Constraining neutron star equation of state using thermonuclear bursts

Juri Poutanen (University of Turku, Finland)

Collaborators:
Valery Suleimanov (Univ. Tübingen, Germany),
Joonas Nättilä (Univ. Turku, Finland)
Andrew Steiner (Univ. Tennessee & Oak Ridge National Lab, USA)
Jari Kajava, Erik Kuulkers (ESAC, Spain)
Duncan Galloway (Monach Univ., Australia)
Plan

• Relation between equation of state of cold dense matter and neutron star parameters

• Astrophysical measurements constraining neutron star mass-radius (M-R)

• Neutron star atmosphere models

• X-ray bursts: dependence on the accretion state

• Constraining neutron star M-R and EoS
The main mystery:
(A) Composition of the core
   Models:
   1. Nucleon matter
   2. Nucleons and hyperons
   3. Pion condensates
   4. Kaon condensates
   5. Quarks (u, d, s)
(B) The pressure of dense matter
   (A)+(B)=The problem of equation of state (EOS)
EoS vs neutron star M-R

Lattimer 2012
Zoo of equations of state

Prohibited by General Relativity

Every EoS gives a different M-R dependence. M and R should be determined from observations.
Modern constraints on EoS and NS mass-radius relation from $M=2M_\odot$.

In order to constrain the EoS, neutron star radii are needed.
Astrophysical measurements of NS radii

- Thermal spectra from lonely NS
- Cooling NS after accretion disc outbursts in transient sources
- Cooling of NSs after X-ray (thermonuclear) bursts
- Oscillations
Neutron star mass-radius relation using blackbody radius at “infinity”

Fitting the bursts spectra with the blackbody we get the temperature $T_{bb}$ and normalization $K$:

$$F_{bol} = \sigma_{SB} T_{bb}^4 K, \quad K = \frac{R_{bb}^2}{D^2}$$

If the distance is known, we can determine apparent radius, which is related to $R$ and $M$ of the neutron star:

$$R_{bb} = R_{\infty} = R(1 + z) = R(1 - R_S / R)^{-1/2}$$

Fig. 4.3. Mass-radius relation for three hypothetical values of the blackbody radius $R_{\infty}$ (5, 10, and 15 km). For clarity, we have not indicated error regions resulting from the uncertainties in the measurements. The straight lines indicate radii $R_*$, equal to the Schwarzschild radius $R_S$, 1.5 $R_S$, and 2.4 $R_S$ (in the text we use $R_S$ instead of $R_*$). The latter could, for example, result from an analysis of a burst with radius expansion (see text), or from the determination of the gravitational redshift of an observed spectral feature. For a given mass, the observed blackbody radius, $R_{\infty}$, has a minimum value $(1.5 \sqrt{3}) R_S$; conversely, for a given blackbody radius $R_{\infty}$ the mass cannot be larger than $R_{\infty}$ (km)/7.7 $M_\odot$.
Comparison of the theoretical X-ray burst spectrum (blue curve) with the black body (red) of the same effective temperature.
Neutron star mass-radius relation using blackbody radius at “infinity”

\[ F = \sigma \, T_{bb}^4 \left( \frac{R_{bb}}{D} \right)^2 = \sigma \, T_{\text{eff},\infty}^4 \left( \frac{R_\infty}{D} \right)^2 \]

\[ f_c = T_{bb} / T_{\text{eff},\infty} \]

\[ K = (R_{bb}/D)^2 \]

\[ R_\infty = R_{bb} \, f_c^2 = D_{10} \sqrt{K} \, f_c^2 \]

\[ D_{10} = D / 10 \text{kpc} \]
X-ray bursts

1. Discovered in the middle of 1970s (e.g. Grindlay et al. 1976).
2. Last for 10-1000 s. Sometimes reach Eddington limit.
3. Originate from accreting neutron stars in low-mass binary systems (LMXBs). About 70 known.
4. Thermonuclear unstable burning of H and He (and maybe C) accreted from the companion in the surface layers of neutron stars.
Rossi X-ray Timing Explorer

