The Overarching Questions
- How did visible matter come into being and how does it evolve?
- How does subatomic matter organize itself and what phenomena emerge?
- Are the fundamental interactions that are basic to the structure of matter fully understood?
- How can the knowledge and technological progress provided by nuclear physics best be used to benefit society?
  - NRC Decadal Study

The Time Scale
- Protons and neutrons formed $10^{-6}$ to 1 second after Big Bang (13.7 billion years ago)
- H, D, He, Li, Be, B formed 3-20 minutes after Big Bang
- Other elements born over the next 13.7 billion years

Topical Collaboration on Neutrinos and Fundamental Symmetries

ππ

$\beta$ decay

allowed $\beta\beta$

neutrinoless $\beta\beta$
Effective Field Theory (pionless, pionful, deltaful,...)

Ab-initio many-body theory: structure & reactions

Computational Science: Fully exploit disruptive technologies

Applied Mathematics: New/improved algorithms

Predictive nuclear theory with quantified uncertainties for practical and fundamental physics
Given a Hamiltonian operator

\[
\hat{H} = \sum_{i<j} \frac{(\vec{p}_i - \vec{p}_j)^2}{2mA} + \sum_{i<j} V_{ij} + \sum_{i<j<k} V_{ijk} + \ldots
\]

solve the eigenvalue problem for wavefunction of \( A \) nucleons

\[
\hat{H} \Psi(r_1, \ldots, r_A) = \lambda \Psi(r_1, \ldots, r_A)
\]

- Expand eigenstates in basis states \( |\Psi\rangle = \sum a_i |\Phi_i\rangle \)
- Diagonalize Hamiltonian matrix \( H_{ij} = \langle \Phi_j | \hat{H} | \Phi_i \rangle \)
- No Core Full Configuration (NCFC) – All \( A \) nucleons treated equally
- Complete basis \( \rightarrow \) exact result
- In practice
  - truncate basis
  - study behavior of observables as function of truncation
**Basis expansion** $\Psi(r_1, \ldots, r_A) = \sum a_i \Phi_i(r_1, \ldots, r_A)$

- Many-Body basis states $\Phi_i(r_1, \ldots, r_A)$ Slater Determinants
- Single-Particle basis states $\phi_\alpha(r_k)$ with $\alpha = (n, l, s, j, m_j)$
- $M$-scheme: Many-Body basis states eigenstates of $\hat{J}_z$

$$\hat{J}_z |\Phi_i\rangle = M |\Phi_i\rangle = \sum_{k=1}^A m_{ik} |\Phi_i\rangle$$

- $N_{\text{max}}$ truncation: Many-Body basis states satisfy

$$\sum_{\alpha \text{ occ.}}^A (2n + l)_\alpha \leq N_0 + N_{\text{max}}$$

- Alternatives:
  - Full Configuration Interaction (single-particle basis truncation)
  - Importance Truncation
    - Roth, PRC79, 064324 (2009)
  - No-Core Monte-Carlo Shell Model
    - Abe *et al*., PRC86, 054301 (2012)
  - SU(3) Truncation
    - Dytrych *et al*., PRL111, 252501 (2013)
Nuclear interaction

Nuclear potential not well-known, though in principle calculable from QCD

\[ \hat{H} = \hat{T}_{\text{rel}} + \sum_{i<j} V_{ij} + \sum_{i<j<k} V_{ijk} + \ldots \]

In practice, alphabet of realistic potentials

- Argonne potentials: AV8', AV18
  - plus Urbana 3NF (UIX)
  - plus Illinois 3NF (IL7)
- Bonn potentials
- Chiral NN interactions
  - plus chiral 3NF, ideally to the same order
- JISP16
- Daejeon16
- ...
Ground state energy of p-shell nuclei with JISP16

Maris, Vary, IJMPE22, 1330016 (2013)

Compare theory and experiment for 24 nuclei

$^{10}$B – most likely JISP16 produces correct $3^+$ ground state, but extrapolation of $1^+$ states not reliable due to mixing of two $1^+$ states

$^{11}$Be – expt. observed parity inversion within error estimates of extrapolation

$^{12}$B and $^{12}$N – unclear whether gs is $1^+$ or $2^+$ (expt. at $E_x = 1$ MeV) with JISP16

