Universality with long-range Coulomb interactions
- A preliminary status report

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in collaboration with
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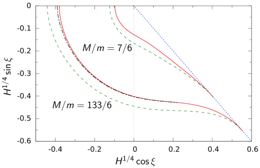
Motivation

- What can we say about a few-body system without knowing much detail about the interaction?
- If interaction is short-range, in first approximation systems can be described by only the scattering length.
- Interesting effects: universal dimer and Efimov trimers if scattering length is large



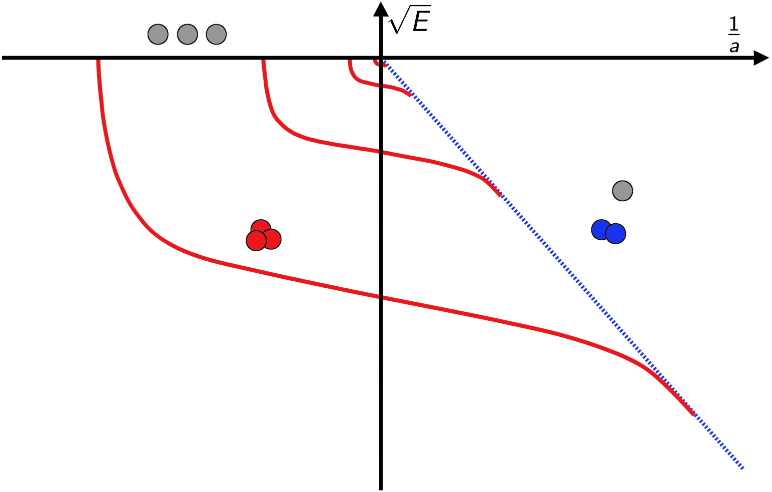
- Explored in many directions: Bosons, Fermions, mass imbalanced systems, N-body systems ...
- Experimental realisation in ultracold atom systems

Motivation

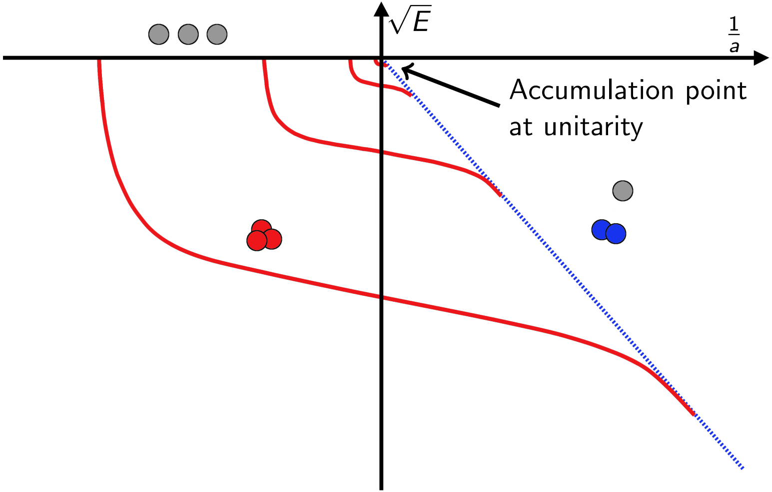
- Efimov predicted effect for nuclei
- Experimentally difficult because scattering length cannot be tuned
- Charge disturbs effects
- What happens to the universal dimer and Efimov trimers if Coulomb potential is added?
- Candidate system: α particles
  - Bosons
  - Large scattering length
  - Experimental data about α clusters available (^{8}Be, Hoyle state..)

Outline

Efimov Effect



Efimov Effect



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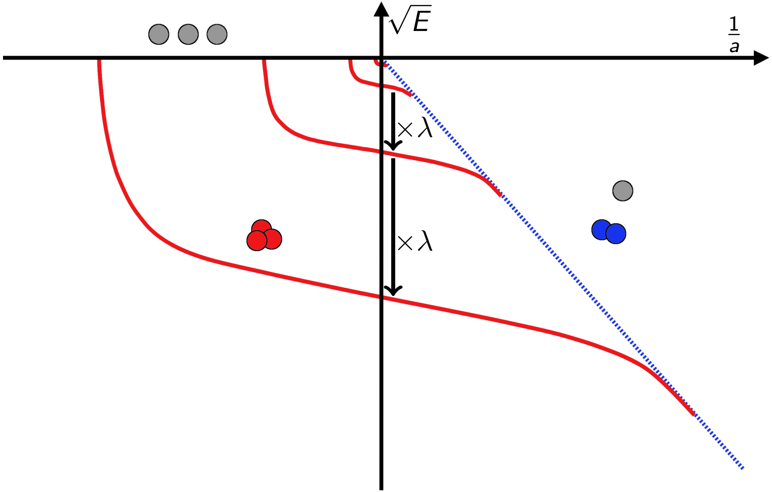

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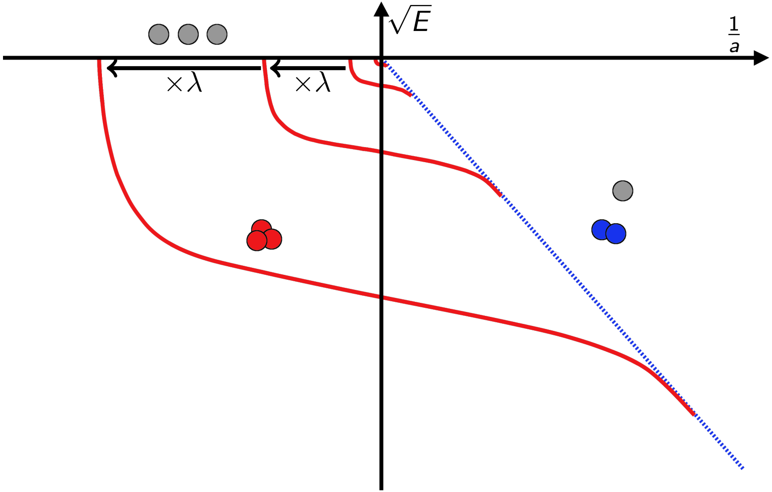


System and Interaction

- N identical bosons (mass m and charge q)
- attractive Gaussian plus repulsive Coulomb potential

$$V_{ij} = \mathbf{V_0} \exp\left(-\frac{r_{ij}^2}{2r_0^2}\right) \qquad V_{ij}^C = \frac{\mathbf{c_c}}{r_{ij}}$$

- Natural length scale: r_0
- Natural energy scale: $E_s = \frac{\hbar^2}{mr_0^2}$
- Natural scale for the strength of the Coulomb potential: $c_s = \frac{\hbar^2}{mr_0}$
- c_c contains q^2

Gaussian Expansion Method

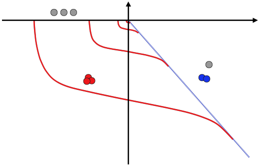
Implementation by Hiyama, Kino, Kamimura *PPNP* 51, 223 (2003)

- Rayleigh-Ritz Variational Method
- Gaussian base functions
- Matrix elements analytic for Gaussian and Coulomb potentials
- Base functions are selected via geometric progression between a minimum and a maximum range
- Fast convergence for a wide range of states
- Find optimized base functions via random sampling

Coulomb-modified scattering length

More Detail in PhD Thesis of S. König (2013)

- Coulomb potential is not short-range
- Scattering length cannot be found by matching inner solution with free waves at zero energy
- Have to use Coulomb Functions as outside solutions
- Cannot calculate for zero energy
- Have to calculate for small energies and extrapolate to zero via fitting



$$C^2_{\eta,0} p \cot \tilde{\delta}_0(p) + \gamma h(\eta) = -\frac{1}{a_C} + \frac{1}{2} r^C_{\mathrm{eff}} p^2 + \ldots$$

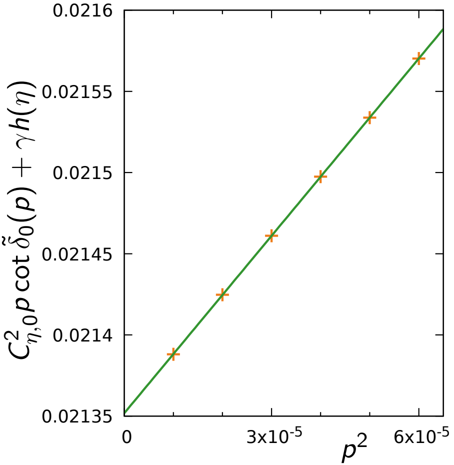
Coulomb-modified scattering length

$$C_{\eta,0}^2 p \cot \tilde{\delta}_0(p) + \gamma h(\eta) = -\frac{1}{a_C} + \frac{1}{2} r_{\text{eff}}^C p^2 + \dots$$

$$\eta = \frac{\gamma}{2p}, \qquad \gamma \propto c_c$$

$$h(\eta) = \text{Re}\,\frac{\Gamma'(i\eta)}{\Gamma(i\eta)} - \log|\eta|$$

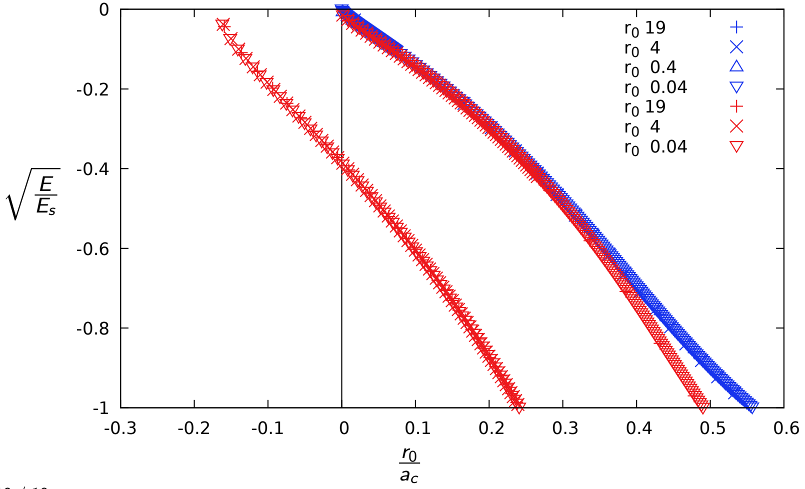
$\tilde{\delta}_0$ is Coulomb-modified scattering phase shift



