Lattice calculation of nucleon EDM

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Outline

- Introduction
  - Motivation of lattice calculation of EDM

- Strategy and method in lattice QCD
  - Lattice QCD
  - Extraction of EDM from correlation function

- Recent update (preliminary)

- Summary and future work
1. Introduction

Neutron EDM

- CPV in QCD and the new physics (NP)
- Since 1960’s, sensitivity of experiment has been developed.
  - Current nEDM upper limit is $|d_N^{\text{exp}}| < 2.9 \times 10^{-26}$ e·cm
- Sensitive observable to NP
  - Naturally QCD has CPV from $\theta$ term, but it seems to be unnaturally small. (strong CP problem)
  - Direct search of CPV from NP
    BSM (SUSY, etc) says the discover is coming soon…
  - Intensity frontier physics
    Alternative direction from high energy collision.
    Precision of the SM calculation is necessary.

http://www.fnal.gov/pub/science/frontiers/
1. Introduction

Nucleon EDM in EW

- Contribution to EDM in weak interaction is very small
  - No CP phase in 1-loop ($|V_{dq}|^2$) and 2-loop diagram (cancelation)
  - Three-loop order (short) and pion loop correction (long):

![Diagram](image)

Czmechi, Krause (1997)
Khriplovich, Zhitnitsky (1982)

\[
d_{N}^{KM\text{ short}} \sim \mathcal{O}(\alpha_s G_F^2) \sim -10^{-34} \text{ e} \cdot \text{cm}
\]

\[
d_{N}^{KM\text{ long}} \sim 10^{-30} - 10^{-32} \text{ e} \cdot \text{cm}
\]

\[
\Rightarrow d_{N}^{KM} = d_{N}^{KM\text{ short}} + d_{N}^{KM\text{ long}} \sim 10^{-30} - 10^{-32} \text{ e} \cdot \text{cm}
\]

which is the 6-order magnitude below the experimental upper limit.
(to confirm, non-perturbative estimate is also needed)
1. Introduction

Nucleon EDM in QCD

- $\theta$ term in QCD Lagrangian
  \[
  \mathcal{L} = \bar{q}_L M q_R + \bar{q}_R M^\dagger q_L + \frac{\theta}{(64\pi^2)} G \tilde{G}
  \]
  \[
  \Rightarrow \mathcal{L}_\theta = \bar{\theta} \frac{1}{64\pi^2} G \tilde{G}, \quad \bar{\theta} = \theta + \arg \det M, \quad \arg \det M \sim \eta \frac{m_u m_d m_s}{m_u m_d + m_d m_s + m_s m_u}
  \]

- Renormalizable and CPV.
- $d_N/\theta \sim 10^{-16}$ e cm (quark model, current algebra, etc)
  $\theta$ and arg det $M$ is *unnaturally* canceled. (Crewther, et al. (1979), Ellis, Gaillard (1979))

- Possible solution
  1. *Massless quark* ($m_u = 0$) from lattice QCD+QED, $m_u = 2.24(35)$ MeV, $m_d = 4.65(35)$ MeV.
    It is hard to explain $\bar{\theta} = 0$.
  2. *Axion model* (*assumption of PQ symm.*), invisible axion model
  3. *Spontaneous CP breaking.* $\theta$ is calculable in loop order.
1. Introduction

Nucleon EDM in BSM

- Higher dimension operators of CPV
  
  \[ H_{CP} = \sum_k C_k(\mu) \mathcal{O}_k \]

  \[ \mathcal{O}_{qEDM} = d_q \bar{q}(\sigma \cdot F)\gamma_5 q \quad \text{: Quark-photon (5-dim)} \]
  
  \[ \mathcal{O}_{cEDM} = d_q^c \bar{q}(\sigma \cdot G)\gamma_5 q \quad \text{: Quark-gluon (5-dim)} \]
  
  \[ \mathcal{O}_{Weinberg} = d^G G G \hat{G} \quad \text{: Pure gluonic (6-dim)} \]

  - In effective Hamiltonian, the new CPV term appears.
  - In BSM, q(c)EDM corresponds to CP phase of heavy particle.
  - \( d_q \) and \( d_q^c \) are determined by BSM

To obtain EDM, we need to estimate QCD effect in nucleon.

