots \right) \]

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2 A. Kisiel, T. Tałuć, W. Broniowski and W. Florkowski
results for the spectra and $v_2$

\[ \frac{dN}{d\vec{p}_T} = 2\pi \rho p_T dp_T dy \]

\[ p_T \leq 1 \] GeV

$\rho$ = 0, 0.01, 0.1, 1, 10, 100, 1000

$\pi^+$, $K^+$, $\rho$

$T_i$ = 330 MeV, $T_i$ = 500 MeV

$\tau$ = 1. fm, $\tau$ = $\tau_0$

$PHENIX \at \sqrt{s_{NN}} = 200$ GeV

$c = 0$ -- 5%

$c = 20$ -- 30%

$T_i$ = 305 MeV, $T_i$ = 460 MeV

$\tau$ = 1. fm, $\tau$ = $\tau_0$

$PHENIX \at \sqrt{s_{NN}} = 200$ GeV

$c = 20$ -- 30%

$\rho$ = 20, 20 + 40%

$\rho$ = 20, 20 + 40%
two-particle method used to calculate the correlation functions (procedure mimics closely the experimental situation),

the wave function calculated in the pair rest frame (PRF) includes Coulomb (option)

correlation function fitted in the Bertsch-Pratt coordinates \((k_T, q_{\text{out}}, q_{\text{side}}, q_{\text{long}})\) with Bowler-Sinyukov correction (option)

\[
C(\vec{q}, \vec{k}) = (1 - \lambda) + \lambda K_{\text{coul}}(q_{\text{inv}}) \left[ 1 + \exp \left( -R_{\text{out}}^2 q_{\text{out}}^2 - R_{\text{side}}^2 q_{\text{side}}^2 - R_{\text{long}}^2 q_{\text{long}}^2 \right) \right],
\]

HBT radii \((R_{\text{out}}, R_{\text{side}}, R_{\text{long}})\) obtained from the fit and compared with data

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3 A. Kisiel, W. Florkowski and W. Broniowski
THERMINATOR

HBT results

STAR @ $\sqrt{s_{NN}} = 200$ GeV

- $c = 0-5\%$
- $T_i = 330$ MeV  $T_i = 500$ MeV
- $\tau = 1$ fm

- $c = 0-5\%$
- $T_i = 305$ MeV  $T_i = 460$ MeV
- $\tau = 1$ fm

$R_{\text{side}}$ [fm]

$R_{\text{out}}$ [fm]

$R_{\text{long}}$ [fm]

$R_{\text{out}}/R_{\text{side}}$

$k_T$ [GeV]
THERMINATOR

oscillations of the HBT radii, PRC 79 (2009) 014902

![Graphs showing oscillations of the HBT radii](image-url)
Conclusions

- 2+1 Cracow hydrodynamical model correctly describes the soft-hadronic data, in our opinion, this is the first successful attempt to solve the RHIC HBT puzzle.

- the things which matter
  - realistic equation of state (no soft point!)
  - initial profile – Gaussian approximation to Glauber, fluctuations of the eccentricity
  - all resonances included
  - two-particle algorithm for femtoscopy