$\longrightarrow$ Calculate a_C, r_{eff}^C for each triplet of V_0, r_0, c_c

Efimov Plot with Coulomb Interaction

Trimers and Dimer for $c_c/c_s = 0.007$ for different Gaussian ranges r_0



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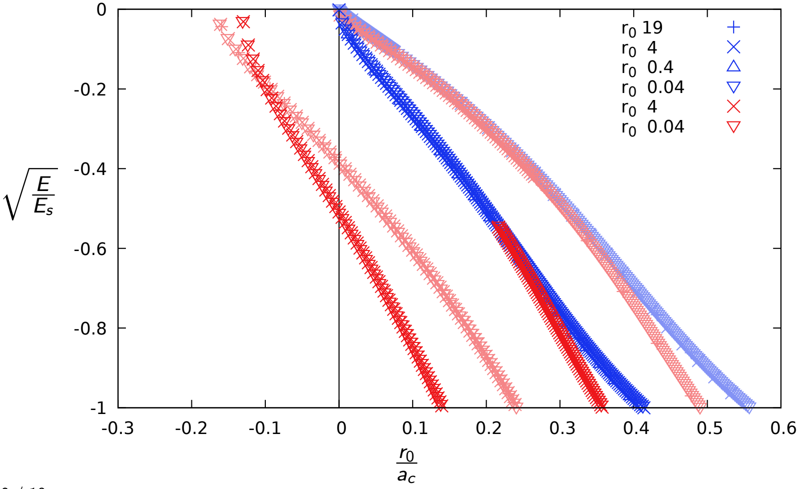
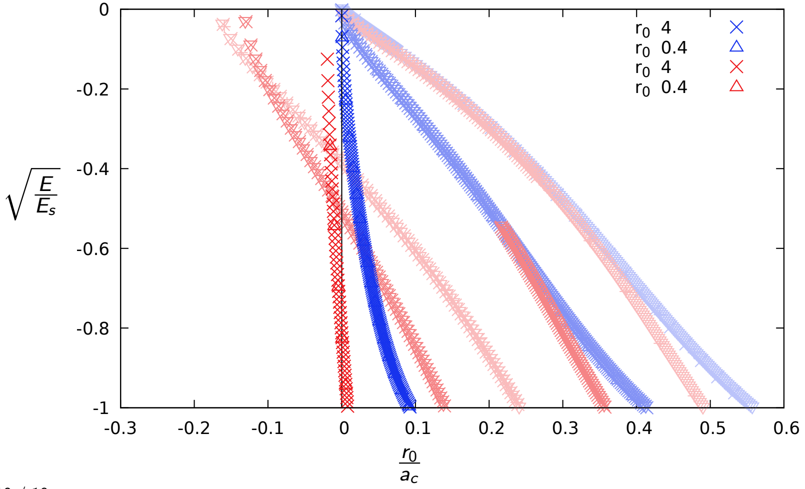


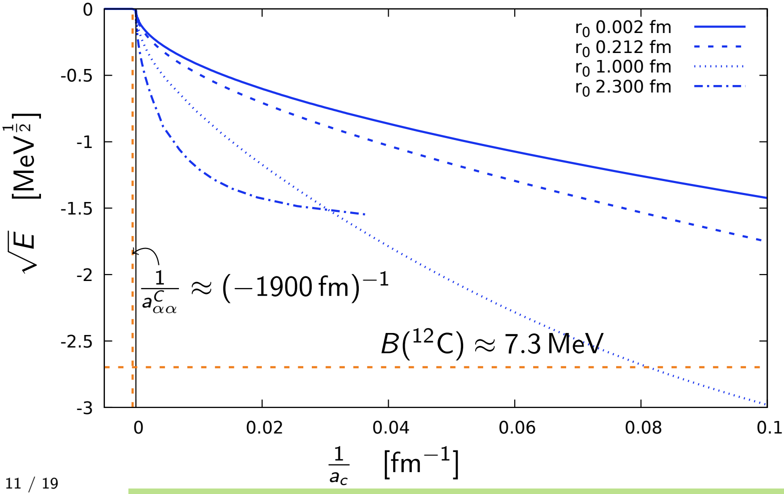
Efimov Plot with Coulomb Interaction

Trimers and Dimer for $c_c/c_s = 0.7$ for different Gaussian ranges r_0



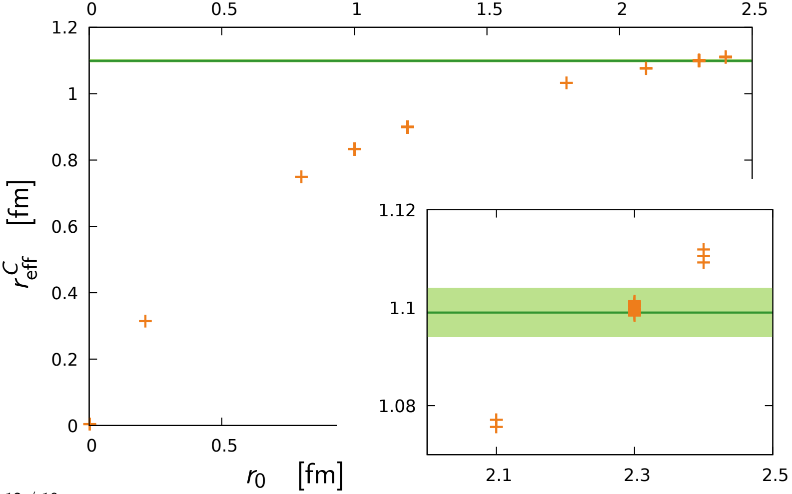
Application to Real System: 3α

$c_c \approx 4 \times 1.44\,\text{MeVfm}$



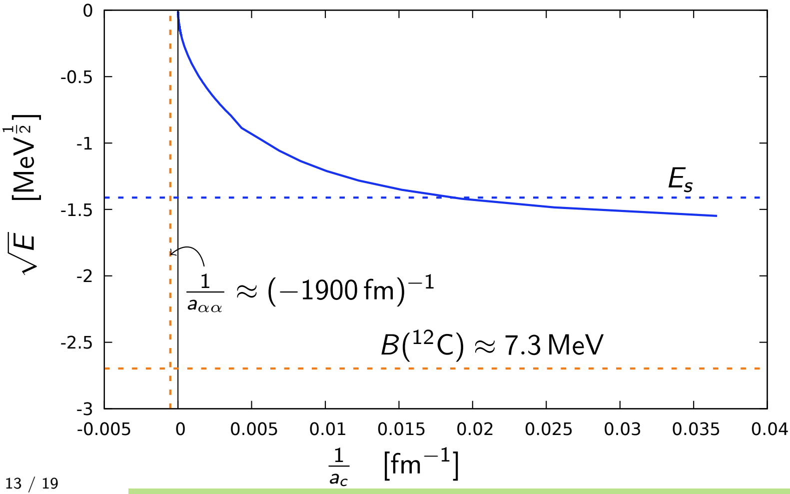
Additionally fix r_{eff}^C for $a_{\alpha\alpha}^C = (-1920 \pm 90)\,\text{fm}$

Values for $a_{\alpha\alpha}^C$ and r_{eff}^C from Higa, Hammer, v. Kolck. *NPA* 809, 171 (2008)



"Efimov Plot" (only Dimer) with $r_0 = 2.3\,\mathrm{fm}$

The ground state of ^{12}C cannot be described!