Hisano, et al. (2009)
1. Introduction

Nucleon EDM in BSM

- Higher dimension operators of CPV

\[ H_{CP} = \sum_k C_k(\mu) O_k \]

\[ O_{qEDM} = d_q \bar{q} (\sigma \cdot F) \gamma_5 q \quad : \text{Quark-photon (5-dim)} \]

\[ O_{cEDM} = d_q \bar{q} (\sigma \cdot G) \gamma_5 q \quad : \text{Quark-gluon (5-dim)} \]

\[ O_{Weinberg} = d^G G G \bar{G} \quad : \text{Pure gluonic (6-dim)} \]

- In effective Hamiltonian, the new CPV term appears.
- In BSM, q(c)EDM corresponds to CP phase of heavy particle.
- \( d_q \) and \( d_q \) are determined by BSM

To obtain EDM, we need to estimate QCD effect in nucleon.

Using baryon CHPT or QCD sum rule, there are several evaluations,

\[ d_N = d_{N}^{QCD} \bar{\theta} + d_N(d_q, d_q^c) + d_N(d^G) \]

\[ \sim 10^{-17}[e \cdot cm] \bar{\theta} + (1.4 - 0.47)d_d - (0.12 - 0.35)d_u + O(10^{-2})d_q^c \]

\[ \sim O(10^{-25} - 10^{-27}) e \cdot cm \]

Hisano, et al. (2009)

Mereghetti, Vries, Hockings, Maekawa, Kolck, Timmermans, ...

Pospelov, Ritz, Hisano, Shimizu, Nagata, Lee, Yang, ...

Hisano, Shimizu (04), Ellis, Lee, Pilaftsis (08), Hisano, Lee, Nagata, Shimizu (12)
1. Introduction

Constraint on nEDM

- EDM experiment
  - (p,d)EDM experiment @ BNL,
  - nEDM experiment @ ORNL, ILL, FRM-2, FNAL, PSI/KEK/TRIUMF, ...
  - Charged particle (d,p)EDM @ COSY
  - Lepton EDM @ J-PARC, FNAL
  - aiming for a sensitivity to $10^{-29}$ e ¥ cm!

- Current estimate of QCD effect is based on quark model or BChPT, and it includes sort of model dependence.

- Non-perturbative contribution of $\theta$ term, qEDM, cEDM, etc is needed.

- $\pi NN$ coupling from lattice QCD is also useful for study of atomic EDM (schiff moment)
1. Introduction

What lattice QCD can do for nEDM

- **In principle**
  - Direct estimate of hadronic contribution to neutron and proton EDM for $\theta$ term, higher dim. CPV operators
  - Matrix elements (or condensate) including higher dimension operators → for QCD sum rule, ChPT, ...

Bhattacharya et al, Lattice 2012
1. Introduction

What lattice QCD can do for nEDM

- In principle
  - Direct estimate of hadronic contribution to neutron and proton EDM for $\theta$ term, higher dim. CPV operators
  - Matrix elements (or condensate) including higher dimension operators
    $\rightarrow$ for QCD sum rule, ChPT, …

- In practice there are some difficulties
  - Statistical noise
    - Gauge background (topological charge, sea quark) and disconnected diagram (flavor singlet contraction) are intrinsically noisy.
1. Introduction

What lattice QCD can do for nEDM

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  - Direct estimate of hadronic contribution to neutron and proton EDM for \( \theta \) term, higher dim. CPV operators
  - Matrix elements (or condensate) including higher dimension operators
    \( \rightarrow \) for QCD sum rule, ChPT, …

- **In practice there are some difficulties**
  - **Statistical noise**
    *gauge background* (topological charge, sea quark) and *disconnected diagram* (flavor singlet contraction) are intrinsically noisy.
  - **Systematic study**
    Volume effect may be significant. (e.g. BChPT discussion)
    
    O’Connell, Savage, PLB633, 319(2006), Guo, Meissner, 1210.5887

Chiral behavior is also important check, \( d_N \sim O(m) \).
Strategy and method in lattice QCD
In lattice regularization, the path integral is computed by Monte-Carlo integral:

$$\langle O \rangle = Z^{-1} \int D\Psi O(\Psi) e^{-S(\Psi)} \approx \sum_{\Psi} O(\Psi) P(\Psi)$$

- **Exact** QCD calculation (enough large number of sampling $N$)
- Gauge invariant
- Translational invariant
- **Ultraviolet cut-off** $a$ (lattice spacing)
- **Infrared cut-off** $V=L_0^D$ (lattice volume)
- Continuum limit and infinite volume are important.
- The development of machine (BG, GPGPU, …) and algorithm, which make much progress.
2. Strategy and method in lattice QCD

Hadron spectrum in lattice QCD

- Good agreement with various lattice action and fermion with experimental results!
  