Operated for 16 years: from 30 Dec, 1995 to 3 Jan, 2012

Main instrument: Proportional Counter Array, 2.5-60 keV

Observed >2000 X-ray bursts
Photospheric Radius
Expansion X-ray bursts

Flux, $F$ \( \left( 10^{-7} \text{ erg s}^{-1} \text{ cm}^{-2} \right) \)

Temperature $T_{bb}$ (keV)

Normalization $K(\text{km/10 kpc})^2$
Photospheric radius expansion (PRE) bursts

PRE bursts provide two observables.

In the “touchdown method” it is assumed that the Eddington flux is reached during “touchdown” (lowest $K$, highest $T_{bb}$).

In addition to the $K$ at the cooling tail, one needs the color-correction to get the apparent radius at infinity. Often it is assumed that $f_c=1.4$. 
\[ F_{\text{Edd}} = \frac{L_{\text{Edd}}}{4\pi D^2} = \frac{GMc}{D^2 \kappa_e (1 + z)} \]

\[ R_\infty = R_{bb} f_c^2 = D_{10} \sqrt{K} f_c^2 \]

**Distance-independent measure**

\[ T_{\text{Edd,}\infty} = \left( \frac{gc}{\sigma_{\text{SB}} \kappa_e} \right)^{1/4} \frac{1}{1 + z} = 6.4 \times 10^9 AF_{\text{Edd}}^{1/4} \]

\[ A = (R_\infty/D_{10})^{-1/2} = K^{-1/4}/f_c \]
Problems with “touchdown method”

Relation between touchdown flux and Eddington flux is not clear. Measurements of the Eddington flux and the apparent area in the tail are decoupled. Not clear whether they are consistent with each other.
What bursts can be used?

We have to be sure that spectral evolution during the cooling tail follows theoretical predictions for a passively cooling atmosphere.
Plane parallel atmosphere model of the burning layer

Emergent radiation

Atmosphere
The emergent spectrum forms here

Radiation diffusion

Thermonuclear burning

NS crust
Atmosphere models

\[ \frac{dP_g}{dm} = g - g_{\text{rad}}, \quad dm = -\rho \, ds, \]  
Hydrostatic equilibrium

\[ \mu \frac{dI(x, \mu)}{d\tau(x, \mu)} = I(x, \mu) - S(x, \mu), \]  
Radiative transfer

\[ \sigma(x, \mu) = \kappa_e \frac{1}{x} \int_{0}^{\infty} x_1 \, dx_1 \int_{-1}^{1} d\mu_1 \, R(x_1, \mu_1; x, \mu) \left( 1 + \frac{C \, I(x_1, \mu_1)}{x_1^3} \right), \]  
Electron opacity

\[ \int_{0}^{\infty} dx \int_{-1}^{+1} \left[ \sigma(x, \mu) + k(x) \right] [I(x, \mu) - S(x, \mu)] \, d\mu = 0, \]  
Energy balance

\[ P_g = N_{\text{tot}} \, kT, \]  
Ideal gas law
Atmosphere models: emerging spectrum

Suleimanov et al. 2011, 2012; Nättilä et al. 2015
Atmosphere models: emerging spectrum

Usually described well by diluted black body (in range 2.5 - 25.0 keV)

\[ F_E = \frac{1}{f_c^4} B_E(T_c = f_c T_{\text{eff}}) \]
Color-correction factor $f_c$

$$K^{-1/4} = A f_c \left( \frac{F}{F_{\text{Edd}}} \right)$$
Data vs. models

- Models are well described by a simple blackbody (with $T$ correction)

- Observations of the cooling are well described by a simple blackbody

We can simplify and only compare the temperature correction!
Color-correction factor $f_c$