Summary of Part I

- In natural units the dimer and trimers are universal even with Coulomb.
  $\rightarrow$ What about tetramer? Under investigation.
- However, for a real system the effective range needs to be fixed as well.
- For the 3α system this leads to the ground state of ^{12}C being out of range of the underlying EFT
- Can we still do something useful with this?
  $\rightarrow$ Look at other systems
  $\rightarrow$ Look at resonances

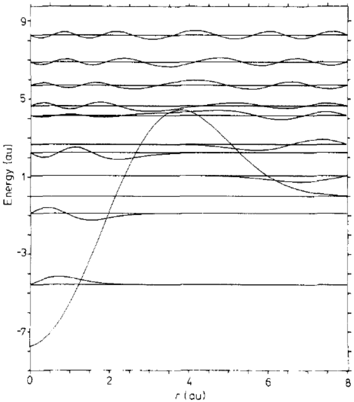
Resonances

Toy model from Maier, Cederbaum, Domcke. JP B 13, 119 (1980)

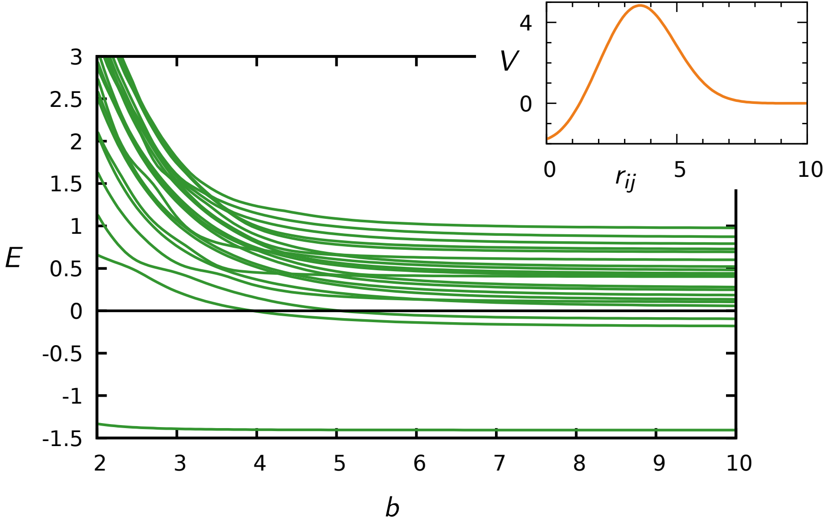
- Stabilisation method
- Put system in a harmonic trap
- Vary size b of the trap
- Resonances appear as avoided crossings
- Toy model:

$$V_{\text{shift}}(r) = Ae^{-\alpha(r-r_{\text{shift}})^2} - Be^{-\beta r^2}$$

- Able to reproduce values from MCD (1980), but only two-body



Resonances for the three-body system



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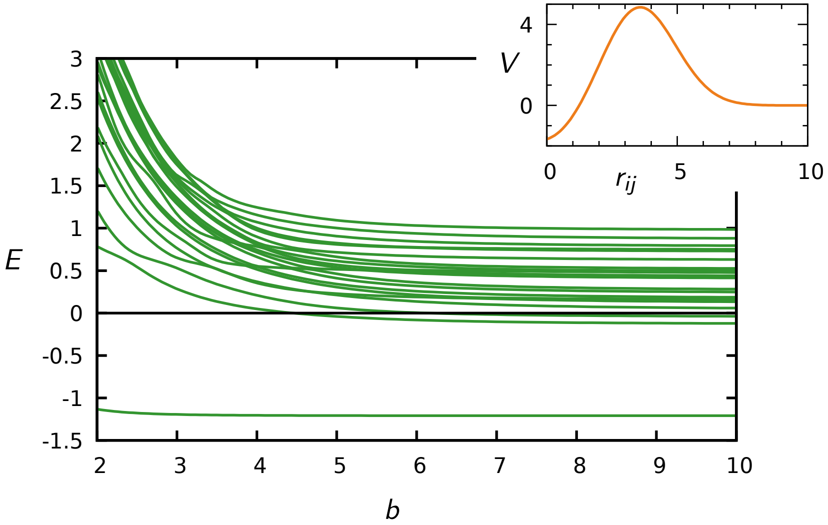

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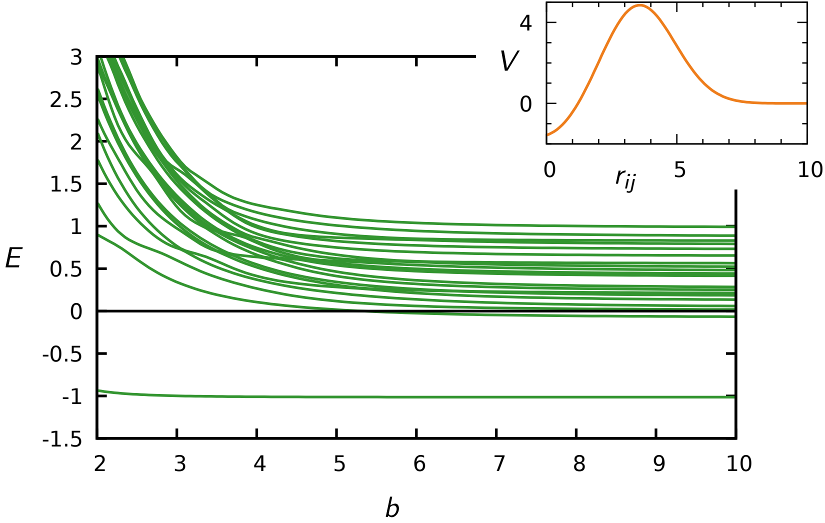

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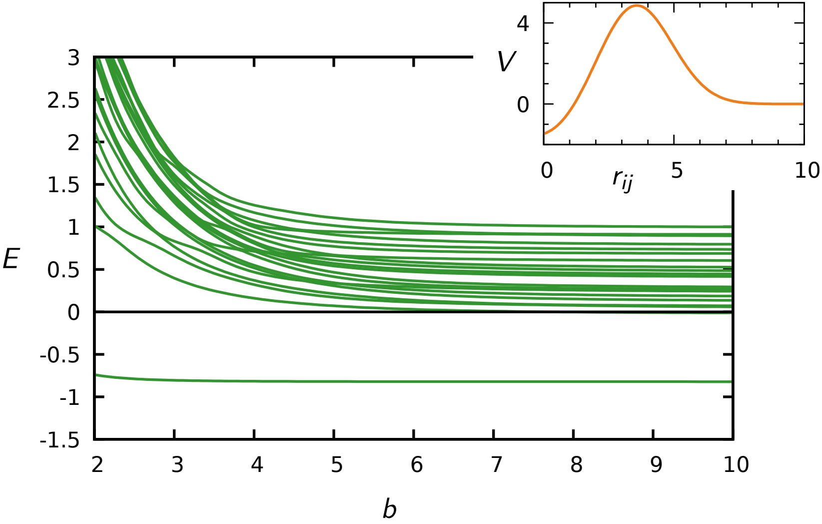

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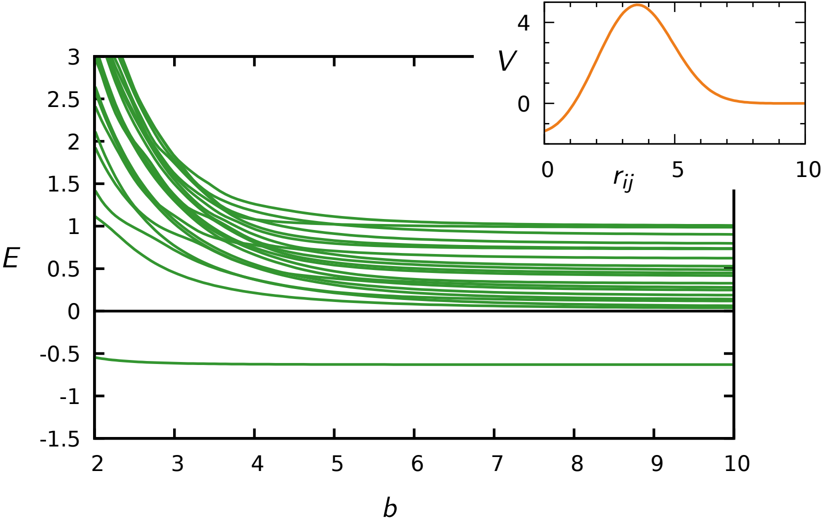

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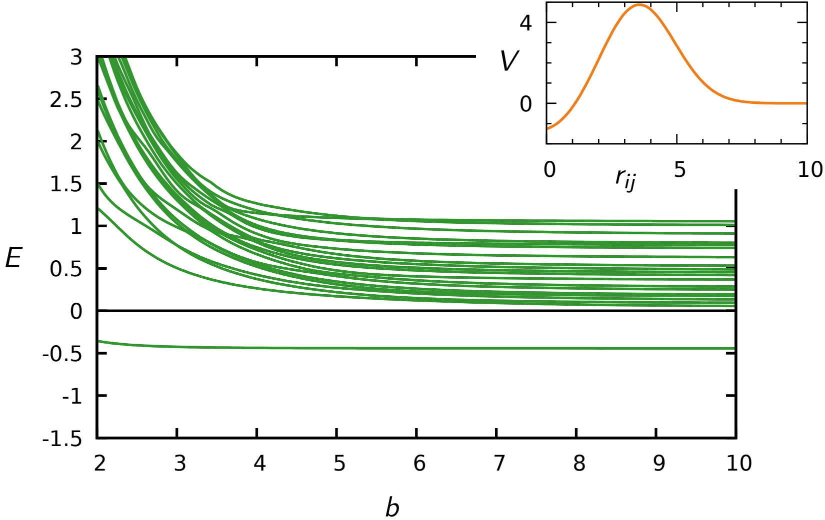

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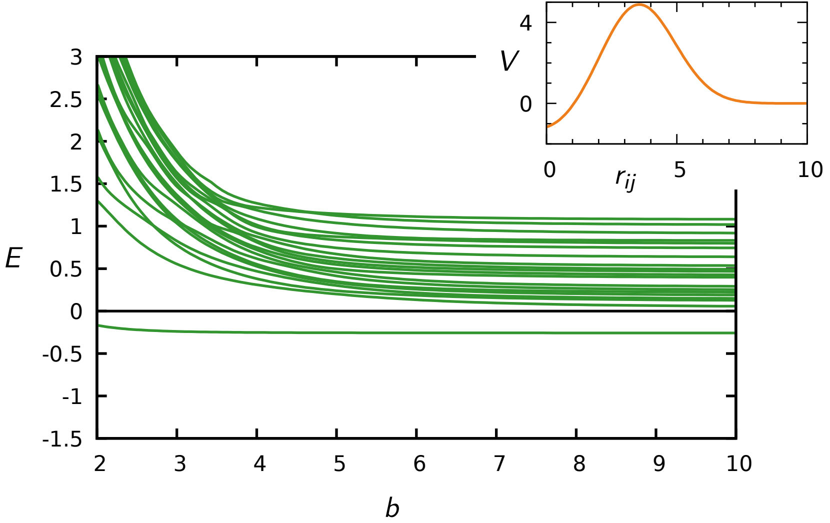