  Kronfeld, 1209.3468
2. Strategy and method in lattice QCD

Choice of lattice fermion

- There are several kinds of fermion definition on the lattice.
- Require “realistic” fermion for the **precise calculation** which has good approximated **chiral symmetry** on the lattice.
- Good suppression of \(O(a)\) effect
- **Domain-wall fermion** is appropriate selection.

**Domain-Wall fermion (DWF)**

- \(L, R\) fermion are localized on boundaries. Exact chiral symmetry is realized if \(L_s \rightarrow \infty\).
- In finite \(L_s\)
  - Violation of chiral symm. is suppressed as \(a m_{res} \sim \exp(-L_s) \ll 1\).
- Because of additional dimension, computational cost is much higher than Wilson fermion or staggered fermion.

[Blum Soni, (97), CP-PACS(99), RBC(00), RBC/UKQCD. (05 --) ]
2. Strategy and method in lattice QCD

Lattice methods of nEDM

- **Spectrum method**

- **Form factor**

- **Imaginary \( \theta \)**
  2. Horsley et al., arXiv:0808.1428 [hep-lat]
2. Strategy and method in lattice QCD

Lattice methods of nEDM

- **Spectrum method**

- **Form factor**

- **Imaginary \( \theta \)**
2. EDM calculation on the lattice

EDM Form factor

\[
\langle n(P_1) | J_{\mu}^{EM} | n(P_2) \rangle_\theta = \bar{u}_N \left[ \frac{F_3(q^2)}{2m_N} \gamma_5 \sigma_{\mu\nu} q_\nu + F_A(q^2) (i q^2 \gamma_\mu \gamma_5 - 2m_N q_\mu \gamma_5) \right]_{P,T\text{-odd}} + F_1(q^2) \gamma_\mu + \frac{F_2(q^2)}{2m_N} \sigma_{\mu\nu} q_\nu \right]_{P,T\text{-even}} u_\theta
\]

2. EDM calculation on the lattice

**EDM Form factor**

\[
\langle n(P_1)|J_{\mu}^{EM}|n(P_2)\rangle_\theta = \bar{u}_N^\theta \left[ \frac{F_3(q^2)}{2m_N} \gamma_5 \sigma_{\mu\nu} q_\nu + \frac{F_A(q^2)}{2m_N} \left( i q_5 \gamma_\mu \gamma_5 - 2m_N q_\mu q_5 \right) \right] P,T-odd
\]

\[
+ F_1(q^2) \gamma_\mu + \frac{F_2(q^2)}{2m_N} \sigma_{\mu\nu} q_\nu \right] u_N^\theta \]

\[
\sum_s u_N^\theta(s) \bar{u}_N^\theta(s) = \frac{ip \cdot \gamma + m_N e^{i\alpha_N^\theta \gamma_5}}{2E_N} \]

CPV phase \( \alpha_N \) in nucleon propagator

2. EDM calculation on the lattice

**EDM Form factor**

\[
\langle n(P_1)|J_\mu^{EM}|n(P_2)\rangle_\theta = \bar{u}_N^\theta \left[ \frac{F_3(q^2)}{2m_N} \gamma_5 \sigma_{\mu\nu} q^\nu + F_A(q^2)(i q^2 \gamma_\mu \gamma_5 - 2m_N q^\mu \gamma_5) \right]_{\text{P,T-odd}}
\]

\[
+ F_1(q^2) \gamma_\mu + \frac{F_2(q^2)}{2m_N} \sigma_{\mu\nu} q^\nu \right] u_N^\theta \right]_{\text{P,T-even}}
\]

\[
\sum_s u_N^\theta(s)\bar{u}_N^\theta(s) = \frac{ip \cdot \gamma + m_N e^{i\alpha_N^\theta \gamma_5}}{2E_N}
\]

**CPV phase** \(\alpha_N\) in nucleon propagator

\[
\langle \theta|\eta_N J_\mu^{EM} \bar{\eta}_N|\theta \rangle = \langle 0|\eta_N J_\mu^{EM} \bar{\eta}_N|0 \rangle + i\theta \langle 0|\eta_N J_\mu^{EM} Q \bar{\eta}_N|0 \rangle
\]