**Ground state magnetic moments with JISP16**

Compare theory and experiment for 23 magnetic moments  Maris, Vary, IJMPE22, 1330016 (2013)

\[
\mu = \frac{1}{J + 1} \left( \langle \mathbf{J} \cdot \mathbf{L}_p \rangle + 5.586 \langle \mathbf{J} \cdot \mathbf{S}_p \rangle - 3.826 \langle \mathbf{J} \cdot \mathbf{S}_n \rangle \right) \mu_0
\]

Only 8B needs reduction

Good agreement with data, given that we do not have any meson-exchange currents
GFMC with AV18 + IL7 interaction
15 magnetic moments compared between theory and experiment
Two-body currents tend to enhance the magnetic moments (exc. $^8$B)

Figure from Rocco Schiavilla
=> Apply extrapolation method:

\[ B(E2; (3^+), 0^-) \text{ (e}^2\text{fm}^4) \]

\[ h\Omega \text{ (MeV)} \]

**6Li**

\[ N_{\text{max}} = 8 \]

**Exp. 10.7**

\[ 10.2 \pm 0.8 \]

\[ 9.8 \pm 0.7 \]

Effective Hamiltonian in the NCSM
Okubo-Lee-Suzuki (OLS) renormalization scheme

- \( n \)-body cluster approximation, \( 2 \leq n \leq A \)
- \( H^{(n)}_{\text{eff}} \) \( n \)-body operator
- Two ways of convergence:
  - For \( P \rightarrow 1 \) \( H^{(n)}_{\text{eff}} \rightarrow H \)
  - For \( n \rightarrow A \) and fixed \( P \): \( H^{(n)}_{\text{eff}} \rightarrow H_{\text{eff}} \)

Adapted from Petr Navratil
Outline of the OLS process

\[ UHU^\dagger = U[T + V]U^\dagger = H_d \]
\[ H_{\text{eff}} = U_{\text{OLS}}HU_{\text{OLS}}^\dagger = PH_{\text{eff}}P = P[T + V_{\text{eff}}]P \]
\[ U^P = PUP \]
\[ \tilde{U}^P = P\tilde{U}^P P = \frac{U^P}{\sqrt{U^P U^P}} \]
\[ H_{\text{eff}} = \tilde{U}^{P\dagger} H_d \tilde{U}^P = \tilde{U}^{P\dagger} UHU^\dagger \tilde{U}^P = P[T + V_{\text{eff}}]P \]
\[ O_{\text{eff}} = \tilde{U}^{P\dagger} UOU^\dagger \tilde{U}^P = P[O_{\text{eff}}]P \]
\[ U_{\text{OLS}} = \tilde{U}^{P\dagger} U \]
Calculation of three-body forces at $N^3$LO

**Goal**

Calculate matrix elements of 3NF in a partial-wave decomposed form which is suitable for different few- and many-body frameworks.

**Challenge**

Due to the large number of matrix elements, the calculation is extremely expensive.

**Strategy**

Develop an efficient code which allows to treat arbitrary local 3N interactions.

(Krebs and Hebeler)
Initial LENPIC Collaboration results: Chiral NN results for $^6$Li by Chiral order

Orange: Chiral order uncertainties; Blue/Green: Many-body method uncertainties


- Ground state energy
  - similar behavior as $^3$H and $^4$He

- Open question:
  - Are chiral uncertainty estimates applicable to excitation energies?

- Need compatible chiral 3NFs
- Need chiral expansion conserved current operator
Ground state energy (MeV)

Experimental data
- LO through $N^2$LO chiral NN potential

S. Binder, et al., LENCIC Collaboration, in preparation
Preliminary LENPIC results with Chiral NN only and $R = 1.0$ fm, IA for operator
S. Binder, et al., LENPIC Collaboration, in preparation

Only $^8$B needs reduction
Consider two nucleons as a model problem with $V = \text{LENPIC}$ chiral NN solved in the harmonic oscillator basis with $\hbar \Omega = 5, 10$ and 20 MeV. Also, consider the role of an added harmonic oscillator quasipotential.