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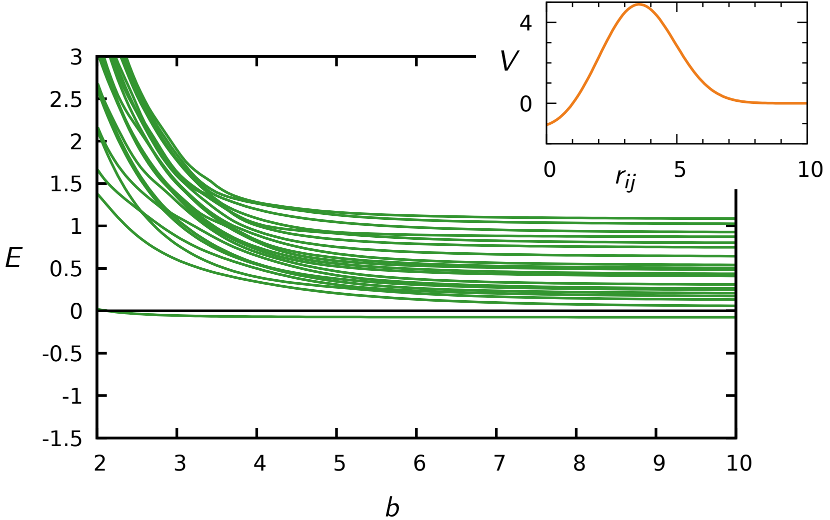

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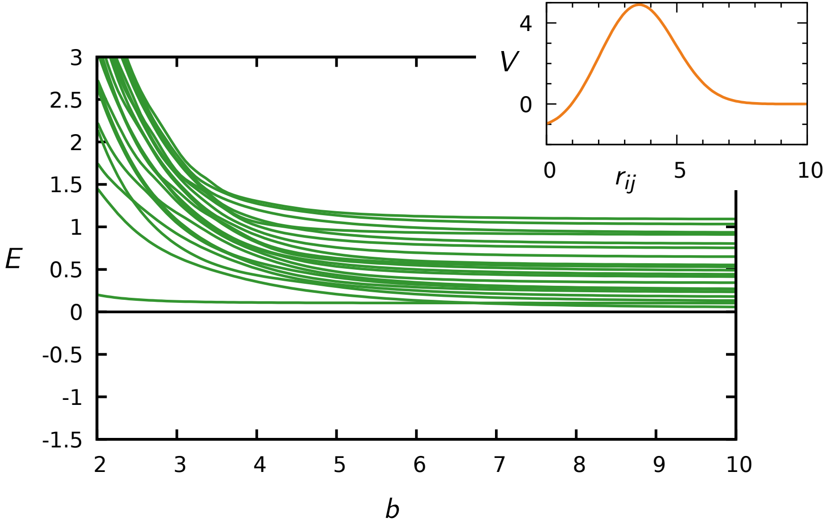

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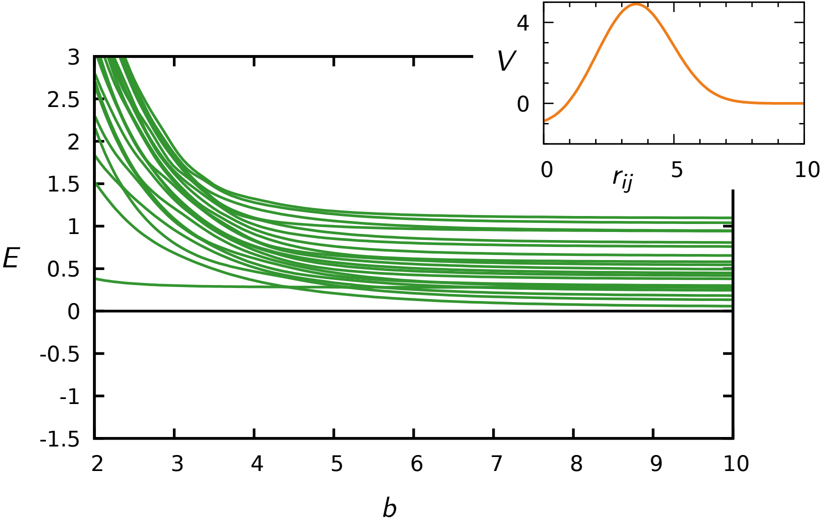

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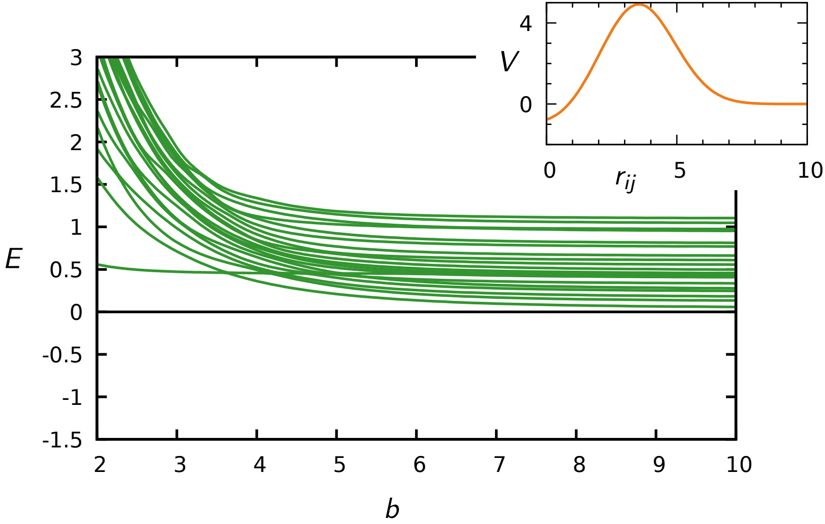

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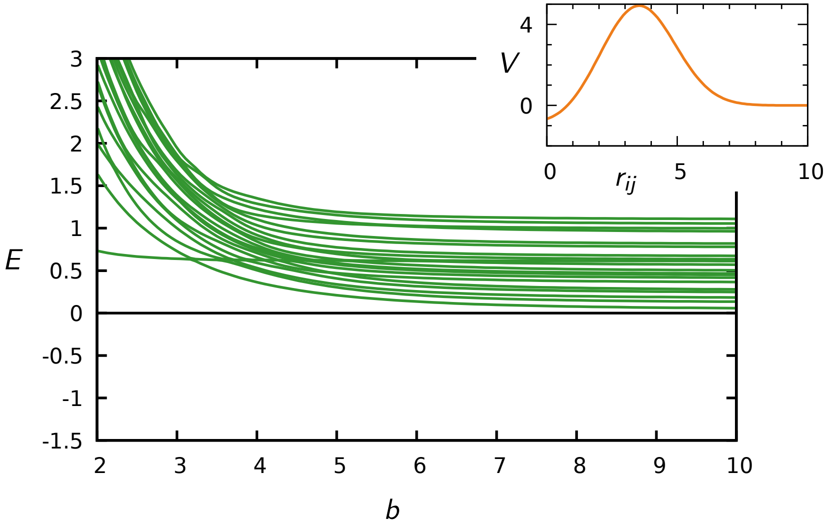

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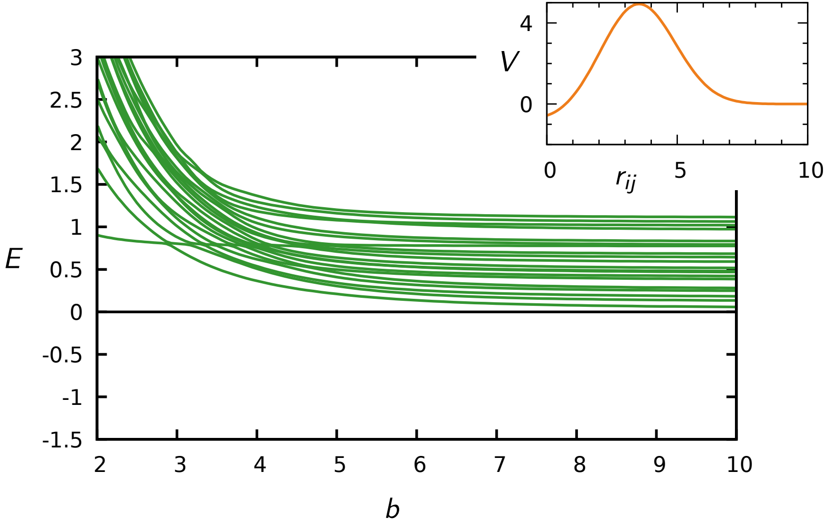