2. EDM calculation on the lattice

**EDM Form factor**

\[
\langle n(P_1)|J_{\mu}^{EM}|n(P_2)\rangle_{\theta} = \bar{u}_N^{\theta} \left[ \frac{F_3(q^2)}{2m_N} \gamma_5 \sigma_{\mu\nu} q_{\nu} + \frac{F_A(q^2)}{2m_N} (iq^2 \gamma_\mu \gamma_5 - 2m_N q_\mu \gamma_5) \right]_{\text{P,T-odd}} \\
+ \frac{F_1(q^2)}{2m_N} \gamma_\mu + \frac{F_2(q^2)}{2m_N} \sigma_{\mu\nu} q_{\nu} \right]_{\text{P,T-even}} u_N^{\theta}
\]

\[
\sum_s u_{N}(s)\bar{u}_{N}(s) = \frac{ip \cdot \gamma + m_N e^{i\alpha_{N}^{\theta} \gamma_5}}{2E_N}
\]

\[
\langle \theta|\eta_N J_{\mu}^{EM} \bar{\eta}_N|\theta \rangle = \langle 0|\eta_N J_{\mu}^{EM} \bar{\eta}_N|0 \rangle + i\theta \langle 0|\eta_N J_{\mu}^{EM} Q \bar{\eta}_N|0 \rangle
\]

\[
\langle 0|\eta_N(t_1) J_{\mu}^{EM}(t) Q \bar{\eta}_N(t_0)|0 \rangle
\]

\[
= \frac{\alpha_{N}}{2} \gamma_5 \left[ F_1 \gamma_\mu + F_2 \frac{q_\nu \sigma_{\mu\nu}}{2m_N} \right] \frac{ip \cdot \gamma + m_N}{2E_N} + \frac{1 + \gamma_4}{2} \left[ F_1 \gamma_\mu + F_2 \frac{q_\nu \sigma_{\mu\nu}}{2m_N} \right] \frac{\alpha_{N}}{2} \gamma_5
\]

\[
+ \frac{1 + \gamma_4}{2} \left[ F_3 \frac{q_\nu \gamma_5 \sigma_{\mu\nu}}{2m_N} + F_A (iq^2 \gamma_\mu \gamma_5 - 2m_N q_\mu \gamma_5) \right] \frac{ip \cdot \gamma + m_N}{2E_N}
\]

• Subtraction of CP-odd phase, \(\alpha_{N}\), in nucleon propagator and CP-even part \(F_{1,2}\)

2. EDM calculation on the lattice

**EDM Form factor**

\[
\langle n(P_1)|J_{\mu}^{EM}|n(P_2)\rangle = \bar{u}_N^\theta \left[ \frac{F_3^\theta(q^2)}{2m_N} \gamma_5 \sigma_{\mu\nu} q_{\nu} + F_A(q^2) (i q^2 \gamma_{\mu} \gamma_5 - 2 m_N q_{\mu} \gamma_5) \right] P,T\text{-odd} + \frac{F_1(q^2) \gamma_{\mu} + F_2(q^2)}{2m_N} \sigma_{\mu\nu} q_{\nu} \right] u_N^\theta \text{ P,T-even}
\]

\[
\sum_s u^\theta_N(s) \bar{u}^\theta_N(s) = \frac{ip \cdot \gamma + m_N e^{i\alpha_N^\theta \gamma_5}}{2E_N}
\]

\[
\langle \theta|\eta_N J_{\mu}^{EM} \bar{\eta}_N |\theta \rangle = \langle 0|\eta_N J_{\mu}^{EM} \bar{\eta}_N |0 \rangle + i\theta \langle 0|\eta_N J_{\mu}^{EM} Q \bar{\eta}_N |0 \rangle
\]

\[
\langle 0|\eta_N(t_1) J_{\mu}^{EM}(t) Q \bar{\eta}_N(t_0) |0 \rangle = \frac{\alpha_N^\theta}{2} \gamma_5 \left[ F_1 \gamma_{\mu} + F_2 \frac{q_\nu \sigma_{\mu\nu}}{2m_N} \right] \frac{ip \cdot \gamma + m_N}{2E_N} + \frac{1 + \gamma_4}{2} \left[ F_1 \gamma_{\mu} + F_2 \frac{q_\nu \sigma_{\mu\nu}}{2m_N} \right] \frac{\alpha_N^\theta}{2} \gamma_5
\]

\[
+ \frac{1 + \gamma_4}{2} \left[ F_3 \frac{q_\nu \gamma_5 \sigma_{\mu\nu}}{2m_N} + F_A(i q^2 \gamma_{\mu} \gamma_5 - 2 m_N q_{\mu} \gamma_5) \right] \frac{ip \cdot \gamma + m_N}{2E_N}
\]