- **Models:**
  \[ F_E = \frac{1}{f_c^4} B(f_c T_{\text{eff}}) \]

- **Observations:**
  \[ F_E = K_{bb} B(T_{bb}) \]

\[ f_c \propto K_{bb}^{-1/4} \]

\[ T_{bb} \propto f_c T_{\text{eff}} \]
The cooling tail method

\[ K = \left( \frac{R_{bb}}{D_{10}} \right)^2 = \frac{1}{f_c^4} \left( \frac{R_\infty}{D_{10}} \right)^2 \]

\[ K^{-1/4} = A f_c \left( \frac{F}{F_{Edd}} \right) \]

\[ A = (R_\infty [\text{km}] / D_{10})^{-1/2} \]

The observed evolution of \( K^{-1/4} \) vs. \( F \) should look similar to the theoretical relation \( f_c \) vs. \( F/F_{\text{Edd}} \)

Two free parameters: \( A \) and \( F_{\text{Edd}} \).
Astrophysical measurements

- The cooling tail method

Neutron star equation of state conference, Montreal 2015:

One of the most promising mass-radius measurement method we have
Photospheric Radius Expansion bursts

- Roughly 2 kinds of bursts
  - Hard state bursts (with low accretion)
  - Soft state bursts (with high accretion)
Bursts from 4U 1608-52 at different accretion rates

@ high persistent flux

@ low persistent flux

used by Guver, Özel

Poutanen et al. (2014)
Why the apparent area is different in different bursts?

Influence of accretion on the burst apparent area and the spectra
Two states of LMXB

Soft/high state - optically thick, cool region

Hard/low state - optically thin, hot region

Barret et al. 2000
Accretion geometry

Hard state  - hot flow / hot optically thin boundary layer

Soft state  - optically thick boundary layer
1. Accretion disk can block nearly $1/2$ of the star.
2. Spreading of matter on NS surface affects the atmosphere structure increasing $f_c$

Inogamov & Sunyaev (1999)  
Suleimanov & Poutanen (2006)

Radiative acceleration/gravitational
radiative/effective

Spectra are nearly diluted blackbodies with color correction

$$f_c = \frac{T_c}{T_{\text{eff}}} = 1.8$$
Can any statistic help us to find correct typical temperature?
Can any statistic help us to find correct typical temperature?
Observations of hard state bursts
Mass and radius constraints from hard state bursts
Parameterized EoS

- Parameters \((a, b, \alpha, \beta)\) of the model are related to nuclear symmetry energy \(S\) and density derivative \(L\):

\[
S \equiv S(n_0) = E(n_0) - E_{\text{nuc}}(n_0) = 16 \text{ MeV} + a + b, \quad (12)
\]

\[
L \equiv 3n_0 \frac{dS(n)}{dn} \bigg|_{n=n_0} = 3(a\alpha + b\beta). \quad (13)
\]
Parameterized EoS

- At high densities we introduce polytropes

\[ P = \epsilon^{1+1/n}, \]

- "Mild" phase transitions

- Or line segments

- "Hard" phase transitions
Parameterized EoS

QMC + Model A

QMC + Model C
Parameterized EoS from the data
Parameterized astrophysical EoS: A probe for nuclear parameters
Conclusions

1. Determining EoS requires measurements not only of the neutron star mass but also of its radius.
2. X-ray (thermonuclear) bursts with photospheric radius expansion are excellent tools to do the job.
3. We have developed detailed atmosphere models to predict the spectral evolution of the X-ray bursts during cooling tails.
4. Spectral evolution of the “hard state” bursts is well described by the theory, while “soft state” bursts are not (and therefore they should not be used for M-R determination).
5. Current burst data (combined with existence of $2M_\odot$ NS) are consistent with the NS radii $11 < R < 13$ km, favoring rather stiff equation of state.
6. There is still some systematic uncertainties related to the data selection (flux intervals), assumption about chemical composition, accounting for rapid rotation, etc.