NUCLEAR CHIRAL THERMODYNAMICS
and PHASES of QCD

Prelude: QCD Phase Diagram (Concepts, Models, Problems)

Main Theme: Nuclear Chiral Thermodynamics
- QCD interface with nuclear physics: Chiral Effective Field Theory
- Nuclear Equation of State and QCD phase diagram
- Density and temperature dependence of the Chiral (Quark) Condensate

Outlook: New constraints from Neutron Stars
Part I: Prelude

QCD PHASE DIAGRAM

Visions & Facts
QCD PHASE DIAGRAM

( theorists’ vision)

Spontaneous Chiral Symmetry Breaking

\[ \langle \bar{q}q \rangle \neq 0 \]

Hadron phase

\[ \langle qq \rangle \neq 0 \]

CSC phases

Quark – gluon phase

Critical point

\[ \mu_B \]

Baryon chemical potential

High Density:

Color Super Conductivity

Cooper pairing

q

[3]

q

\[ \bar{q}q \]

Condensation

\[ q \]

[1]
QCD PHASE DIAGRAM

Lattice QCD

T
[GeV]

hadron phase

\langle \bar{q}q \rangle \neq 0

quark – gluon phase

\langle q q \rangle \neq 0

\bar{q}q condensation

Constraints from Nuclear Physics

New constraints from Neutron Stars

\textit{Constraints from Nuclear Physics}

\textit{New constraints from Neutron Stars}
MODELING the QCD PHASE DIAGRAM

Guiding principle: 
QCD symmetries and symmetry breaking patterns

Spontaneously broken chiral symmetry 
\( SU(N_f)_R \times SU(N_f)_L \)

non-local PNJL model
Centre \( Z(3) \) of \( SU(3)_c \) gauge group

- **chiral** and deconfinement crossover transitions (3 flavor PNJL model)

\[
\frac{\langle \bar{q}q \rangle_T}{\langle \bar{q}q \rangle_0} \quad \text{Polyakov loop}
\]

\[
\frac{L_{\text{QCD}}}{{T_{\text{MeV}}}} \quad \frac{N_f}{c} = 2\pi \quad 2+1 \text{ flavors}
\]

**pure gauge**

\[
\frac{N_f}{c} = 6, p^4 \quad N_f = 8, 12, 16, \text{ stout}
\]

\[
N_f = 4, 8 \quad \text{(pure gauge)}
\]

---


Technische Universität München
The orange band shows the confinement-deconfinement crossover transition as described by the Polyakov loop in the range $k_l < \Phi < k_n$. The dashed black line corresponds to the chiral crossover $\bar{\psi}\psi / \bar{\psi}\psi < 0 < k_n$. The solid black line indicates the chiral first-order transition. The temperature scale is set by $T_c = 270$, $218$ MeV. Including wavefunction renormalization effects requires a careful reassessment of chiral low-energy theorems. Pseudoscalar meson masses and corresponding decay constants at zero temperature have been derived. The results clearly show that the formalism incorporates fundamental chiral relations such as the Gell-Mann–Oakes–Renner and Goldberger-Treiman relations. In the three-flavor case, the inclusion of the 't Hooft-Kobayashi-Maskawa interaction leads to the correct mass splitting between the $\eta$ and the $\eta'$. The PNJL thermodynamics has now been developed with systematic inclusion of the quark quasiparticle renormalization factor $Z_b$. The temperature dependence of the chiral condensate and of the Polyakov loop has been calculated, indicating chiral and deconfinement crossover transitions. We have compared our results with recent lattice QCD computations. Finally, a quark chemical potential has been introduced that enables extensions to the finite-density region of the QCD phase diagram. The impact of the wavefunction renormalization factor $Z_b$ compared to previous calculations [llf lm] setting $Z_b \equiv 1$ is generally quite small over the whole relevant momentum range. This can be understood considering the gap equations at zero temperature, since $Z_b$ deviates significantly from unity only in the momentum range $p < l$ GeV, its effect does not contribute much to the relevant integrals because of its suppression by the integration measure. With inclusion of $Z_b$, the chiral and deconfinement crossover transitions tend to become smoother compared to our previous investigations. The flavor dependence of the deconfinement temperature scale is an important issue in lowering chiral transition temperatures in accordance with the tendency recently

---

**Figure 1:** Phase diagram for the 3-flavor nonlocal PNJL model at meanfield level. The orange band shows the confinement-deconfinement crossover transition as described by the Polyakov loop in the range $k_l < \Phi < k_n$. The dashed black line corresponds to the chiral crossover $\bar{\psi}\psi / \bar{\psi}\psi < 0 < k_n$. The solid black line indicates the chiral first-order transition. The temperature scale is set by $T_c = 270$, $218$ MeV.