- Subtraction of CP-odd phase, $\alpha_N$, in nucleon propagator and CP-even part $F_{1,2}$
- EDM is given in zero momentum transfer limit $d_N = \lim_{Q^2 \to 0} F_3(Q^2)/2m_N$

2. EDM calculation on the lattice

**3-pt function**

- (Nucleon)-(EM current)-(Nucleon)
- Location of operators
  - $t = 0$ and $t = t_N$: nucleon op.
  - EM current inserts between nucleon ops.
- Comparison of different $t_{sep}$ is a good check of excited state contamination.
- Ratio of 3-pt and 2-pt

The signal appears as a plateau.

\[ t_N = 0 \]
\[ J_\mu \]
\[ t_N = t_{sep} \]
Recent update
3. Recent update (preliminary)

Recent work on EDM from lattice

- $\theta$ term
  - New developed algorithm, called as **AMA method**, which is error reduction techniques without additional cost.
    

- Extremely high statistics for form factors in DWF

- Ingredients to extract EDM form factor: ES, Blum, Izubuchi, Lattice 2013
  - EM form factor
  - topological charge distribution
  - CPV phase of nucleon wave function, $\alpha_N$
  - 3pt function in CP-odd sector
3. Recent update (preliminary)

Parameters

- **DWF**
  - \(24^3 \times 64\) lattice, \(\sigma^{-1} = 1.73\) GeV (~3 fm\(^3\) lattice)
  - \(L_s = 16\) and \(am_{\text{res}} = 0.003\)
  - \(m = 0.005, 0.01\) corresponding to \(m_\pi = 0.33, 0.42\) GeV
  - Two temporal separation of N sink and source in 3 pt. function
    \[t_{\text{sep}} = 12\ (t_{\text{source}} = 0, t_{\text{sink}} = 12), t_{\text{sep}} = 8\ (t_{\text{source}} = 0, t_{\text{sink}} = 8)\]
  - \# configs = 751 \((m=0.005)\), 700 \((m=0.01)\) \([t_{\text{sep}} = 12]\)
  - \# configs = 180 \((m=0.005)\), 132 \((m=0.01)\) \([t_{\text{sep}} = 8]\)
  - Comparison to check the higher excited state contamination

- **AMA**
  - \# of low-mode: \(N_\lambda = 400\ (m=0.005), 180\ (m=0.01)\)
  - Stopping condition, \(|r| < 0.003\)
  - \(N_G = 32\) (2 separation for spatial, 4 separation for temporal direction of source location) \(\rightarrow\) effectively \(O(10^4)\) statistics
3. Recent update (preliminary)

**EM form factor**

- By using AMA algorithm, statistical error of these observables achieve below 5% level.

- Compared with previous works (RBC PhysRevD79(2009)), computational time can be reduced by factor 5 and more. ⇒ higher precision
3. Recent update (preliminary)

Topological charge distribution

- **Topological susceptibility**

  \[
  \langle Q^2 \rangle / V = 3.0(1) \times 10^{-4} \text{ GeV}^4 \quad (m = 0.005) \\
  = 4.6(2) \times 10^{-4} \text{ GeV}^4 \quad (m = 0.01)
  \]

  Suppression by quark mass as expected in ChPT

---

![Graphs showing distribution of $Q_{\text{top}}$ for $m=0.005$ and $m=0.01$]
3. Recent update (preliminary)

\( \alpha_N : \text{CP-odd phase of wave function} \)

- Projection with \( \gamma_5 \) for 2 pt in \( \theta \) term, perform global fitting
  \[
  \text{tr} \left[ \gamma_5 \langle N(t)\bar{N}(0)Q \rangle \right] = Z_N \frac{2m_N}{E_N} \alpha_N e^{-E_N t} + O(e^{-E_N^*})
  \]
- By using AMA, this factor is determined within 15% error.
- It does not depend on smearing function and momentum (mass dependence is not so clear)
3. Recent update (preliminary)

Subtraction term and 3pt function

- Splitting EDM form factor into two parts:
  \[ F_3 = F_Q + F_\alpha, \quad F_Q = C(m_N)\langle NJ_t^{EM} \bar{N}Q\rangle, \quad F_\alpha = F(\alpha_N, F_{1,2}) \]

- \( F_\alpha \) is good precision, and fluctuation of \( F_Q \) is large.
The sink and source separation in 3pt function enables us to control the statistical noise and excited state contamination.