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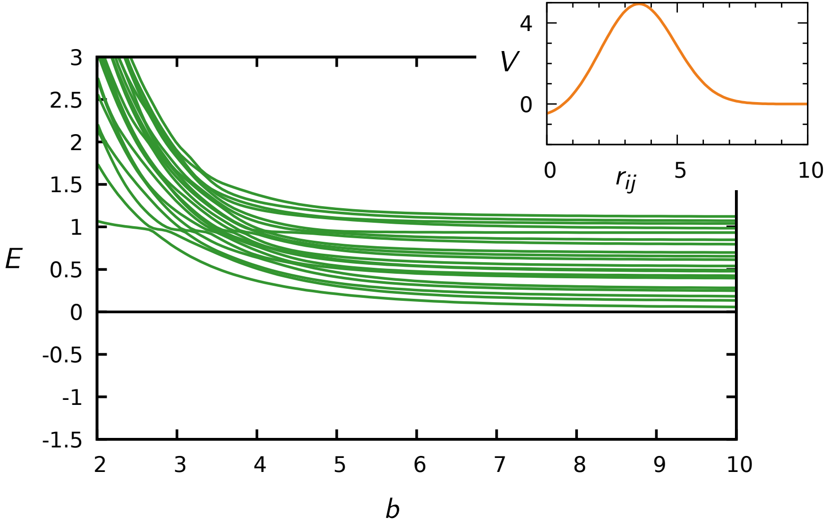

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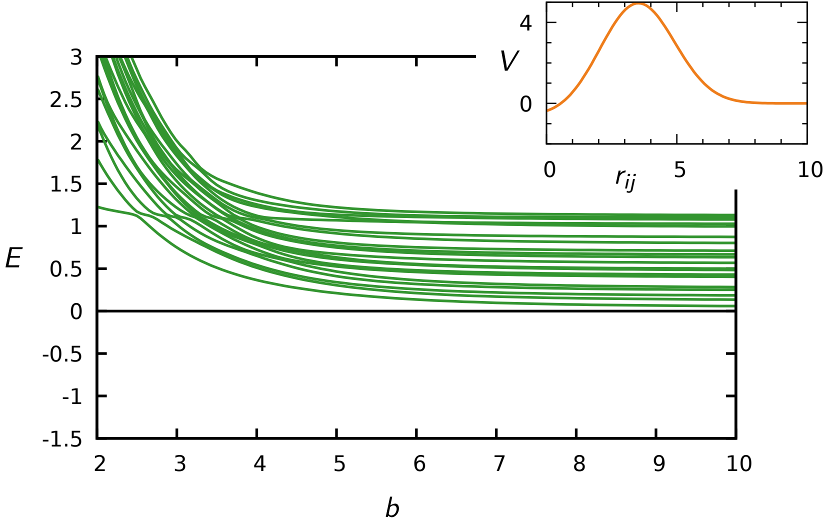

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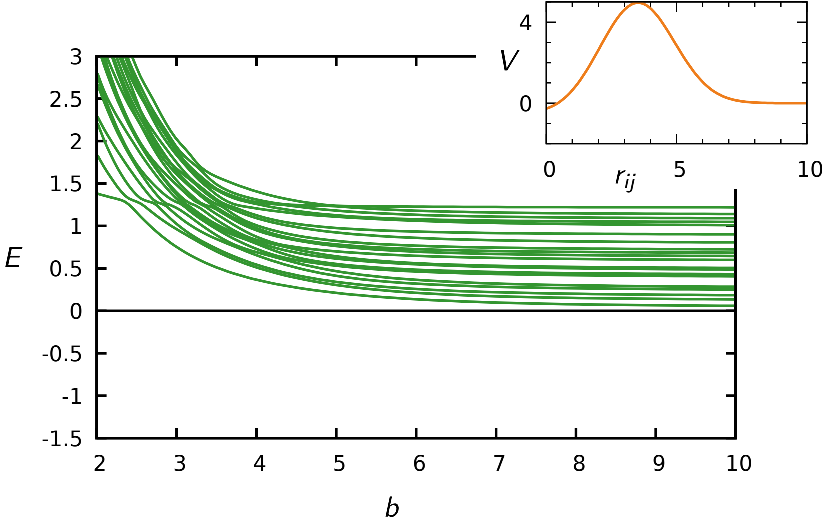

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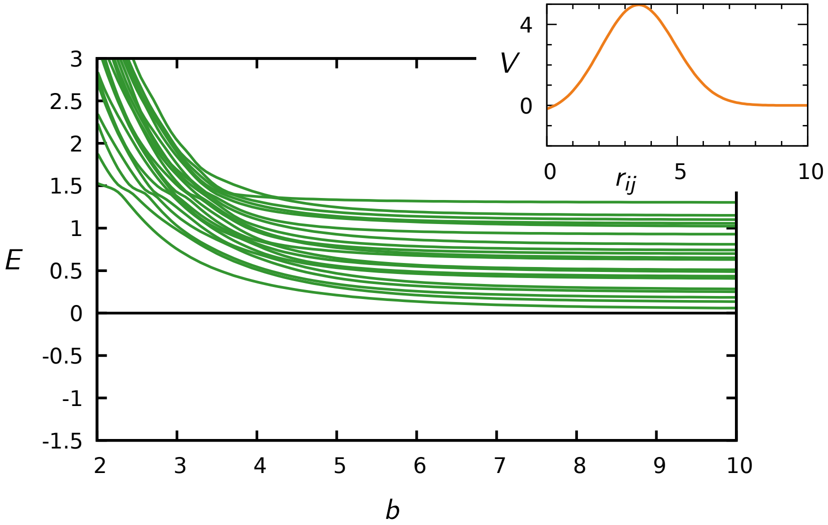

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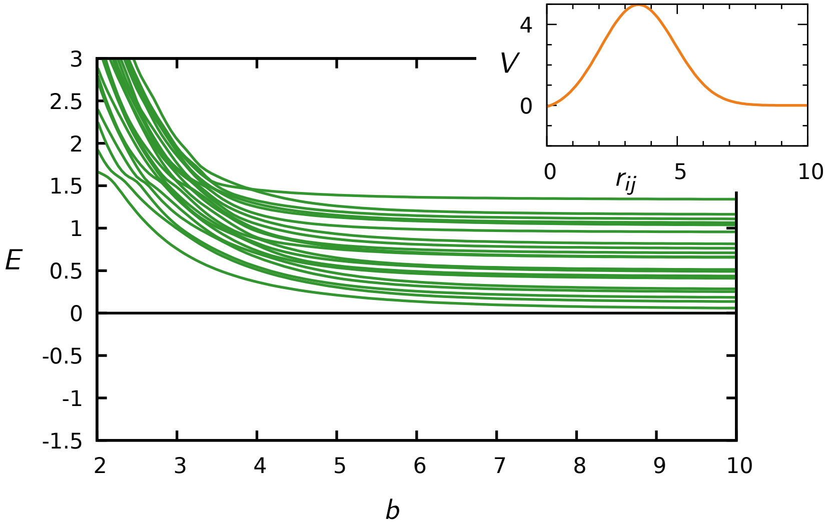

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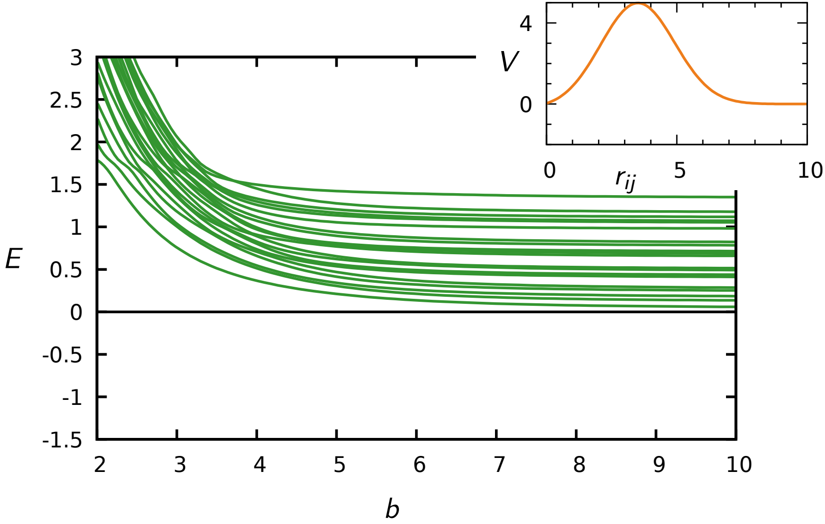

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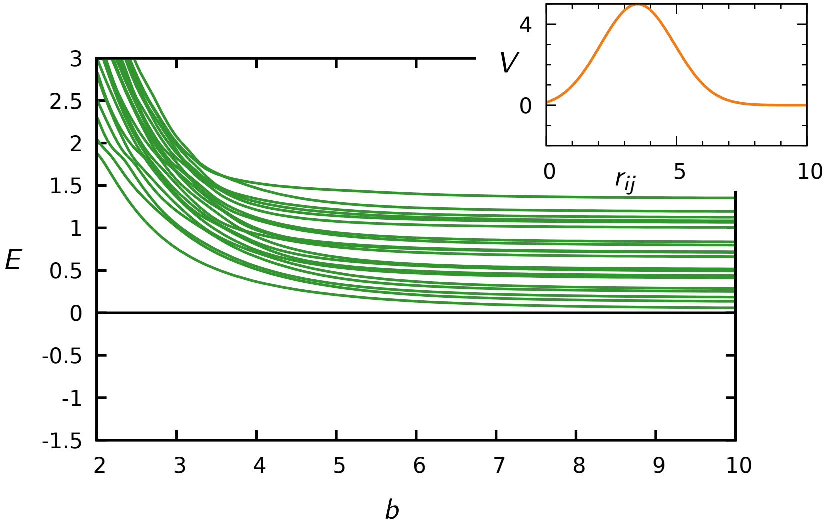