**Figure 2:** The $T$ dependence of the $\bar{\sigma}$ and $\text{Re}\Phi$. The dotted and solid lines are results with $T_0 = 270$ and $218$ MeV, respectively.

---

**References:**

PHASE DIAGRAM (contd.)

- PNJL analysis of Lattice QCD phase diagram at imaginary chemical potential suggests significant isoscalar vector term in the effective quark-quark interaction
  \[ \delta \mathcal{L}_V = -G_V (\bar{\psi} \gamma_\mu \psi)(\bar{\psi} \gamma^\mu \psi) \]

- Existence and location of critical point: extremely sensitive to
  a) Strength of vector interaction
  b) Axial U(1) breaking interaction

Trajectories of critical point

PNJL calculations
input:

input:


preliminary
The entropy jumps. The bump around the first-order transition from below. At the transition, the quark density approaches the SB-limits (dashed lines) are also shown. In Fig. 9, the susceptibility is more pronounced towards the CEP. In Fig. 10, the scaled quark number density (left panel) and for different chemical potentials. The CEP is located approximately at \( T \approx 100 \) MeV, followed by the coexistence region of a first order transition. Thus, we find the comparatively large value of \( \frac{\partial \rho}{\partial T} \) at the quark number density approaches much faster the critical endpoint, it happens at large chemical potential, our truncation scheme may no longer count. Expectations from recent lattice calculations in the PQM model, certainly, at such large values of the chemical potential, the susceptibility is more pronounced towards the CEP.

Certainly, at such large values of the chemical potential, the susceptibility is more pronounced towards the CEP. This statement agrees with the result of corresponding calculations in the PQM model, certainly, at such large values of the chemical potential, the susceptibility is more pronounced towards the CEP.
Part II:

NUCLEAR CHIRAL THERMODYNAMICS
NUCLEAR MATTER and QCD PHASES

Scales in nuclear matter:

- momentum scale: \( k_F \simeq 1.4 \text{ fm}^{-1} \simeq 2m_\pi \)
- NN distance: \( d_{NN} \simeq 1.8 \text{ fm} \simeq 1.3 \text{ m}_\pi^{-1} \)
- energy per nucleon: \( \frac{E}{A} \simeq -16 \text{ MeV} \)
- compression modulus: \( K = (260 \pm 30) \text{ MeV} \simeq 2m_\pi \)
PIONS and NUCLEI in the context of LOW-ENERGY QCD

- CONFINEMENT of quarks and gluons in hadrons
- Spontaneously broken CHIRAL SYMMETRY

LOW-ENERGY / LOW-TEMPERATURE QCD: Effective Field Theory of weakly interacting Nambu-Goldstone Bosons (PIONS) representing QCD at (energy and momentum) scales $Q \ll 4\pi f_\pi \sim 1$ GeV

\[ m_\pi^2 f_\pi^2 = -m_q \langle \bar{\psi}\psi \rangle + O(m_q^2) \]

\[ f_\pi = 92.4 \text{ MeV} \]
CHIRAL EFFECTIVE FIELD THEORY

- Systematic framework at interface of QCD and Nuclear Physics

- Interacting systems of **PIONS** (light / fast) and **NUCLEONS** (heavy / slow):

\[ \mathcal{L}_{eff} = \mathcal{L}_\pi(U, \partial U) + \mathcal{L}_N(\Psi_N, U, ...) \]

\[ U(x) = \exp\left[ i\tau_a \pi_a(x)/f_\pi \right] \]

- Construction of Effective Lagrangian: **Symmetries**

  short distance dynamics: **contact terms**
Explicit $\Delta(1230)$ DEGREES of FREEDOM