Comparison

- \( t_{\text{sep}} = 12 \) (blue), \([N_{\text{conf}} = 751]\)
- \( t_{\text{sep}} = 8 \) (green), \([N_{\text{conf}} = 180]\)

- Good consistency between them.
- Precision in \( t_{\text{sep}} = 8 \) is much better.
3. Recent update (preliminary)

$q^2$ dependence

- Fitting data of EDM form factor at each momentum
  - Open ($t_{\text{sep}}=8$) [$N_{\text{conf}} = 751$], filled ($t_{\text{sep}}=12$) [$N_{\text{conf}} = 180$]
- Fitting function
  - 3 point linear:
    - $-q^2 < 0.55 \text{ GeV}^2$
  - 2 point linear:
    - $-q^2 < 0.4 \text{ GeV}^2$
- Estimate of systematic error in extrapolation
- Fitting with BChPT is interesting extension.

$\Rightarrow$ estimate of LECs corresponding to $\pi\text{NN}$ coupling
3. Recent update (preliminary)

Mass dependence

- **Comparison**
  - Statistical error is still dominant rather than systematic one.
  - Central value is 10 times larger than models.
  - $M_{\pi}^2$ dependence is not clear, however its sign is consistent with magnetic moment, $d_N \sim \mu_m m_{\pi}^2 \Delta m$

Abada et al., PLB256(1991), Aoki, Hatsuda PRD45(1992)
3. Recent update (preliminary)

**Statistical error**

- Comparison between AMA error reduction and number of configurations.
- Number of configurations: reduce stat. error and relating to Q distribution
  - AMA error reduction: reduce stat. error

**Error rate**

\[
\text{Error rate} = \frac{\text{Error(full)}}{\text{Error(N)}}
\]

- AMA works well
- Reduction rate when increase of configs. is slightly better.

**Full statistics →**

<table>
<thead>
<tr>
<th>Config</th>
<th>Error Rate</th>
</tr>
</thead>
<tbody>
<tr>
<td>100 config N_G=32</td>
<td>36%</td>
</tr>
<tr>
<td>200 config N_G=32</td>
<td>52%</td>
</tr>
<tr>
<td>400 config N_G=32</td>
<td>73%</td>
</tr>
<tr>
<td>751 config N_G=4</td>
<td>35%</td>
</tr>
<tr>
<td>751 config N_G=8</td>
<td>50%</td>
</tr>
<tr>
<td>700 config N_G=4</td>
<td>40%</td>
</tr>
<tr>
<td>700 config N_G=8</td>
<td>58%</td>
</tr>
</tbody>
</table>

- Full statistics
  - 100 config N_G=32: 33%
  - 200 config N_G=32: 51%
  - 400 config N_G=32: 76%
4. Summary

Summary and future plan

- Nucleon EDM in $N_f = 2+1$ DWF in $\theta$ vacuum
  - Signal of EDM within 40% statistical error using AMA techniques.
  - 3-pt function is still noisy.
  - Short $t_{sep}$ allows us to reduce the statistical error without large excited state contamination effect.
4. Summary

Summary and future plan

- Nucleon EDM in $N_f = 2+1$ DWF in $\theta$ vacuum
  - Signal of EDM within 40% statistical error using AMA techniques.
  - 3-pt function is still noisy.
  - Short $t_{\text{sep}}$ allows us to reduce the statistical error without large excited state contamination effect.

- (Near) physical point of DWF configurations
  - Ensembles near physical points and large volume are available.
  - AMA with Möbius-DWF approximation is helpful.  
  