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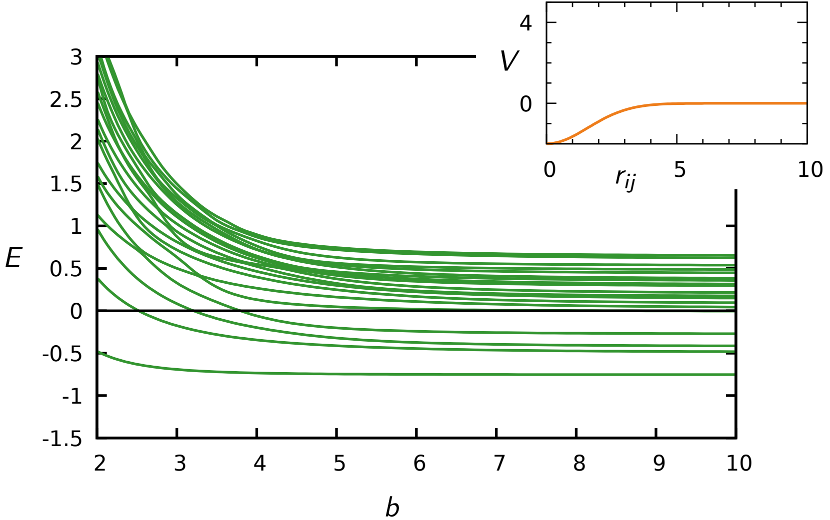

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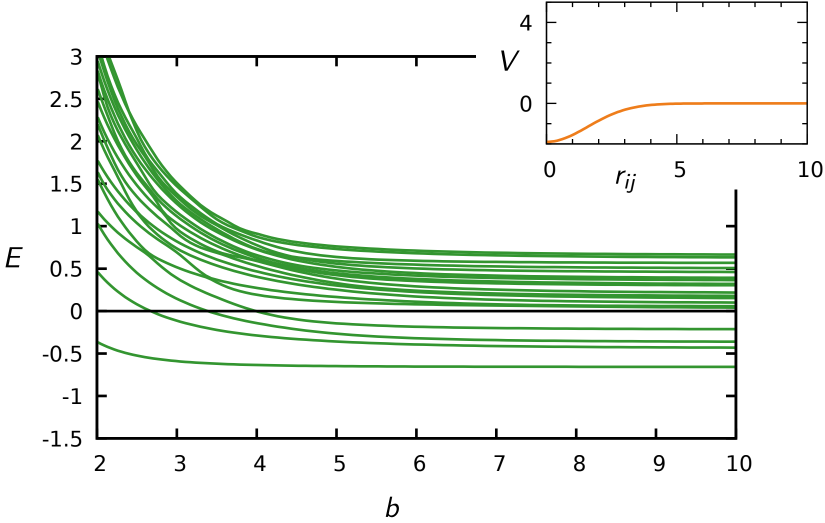

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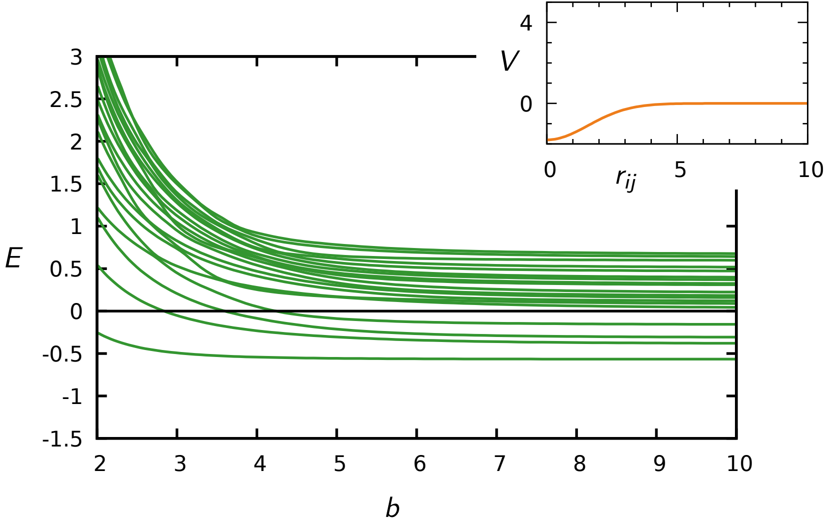

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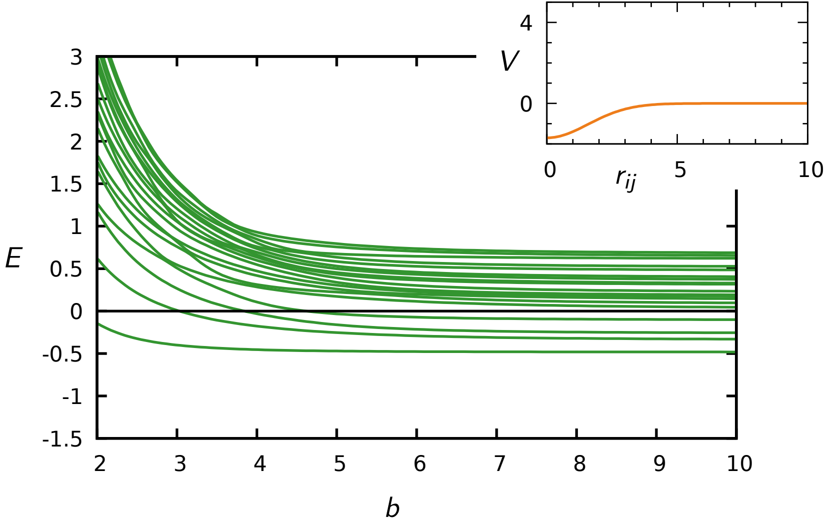

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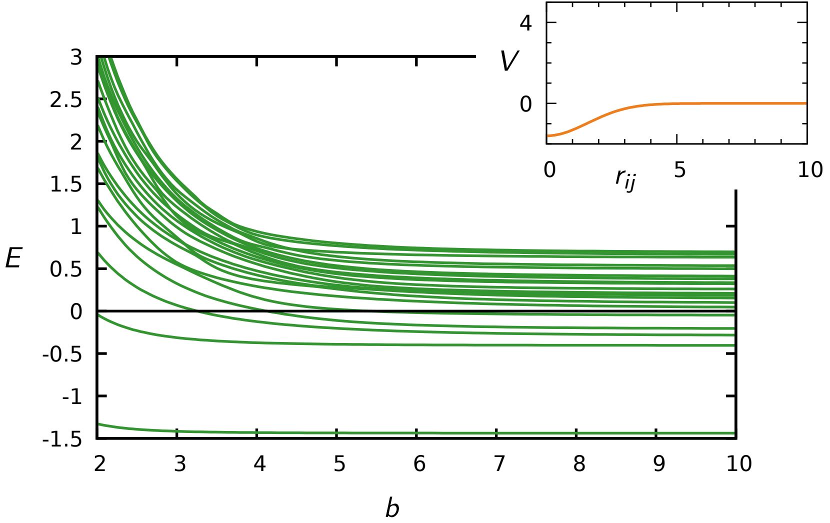

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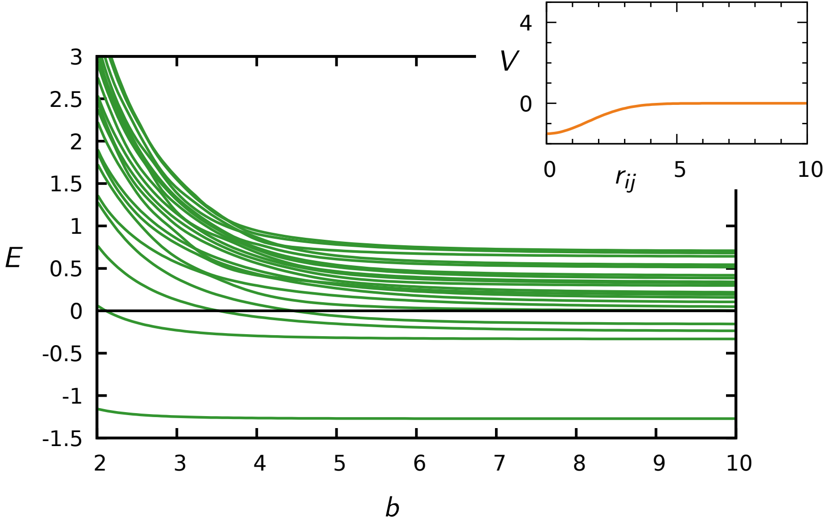

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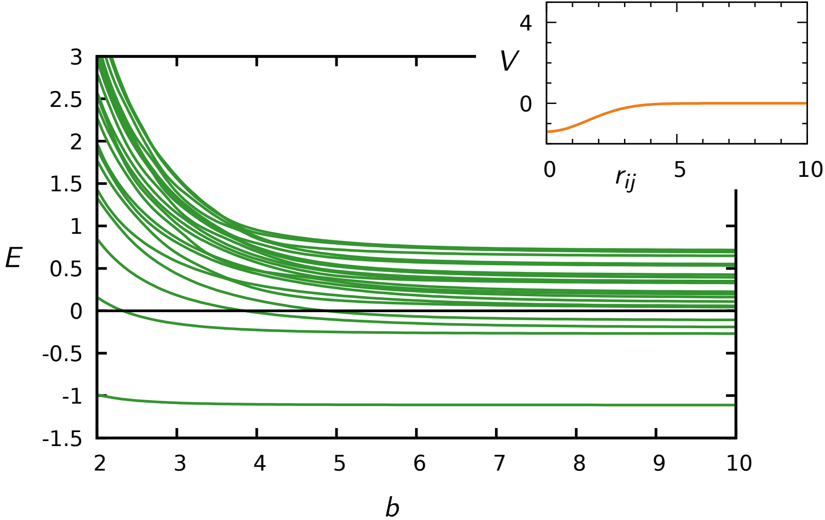