- **Large spin-isospin polarizability** of the Nucleon

  example: polarized Compton scattering

  \[
  \beta_\Delta = \frac{g_A^2}{f^2 \pi (M_\Delta - M_N)} \sim 5 \text{ fm}^3
  \]

  \[
  M_\Delta - M_N \simeq 2 m_\pi < < 4\pi f_\pi
  \]

  (small scale)

- **Pionic Van der Waals** - type intermediate range central potential


  N. Kaiser, S. Fritsch, W.W., NPA750 (2005) 259

  \[
  V_c(r) = -\frac{9 g_A^2}{32\pi^2 f_\pi^2} \beta_\Delta \frac{e^{-2m_\pi r}}{r^6} P(m_\pi r)
  \]

  strong 3-body interaction

  J. Fujita, H. Miyazawa (1957)

  Pieper, Pandharipande, Wiringa, Carlson (2001)
Important pieces of the CHIRAL NUCLEON-NUCLEON INTERACTION

**ISOVECTOR TENSOR FORCE**

- $S_1, V_T, S_2$
- Note: **no** $\rho$ meson

**CENTRAL ATTRACTION** from **TWO-PION EXCHANGE**

- Note: **no** $\sigma$ boson

Van der WAALS - like force:

$$V_c(r) \propto -\frac{\exp[-2m_\pi r]}{r^6} P(m_\pi r)$$

... at intermediate and long distance

CHIRAL DYNAMICS and the NUCLEAR MANY-BODY PROBLEM


- Small scales: \[ k_F \sim 2 m_\pi \sim M_\Delta - M_N << 4\pi f_\pi \]

- PIONS (and DELTA isobars) as explicit degrees of freedom

**IN-MEDIUM CHIRAL PERTURBATION THEORY**

Pion exchange processes in presence of filled Fermi sea

2nd order **TENSOR** force + nucleon’s **SPIN-ISOSPIN** polarizability

Short-distance dynamics: contact interactions (incl. resummations)
**IN-MEDIUM CHIRAL PERTURBATION THEORY**

- Loop expansion of *(In-Medium) Chiral Perturbation Theory*

  Systematic expansion of **ENERGY DENSITY** $\mathcal{E}(k_F)$ in powers of Fermi momentum [modulo functions $f_n(k_F/m_\pi)$]
  
  (works for $k_F << 4\pi f_\pi \sim 1$ GeV)

- Finite nuclei $\leftrightarrow$ energy density functional


  many quantitatively successful applications throughout the nuclear chart

  e.g. P. Finelli et al.: Nucl. Phys. A 770 (2007) 1

- Nuclear **thermodynamics**: compute **free energy density**

  (3-loop order)


  in-medium nucleon propagators incl. Pauli blocking
**NUCLEAR MATTER**

- **In-medium ChPT**
  3-loop \((\pi, N, \Delta)\)

- **Input** parameters:
  two contact terms

- basically:
  analytic calculation

- **Output:**
  - Binding & saturation
    \[ E_0/A = -16 \text{ MeV} , \quad \rho_0 = 0.16 \text{ fm}^{-3} , \quad K = 290 \text{ MeV} \]
  - Realistic (complex, momentum dependent) single-particle potential
    ... satisfying Hugenholtz - van Hove and Luttinger theorems (!)
  - Asymmetry energy \( A(k_F^0) = 34 \text{ MeV} \)
  - Quasiparticle interaction and Landau parameters

\[ \text{S. Fritsch, N. Kaiser, W.W.}
\text{ Nucl. Phys. A 750 (2005) 259} \]

\[ \text{J.W. Holt, N. Kaiser, W.W.}
\text{ Nucl. Phys. A 870 (2011) 1,}
\text{ arXiv: 1111.1924 [nucl-th]}
\text{(NPA (2012), in print)} \]
NUCLEAR THERMODYNAMICS

NUCLEAR CHIRAL (PION) DYNAMICS

BINDING & SATURATION:
Van der Waals + Pauli

\[ \begin{align*}
\pi & \ \ \ \ \ \ \ \ \ \ \ N, \Delta \\
N & \rightarrow \pi \rightarrow N
\end{align*} \]