- Remove chiral extrapolation $\rightarrow$ less than 10% precision

<table>
<thead>
<tr>
<th>Lattice size</th>
<th>Physical size</th>
<th>$a$</th>
<th>$L_s$</th>
<th>Gauge action</th>
<th>Pion mass</th>
</tr>
</thead>
<tbody>
<tr>
<td>$32^3 \times 64$</td>
<td>4.6 fm$^3$</td>
<td>0.135 fm</td>
<td>32</td>
<td>DSDR</td>
<td>171 -- 241 MeV</td>
</tr>
<tr>
<td>$48^3 \times 96$</td>
<td>5.5 fm$^3$</td>
<td>0.115 fm</td>
<td>16</td>
<td>Iwasaki</td>
<td>135 MeV</td>
</tr>
</tbody>
</table>
Thank you for your attention!
Backup
Plan to do extension toward BSM action

Matrix element including BSM operator, quark EDM and chromo EDM (PQ symmetry is assumed)

The q(c)EDM term is CP-violating tensor charge of nucleon, connected diagram should be leading contribution → statistical signal will be clear.

External E field method may be easy way.

qEDM

Tensor charge matrix element + matrix element with qEDM:

\[\partial A_\mu \langle N | d_q (\bar{q} \gamma_5 \sigma \cdot F q) | N \rangle_E\]

\[= \langle N | d_q (\bar{q} \gamma_5 q^\nu \sigma_{\mu\nu} q) | N \rangle + \langle N | J_\mu d_q (\bar{q} \gamma_5 \sigma \cdot F q) | N \rangle\]

Quark chromo EDM

Matrix element with chromo EDM term:

\[\partial A_\mu \langle N | d_{cq} (\bar{q} \gamma_5 \sigma \cdot G q) | N \rangle_E = \langle N | J_\mu d_{cq} (\bar{q} \gamma_5 \sigma \cdot G q) | N \rangle\]
Examples of CAA

- **Lowmode averaging (LMA)**
  - Using lowlying eigenmode of Dirac operator to approximate propagator:
    \[
    \mathcal{O}^{(\text{appx})} = \sum_{\lambda} \mathcal{O}_\lambda^{\text{low}}
    \]
    where $N_\lambda$ is number of lowmode computed by Lanczos.
    Except for computational cost of eigenmode, $\text{Cost}(\text{LMA}) \approx 0$, but approximation is only lowmode part (long distance contribution).

- **All-mode averaging (AMA)**
  - Using sloppy CG (loose stopping condition),
    \[
    \mathcal{O}^{(\text{appx})} = \mathcal{O}^{\text{sloppy}}
    \]
    If stopping cond. is 0.003, $\text{Cost}(\text{AMA}) \approx \text{Cost}(\text{CG})/50$ (without deflation).
    Approximation becomes better than LMA for other than lowmode dominated observables (nucleon, finite momentum hadron, ...).
Volume effect?

- BChPT analysis

\[ \text{CP violating coupling} \]

In LO, NLO BChPT analysis, there may be more than 20% finite size effect.
3. Recent update (preliminary)

**Error reduction techniques**

- **Covariant approximation averaging (CAA)**
  - For original correlator $O$, (unbiased) improved estimator is defined as
    \[ O^{(\text{imp})} = O^{(\text{rest})} + \frac{1}{N_G} \sum_{g \in G} O^{(\text{appx})}_g, \quad O^{(\text{rest})} = O - O^{(\text{appx})} \]
  - $<O> = <O^{(\text{imp})}>$ if approximation has **covariance under lattice symmetry $g$**
  - Improved error $\text{err}^{\text{imp}} \simeq \text{err} / \sqrt{N_G}$
  - Computational cost of $O^{(\text{imp})}$ is cheap.

- **All-mode-averaging (AMA)**
  - Relaxed CG solution for approximation
    \[ O^{(\text{appx})} = O[S_l], \quad S_l = \sum_{\lambda=1}^{N_\lambda} v_{\lambda} v_{\lambda}^\dagger + P_n(\lambda)|_{|\lambda|>N_\lambda} \]
  - $P_n(\lambda)$ is polynomial approximation of $1/\lambda$
    - Low mode part : # of eigen mode
    - Mid-high mode : degree of poly.

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Blum, Izubuchi, ES, 1208.4349, ES (lattice 2012), Chulwoo, plenary on Fri
EM form factor
3. Recent update (preliminary)

**Comparison with $\mu = t, z$**

- EDM form factor is given from two directions of EM current
- Two signals are consistent, and data in $t$ direction is much stable.