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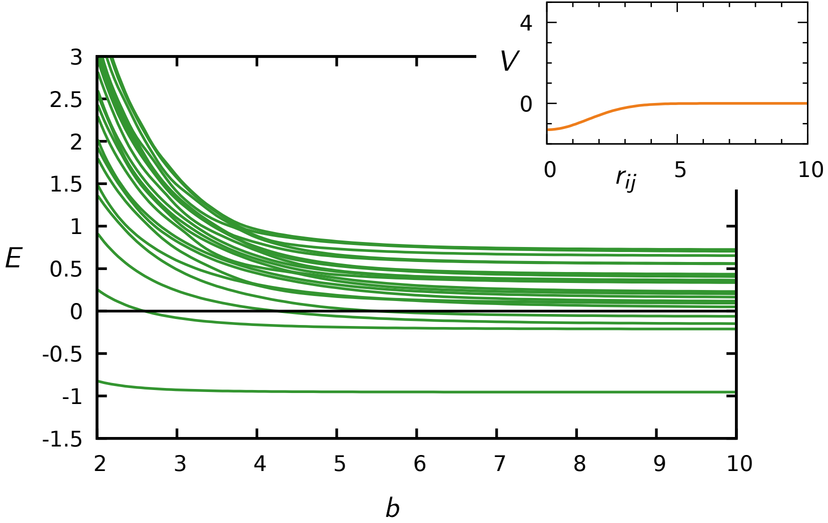

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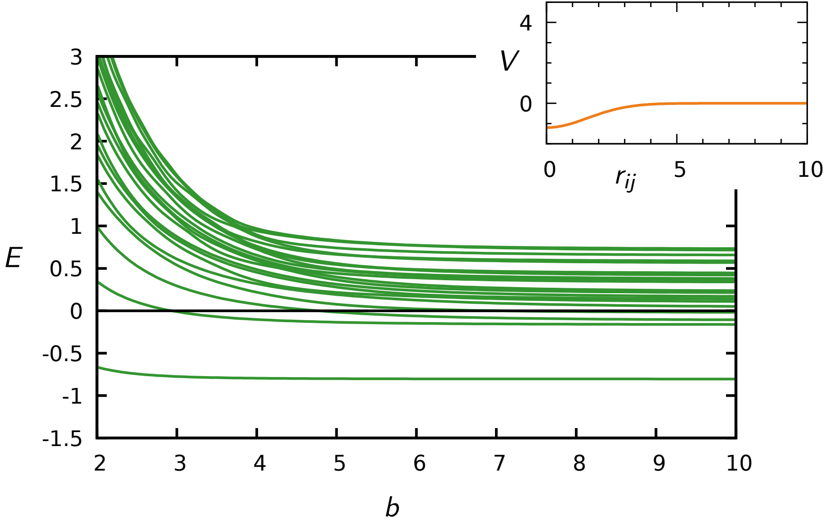

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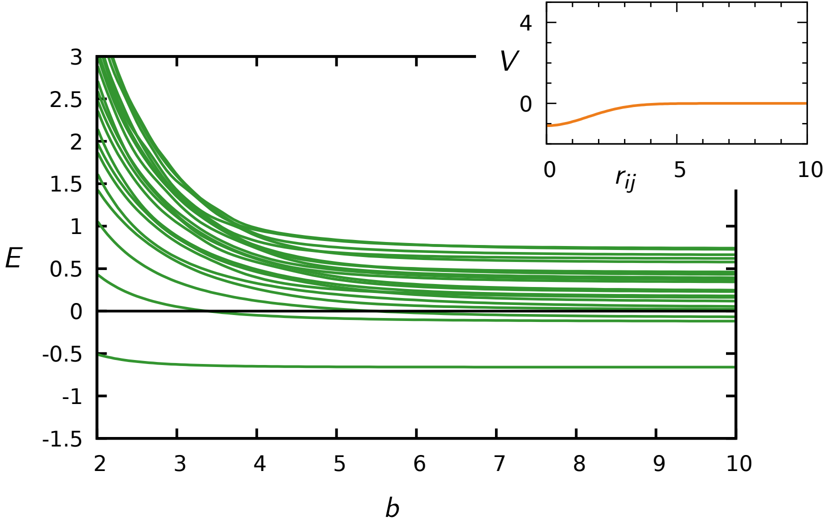

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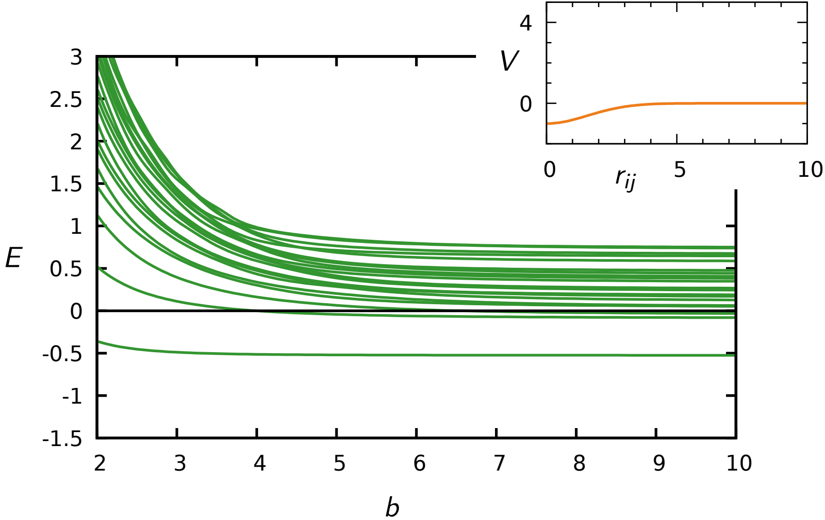

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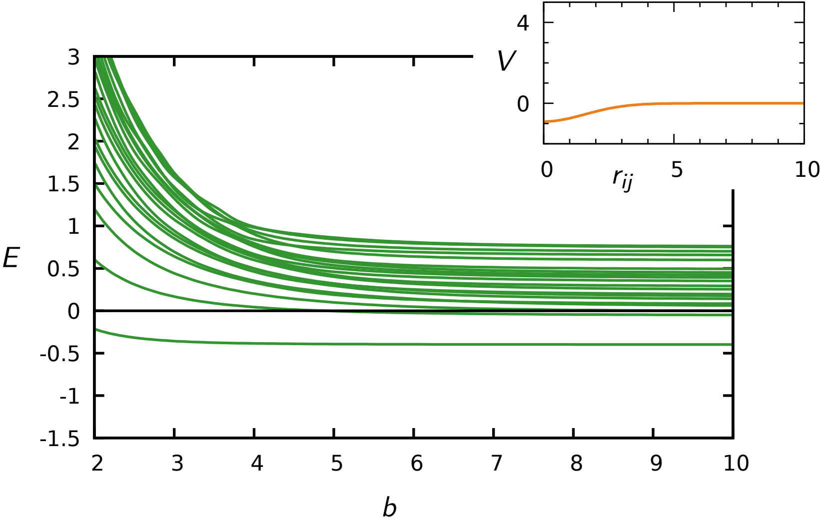

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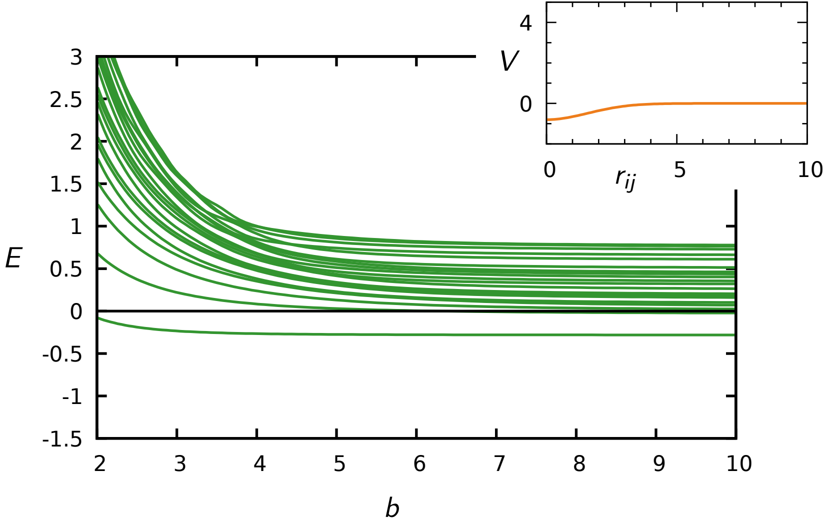