+ 3-body forces

\[ \begin{align*}
\pi & \ \ \ \ \ \ \ \ \ \ \ N, \Delta \\
N & \rightarrow N
\end{align*} \]

contact terms

nuclear matter: equation of state

pressure

3-loop in-medium ChEFT

\[ T = 25 \text{ MeV} \]

\[ T = 20 \text{ MeV} \]

\[ T = 15 \text{ MeV} \]

\[ T = 10 \text{ MeV} \]

\[ T = 5 \text{ MeV} \]

\[ T = 0 \text{ MeV} \]

\[ \rho \ [\text{fm}^{-3}] \]

baryon density

Liquid - Gas Transition at Critical Temperature \( T_c = 15 \text{ MeV} \)

(empirical: \( T_c = 16 \text{ - } 18 \text{ MeV} \))
**PHASE DIAGRAM of NUCLEAR MATTER**

In-medium

**chiral effective field theory**

(3-loop calculation of free energy density)


S. Fiorilla, N. Kaiser, W.W.


- Pion-nucleon dynamics incl. delta isobars
- Short-distance NN contact terms
- Three-body forces

---

**Phase Diagram**

- Gas
- Liquid
- Critical point
- Symmetric ($N = Z$) nuclear matter

**Variables**

- Temperature $T$ [MeV]
- Baryon chemical potential $\mu_B$ [MeV]
- Pressure $P$ [MeV]
- Density $\rho$ [fm$^{-3}$]

---

Technische Universität München
PHASE DIAGRAM of NUCLEAR MATTER

Trajectory of CRITICAL POINT for asymmetric matter as function of proton fraction $Z/A$

...determined almost entirely by isospin dependent (one- and two-) pion exchange dynamics

In-medium chiral effective field theory (3-loop) with resummation of short distance contact terms (large nn scattering length, $a_s = 19 \text{ fm}$)


Perfect agreement with sophisticated many-body calculations.
CHIRAL CONDENSATE at finite BARYON DENSITY

- Chiral (quark) condensate $\langle \bar{q}q \rangle$:
  Order parameter of spontaneously broken chiral symmetry in QCD
  \[ m_\pi^2 f_\pi^2 = -2 m_q \langle \bar{q}q \rangle \]

- Hellmann - Feynman theorem:
  \[ \langle \Psi | \bar{q}q | \Psi \rangle = \langle \Psi | \frac{\partial H_{\text{QCD}}}{\partial m_q} | \Psi \rangle = \frac{\partial E(m_q; \rho)}{\partial m_q} \]

\[ \frac{\langle \bar{q}q \rangle_\rho}{\langle \bar{q}q \rangle_0} = 1 - \frac{\rho}{f_\pi^2} \left[ \frac{\sigma_N}{m_\pi^2} \left( 1 - \frac{3 p_F^2}{10 M_N^2} + \ldots \right) + \frac{\partial}{\partial m_\pi^2} \left( \frac{E_{\text{int}}(p_F)}{A} \right) \right] \]

- (free) Fermi gas of nucleons
- Nuclear interactions (dependence on pion mass)

sigma term
in-medium chiral effective field theory

Technische Universität München
CHIRAL CONDENSATE: DENSITY DEPENDENCE

- **In-medium Chiral Effective Field Theory** (NLO 3-loop)

  constrained by **realistic nuclear equation of state**


- Substantial **change** of **symmetry breaking scenario**
  between chiral limit $m_q = 0$ and physical quark mass $m_q \sim 5$ MeV

- **Nuclear Physics** would be **very different** in the **chiral limit**!

\[ \frac{\langle \bar{\psi} \psi \rangle(\rho)}{\langle \bar{\psi} \psi \rangle(\rho = 0)} \]

\[ \rho \] [fm$^{-3}$]

\[ m_\pi = 0.14 \text{ GeV} \]

\[ T = 0 \]

\[ m_\pi \rightarrow 0 \]

\[ m_q \rightarrow 0 \]

\[ \rho_0 \]

\[ \langle \bar{\psi} \psi \rangle(\rho) \]

\[ \langle \bar{\psi} \psi \rangle(\rho = 0) \]
**CHIRAL CONDENSATE: DENSITY and TEMPERATURE DEPENDENCE**

- Free energy density
  \[ \mathcal{F}(m_q; \rho, T) \]

- **In-medium Chiral Effective Field Theory**
  (NLO 3-loop)