Hamiltonian #1

$H = T + V$

Hamiltonian #2

$H = T + U_{\text{osc}} (\hbar \Omega_{\text{basis}}) + V$

Other observables:
Root mean square radius $R$
Magnetic dipole operator $M1$
Electric dipole operator $E1$
Electric quadrupole moment $Q$
Electric quadrupole transition $E2$
Gamow-Teller $GT$
Neutrinoless double-beta decay $M(0\nu2\beta)$

Dimension of the “full space” is 400 for all results depicted here.
Deuteron gs energy: truncation vs OLS

\[
\text{Fract. Diff.} = \frac{E_{\text{model}} - E_{\text{exact}}}{|E_{\text{exact}}|}
\]

Insets: Semilog plots of high \(N_{\text{max}}\) region

OLS gives exact results for all cases (green lines at Fract. Diff. = 0)

Convergence patterns sensitive to chiral order

Even unbound cases (Fract. Diff. > 1.0) are accurately treated with OLS
Consider a 2-body contribution within EFT to 0νββ-decay at N²LO


\[ M^0 = \langle \Psi_{A,Z+2} | \sum_{i,j} \frac{R}{r_{ij}} \left[ F_1(x_{ij}) \hat{\sigma}_i \hat{\sigma}_j + F_2(x_{ij}) T_{ij} \right] \tau_i^+ \tau_j^+ | \Psi_{A,Z} \rangle \]

\[ F_1(x) = (x - 2)e^{-x}, \quad F_2(x) = (x + 1)e^{-x}, \quad x = m_\pi |\vec{r}| \]

\[ T_{ij} = 3 \hat{\sigma}_i \hat{r}_{ij} \hat{\sigma}_j \hat{r}_{ij} - \hat{\sigma}_i \hat{\sigma}_j \]

Regulator applied to 0νββ-decay operator for consistency with LENPIC interaction

\[ f \left( \frac{r}{R} \right) = \left( 1 - \exp \left( - \frac{r^2}{R^2} \right) \right)^6 \]

\[ R = 1.0 \text{ fm for these results} \]

Additional operators under development – stay tuned
Two nucleons in a Harmonic Oscillator trap with trap $\hbar \Omega = \text{basis } \hbar \Omega$
LENPIC Chiral NN interaction at $N^2LO$ with $R = 1.0$ fm
Comparison of GT and 0n2b-decay matrix elements from truncation with Exact/OLS

Recast Fract. Diff. (FD) results as a Quenching Factor (QF)
$QF = \frac{\text{Exact}}{\text{Model}} = 1 - \frac{(FD \times |\text{Exact}|)}{\text{Model}}$

$(a)$ GT ($V_{NLO}$)

$(b)$ GT ($V_{N2LO}$)

$(c)$ 0ν2β ($V_{NLO}$)

$(d)$ 0ν2β ($V_{N2LO}$)

$^{1}S_0(nn \ gs) \rightarrow ^{3}S_1 - ^{3}D_1\text{ (deuteron gs)}$

$^{1}S_0(nn \ gs) \rightarrow ^{1}S_0(pp \ gs)$
Outlook:

Implement in finite nuclei:

- Perform benchmark A=6 calculations with UNC group (underway)
- Evaluate/save density matrices (static and transition) and use them to evaluate consistent OLS’d or SRG’d observables

Expand treatment to wider range of EW operators within Chiral EFT at NLO & N2LO

Extend to 3-body interactions with OLS or SRG on operators at the 3-body level

Extend to medium weight nuclei with “Double OLS” approach
Partial list of projects underway – keep on your radar screens

Iteratively improved natural orbitals  (with Notre Dame Univ)

Ab initio nuclear reactions
  multiple-scattering with realistic 1-body density matrices (with Ohio Univ)
  non-perturbative time-dependent Coulomb excitation (with IMP-Lanzhou)

Benchmarking neutrinoless double-beta decay (with UNC)

Consistent electroweak operators (LENPIC)
  moments
  transitions
  double-beta decay

Valence effective interactions (with S. Korea, France, Russia)

Artificial Neural Network developments and applications to NCSM (with LBNL)
Collaborators at Iowa State University
Members of NUCLEI and Topical Collaboration Teams

Robert Basili (grad student)
Weijie Du (grad student)
Matthew Lockner (grad student)
Pieter Maris
Soham Pal (grad student)
Shiplu Sarker (grad student)

New faculty position at Iowa State in Nuclear Theory
Supported, in part, by the Fundamental Interactions
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