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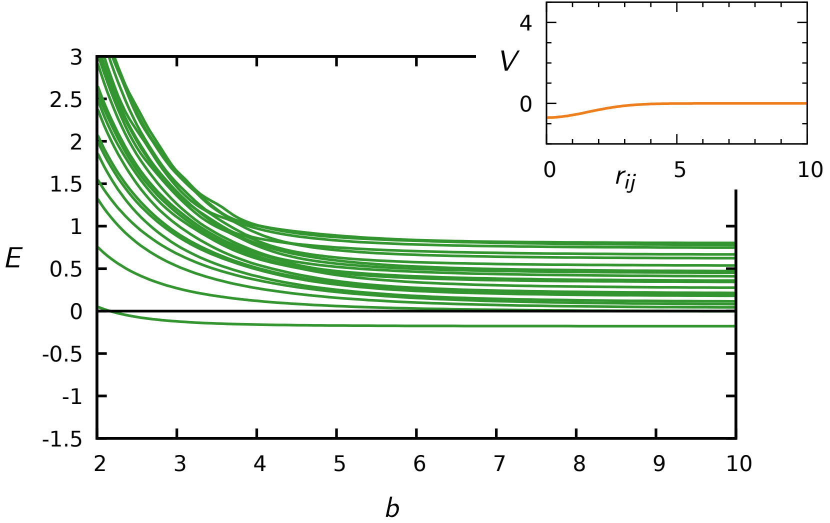

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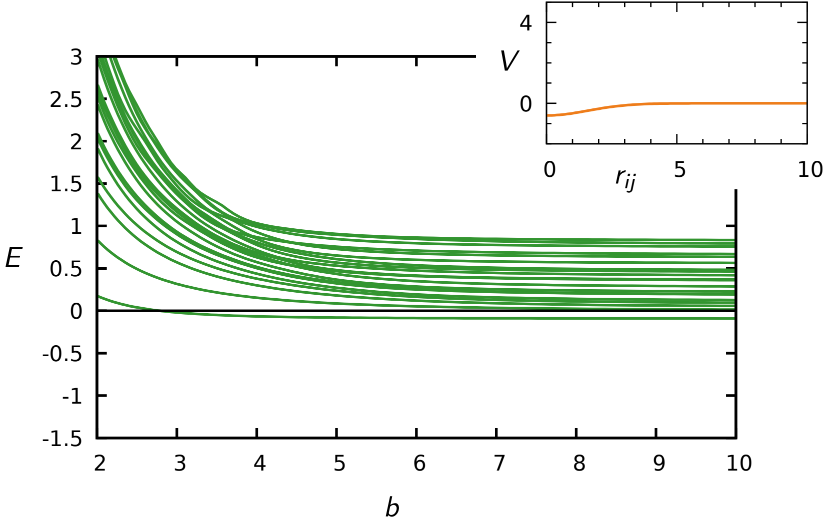

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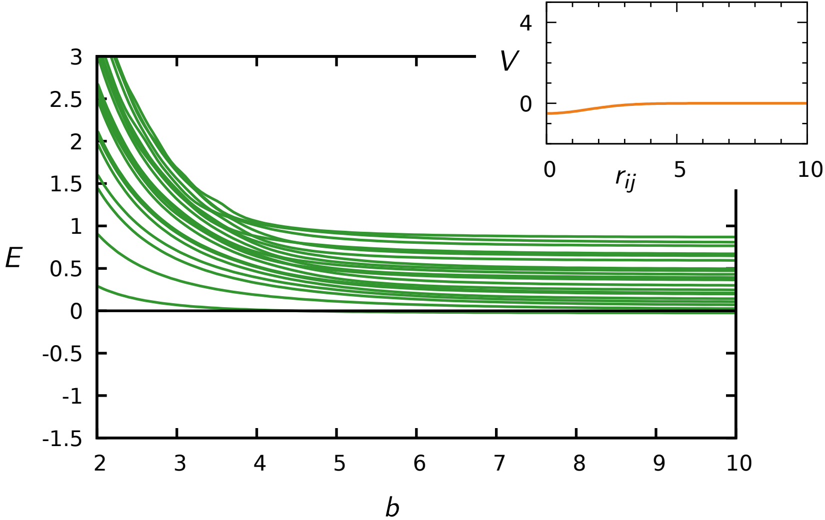

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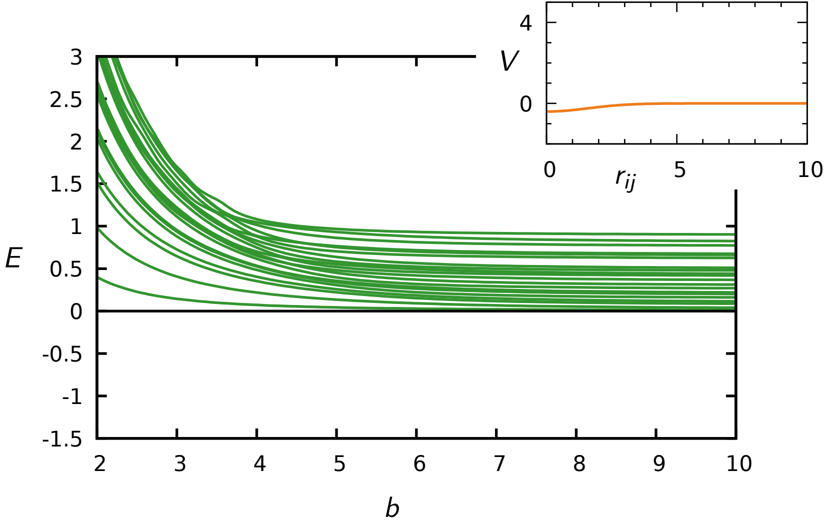

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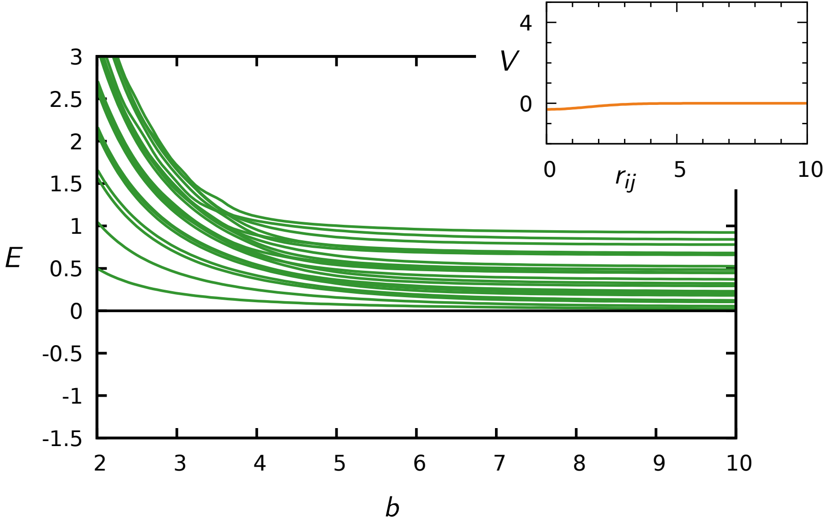

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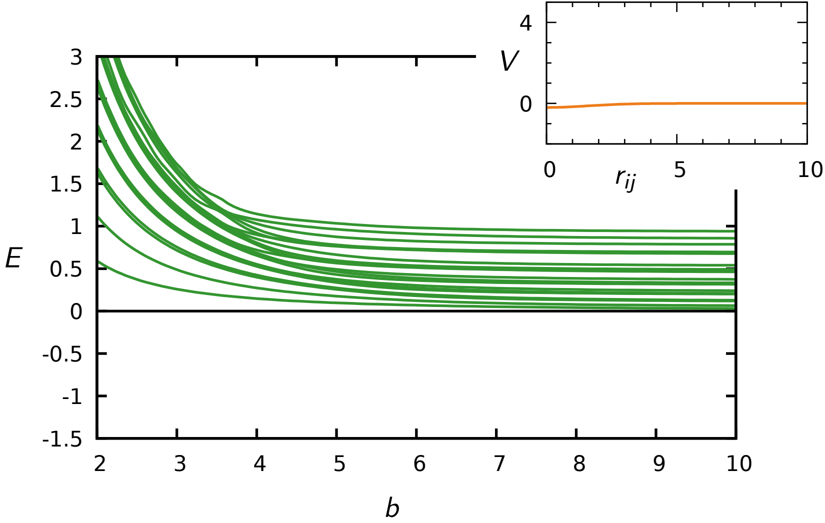

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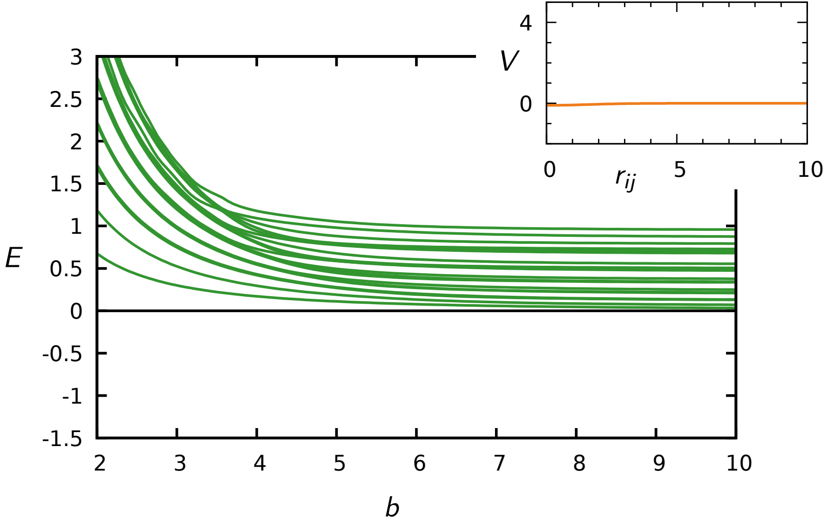

Outlook

- Three- and four-body calculations for the bound state sector
- Resonances with Coulomb barrier
- Investigate ^{8}Be, the Hoyle state and ^{16}O
- Look for other systems with charged bosons (e.g. ions)

Thank you for your attention!