- constrained by realistic nuclear equation of state

\[ \langle \Psi | \bar{q} q | \Psi \rangle_{\rho, T} = \frac{\partial \mathcal{F}(m_q; \rho, T)}{\partial m_q} \]

No indication of first order chiral phase transition for
\[ \rho \lesssim 2 \rho_0 \quad \text{and} \quad T \lesssim 100 \text{ MeV} \]


\[ \frac{\langle \bar{q} q \rangle_{\rho, T}}{\langle \bar{q} q \rangle_0} \]

symmetric nuclear matter
\[ N = Z \]

\[ T = 0 \quad \text{and} \quad T = 100 \text{ MeV} \]

\[ \rho \quad \text{[fm}^{-3} \text{]} \]

[Graph showing the dependence of \( \langle \bar{q} q \rangle \) on density at different temperatures]

Technische Universität München
CHIRAL CONDENSATE: Dependence on TEMPERATURE and BARYON CHEMICAL POTENTIAL

- Liquid-gas phase transition leaves its signature also in chiral condensate
- But: no tendency toward chiral first order transition in the range $\mu_B \lesssim 1$ GeV

$\langle \bar{q}q \rangle_{\rho,T}$

Outlook:

New Constraints from
NEUTRON STARS
A two-solar-mass neutron star measured using Shapiro delay

direct measurement of neutron star mass from increase in travel time near companion

J1614-2230 most edge-on binary pulsar known (89.17°) + massive white dwarf companion (0.5 $M_{\text{sun}}$)

heaviest neutron star with $1.97 \pm 0.04 \, M_{\text{sun}}$
TWO-SOLAR-MASS NEUTRON STAR

... observed using Shapiro delay

P.B. Demorest et al., Nature 467 (2010) 1081
**New constraints from EFT and neutron star observables**

- **Exotic** equations of state ruled out?

---

**News from NEUTRON STARS**

K. Hebeler, J. Lattimer, C. Pethick, A. Schwenk  
PRL 105 (2010) 161102

---

A.W. Steiner, J. Lattimer, E.F. Brown  

---

- realistic “nuclear” EoS (Illinois)

---

- \textbf{New constraints from EFT and neutron star observables}

- **"Exotic"** equations of state ruled out?
NEUTRON STAR MATTER

Equation of State

Including new neutron star constraints plus Chiral Effective Field Theory at lower density.

Low-density (crust) + ChEFT (FKW)
Constrained extrapolation (polytropes)
Akmal, Pandharipande, Ravenhall (1998)

\[ P = \text{const} \cdot \rho^\Gamma \]

astrophysics constraints \( M(R) \)

nuclear physics constraints

Figure 6.3: PSR J1614-2230 and the observational constraints to the radii due to Steiner et al. delimits the range for the physical EoS into the green area wfor further explanation see textx. The horizontal dashed lines are the limits previously given for \( P^2 \) according to [19]. The APR EoS [20] is shown for comparison.
SUMMARY

• Exploration of **QCD phase diagram**: progress concerning basic symmetry breaking patterns
  ▶ Lattice QCD (restricted to small quark chemical potentials)
  ▶ Models (PNJL, PQM) (but: nuclear physics constraints missing)
  ▶ Dyson-Schwinger QCD ( -- same problem -- )

• **Nuclear thermodynamics** based on
  In-medium **Chiral Effective Field Theory**
  Fermi liquid ↔ interacting Fermi gas (1st order transition)
  ▶ No indication of first order **chiral** transition in the range
    \[ \rho \lesssim 2 \rho_0 , \ T \lesssim 100 \text{ MeV} \]
  ▶ Major challenge: design **QCD phase diagram** that is consistent with established hadronic and nuclear physics

• New **dense & cold matter** constraints from **neutron stars**:
  ▶ Mass - radius relation; observation of two-solar-mass n-star
  ▶ “Non-exotic” equation of state works best!
The End

thanks to:

Nino Bratovic     Salvatore Fiorilla     Thomas Hell
Jeremy Holt       Norbert Kaiser        Kouji Kashiwa
Skyrme phenomenology

Multifragmentation and fission analysis

G. Sauer, H. Chandra, U. Mosel
Nucl. Phys. A 264 (1976) 221

V.A. Karnaukhov et al.: