QUANTUM

THE ANOMALY STRIKES BACK!

WARS

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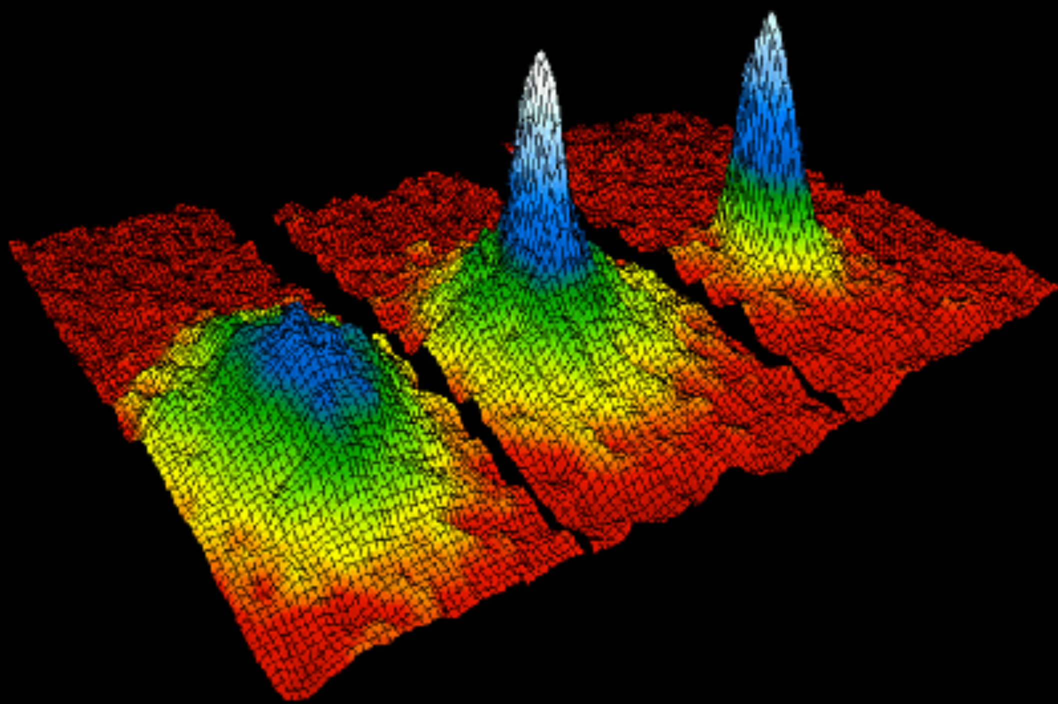
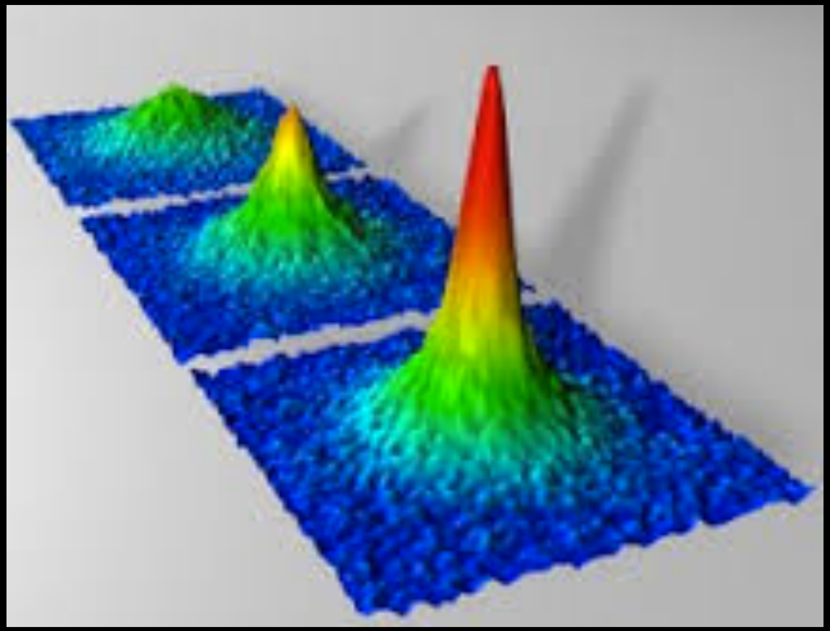
Hirschegg, January 2018

Ultracold atoms



Bose-Einstein
condensates
(1995)



Fermionic condensates
(2004)

Ultracold atoms

Astonishing degree of control…

- **Temperature** (Superfluid transitions)
- Polarization (LOFF-type phases, polarons)
- **Coupling** (BEC-BCS crossover)
- Shape of external trapping potential
- Mass imbalance (different isotopes)
- **Dimension** (highly anisotropic traps & lattices)
- Bosons, fermions, mixtures: Li, K, Sr, Yb, Dy, Er,…

… and astonishing degree of measurement/detection…

- Thermodynamics
- Phase transitions
- Collective modes
- Spin response
- Hydrodynamic response
- Entanglement
- Time-dependent dynamics
- …

3D Fermions: Hamiltonian & scales

Two species of fermions with a contact two-body force

$$\hat{H} = \int d^3x \left[\sum_{s=\uparrow,\downarrow} \hat{\psi}_s^\dagger(\mathbf{x}) \left(-\frac{\hbar^2 \nabla^2}{2m} \right) \hat{\psi}_s(\mathbf{x}) - g \hat{n}_\uparrow(\mathbf{x}) \hat{n}_\downarrow(\mathbf{x}) \right]$$

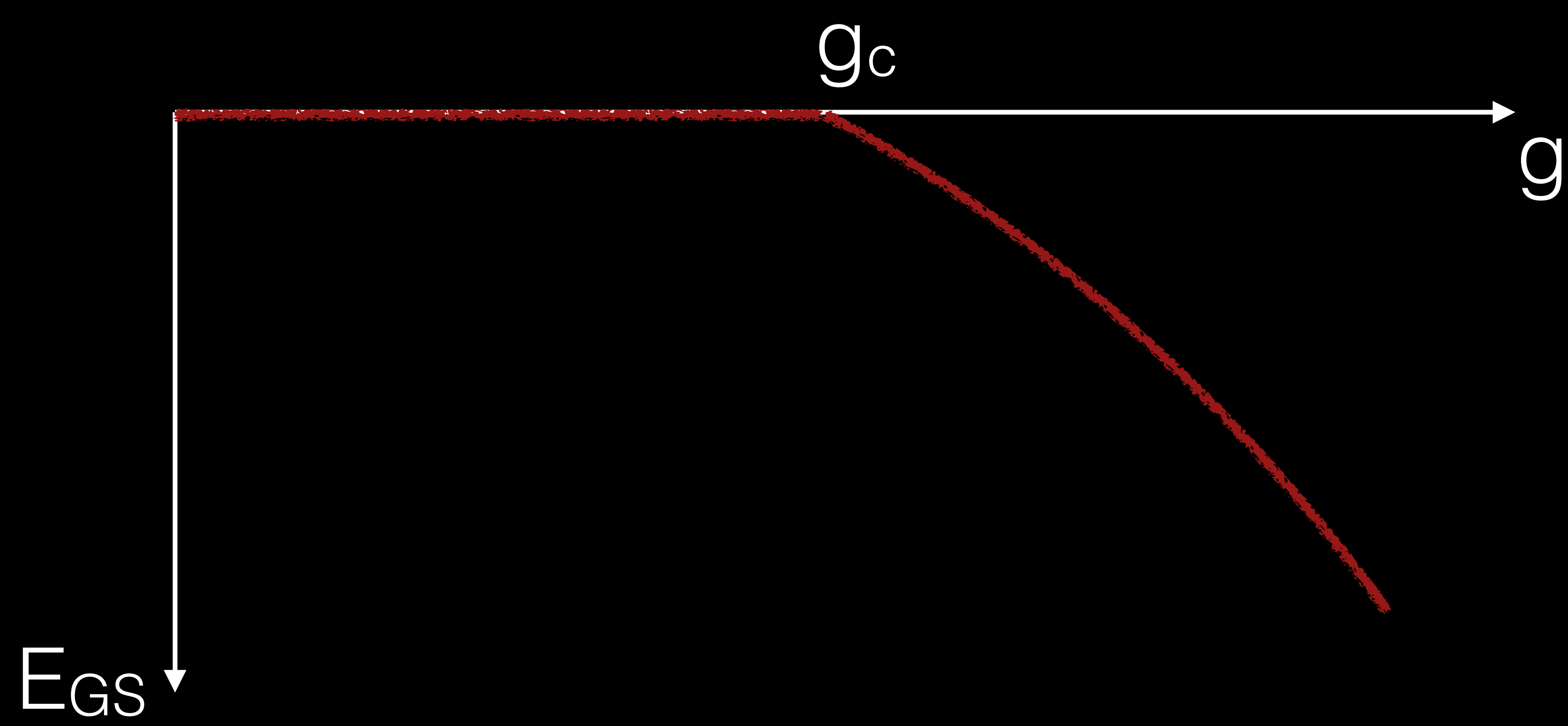
Coupling is dimensionful

$$[g] = L$$

Renormalize by solving the two-body problem and relating bare coupling to scattering length

Two-body problem in 3D

Bound state appears at a critical attractive coupling



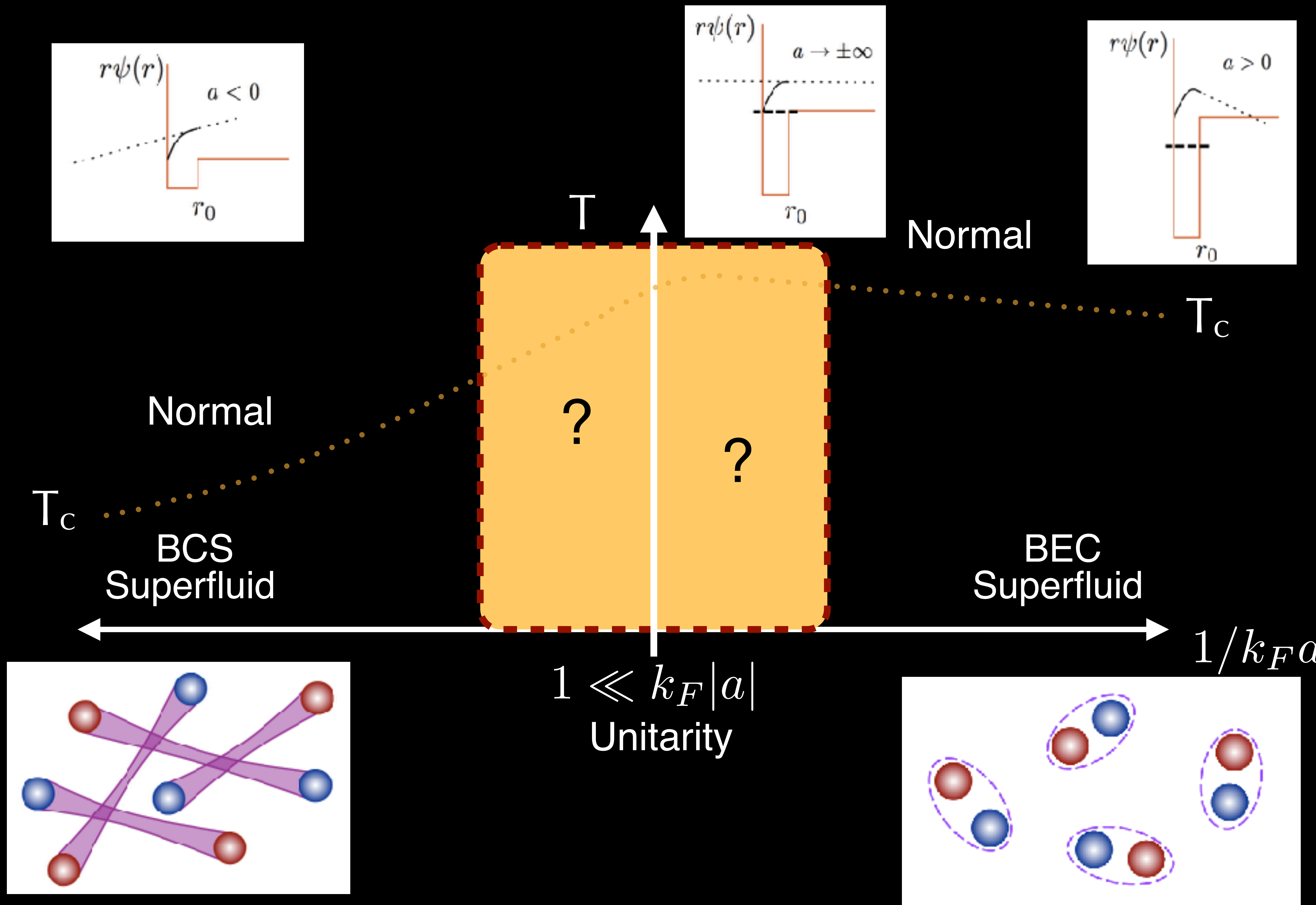
Scattering length and density determine the **physical dimensionless coupling**

$k_F \propto n^{1/3}$

a : scattering length

$$\boxed{1/(k_F a)}$$

The 3D BCS-BEC crossover



Outline

- Introduction (ultracold atoms in **3D**)

- Scale anomalies
    - In nonrelativistic **2D** fermions
        - Selected results

    - “A new hope” in **1D**
        - Exact mappings
        - Thermodynamics
        - Tan’s contact

- Summary and Conclusions

Scale anomalies

Ultracold atoms in 2D

There is work by multiple experimental groups around the world

Heidelberg (Jochim), Hamburg (Moritz), Bonn (Köhl),

Moscow (Turlapov)

Melbourne (Vale)

Toronto (Thywissen)

Cambridge (Zwierlein)

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2D Fermions: Hamiltonian & scales

Two species of fermions with a contact two-body force

$$\hat{H} = \int d^2x \left[\sum_{s=\uparrow,\downarrow} \hat{\psi}_s^\dagger(\mathbf{x}) \left(-\frac{\hbar^2 \nabla^2}{2m} \right) \hat{\psi}_s(\mathbf{x}) - g \hat{n}_\uparrow(\mathbf{x}) \hat{n}_\downarrow(\mathbf{x}) \right]$$

Coupling is dimensionless

$$[g] = 1$$

Classically scale invariant!

Factor out center-of-mass motion and solve "relative" problem:

$$\left[\frac{-\nabla^2}{2\bar{m}} - g\delta(\mathbf{x}) \right] \phi(\mathbf{x}) = E_r \phi(\mathbf{x})$$

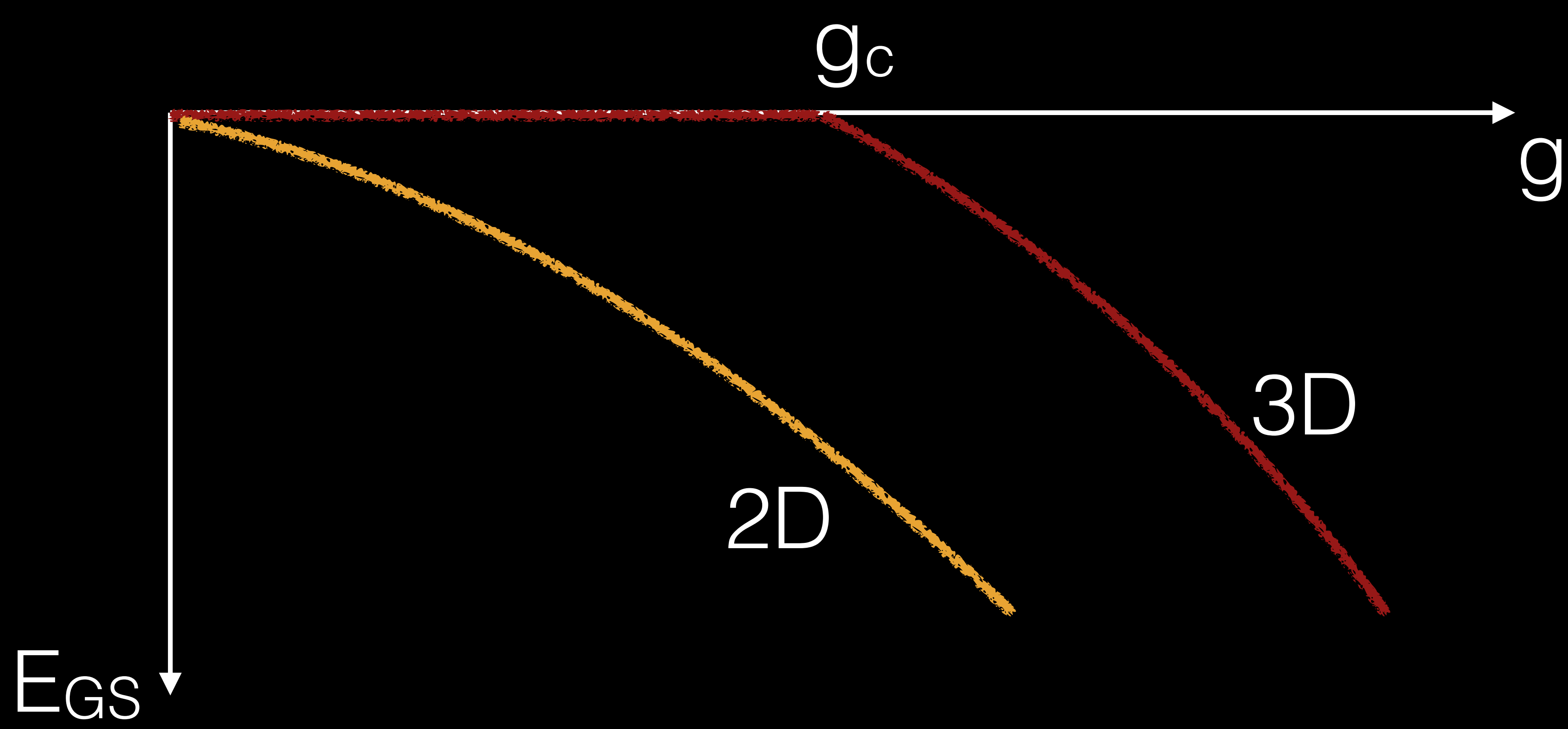
Find bound-state!

$$\boxed{\epsilon_B = \Lambda e^{-4\pi/|g|}}$$

Quantum mechanically **not** scale invariant

Two-body problem & anomaly

Cutoff required, bound state exists for all attractive couplings



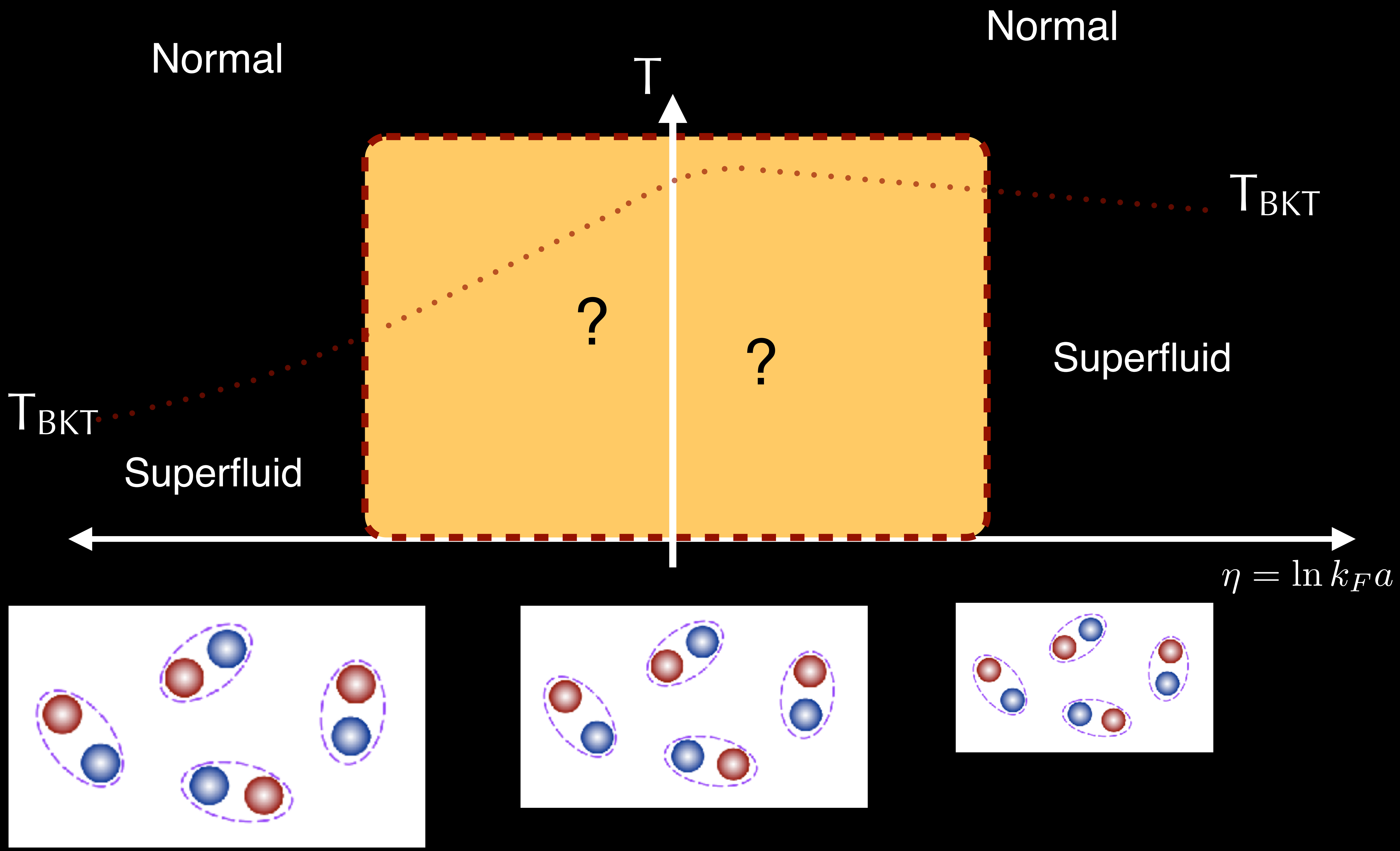
The binding energy represents a **scale anomaly.**

Binding energy and density determine the **dimensionless physical coupling**

$\epsilon_F \propto n$

$$\boxed{\eta = \frac{1}{2}\ln(2\epsilon_F/\epsilon_B)}$$

The 2D BCS-BEC crossover



Selected results in 2D

Ground state, thermodynamics, contact

G. Bertaina and S. Giorgini,
Phys. Rev. Lett. **106**, 110403 (2011).

H. Shi, S. Chiesa, and S. Zhang,
Phys. Rev. A **92**, 033603 (2015).

A. Galea, H. Dawkins, S. Gandolfi, A. Gezerlis,
Phys. Rev. A **93**, 023602 (2016).

E. R. Anderson, J. E. Drut
Phys. Rev. Lett. **115**, 115301 (2015).

L. Rammelmüller, W. J. Porter, J. E. Drut
Phys. Rev. A **93,** 033639 (2016).

Z.-H. Luo, C. E. Berger, J. E. Drut
Phys. Rev. A **93**, 033604 (2016).

M. Bauer, M. M. Parish, and T. Enss,
Phys. Rev. Lett. **112**, 375 135302 (2014).

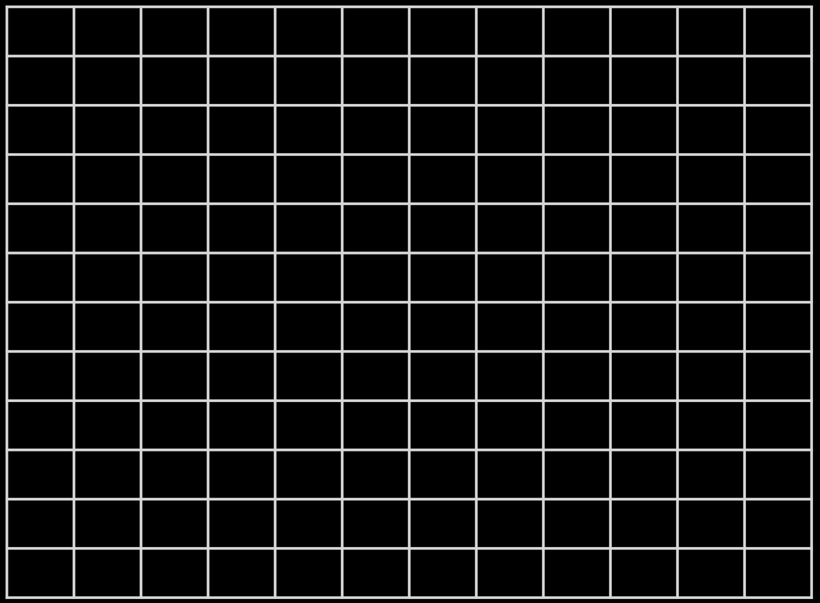
J. Hofmann,
Phys. Rev. Lett. **108**, 185303 (2012).

E. Taylor and M. Randeria,
Phys. Rev. Lett. **109**, 135301 (2012).

Technical aspects: lattice MC

Scales & continuum limit

$$1 = \ell \ll \quad \lambda_F,\ \lambda_T \quad \ll L = N_x$$



$$\lambda_T = \sqrt{2\pi\beta}$$

$$\lambda_F = n^{-1/2}$$

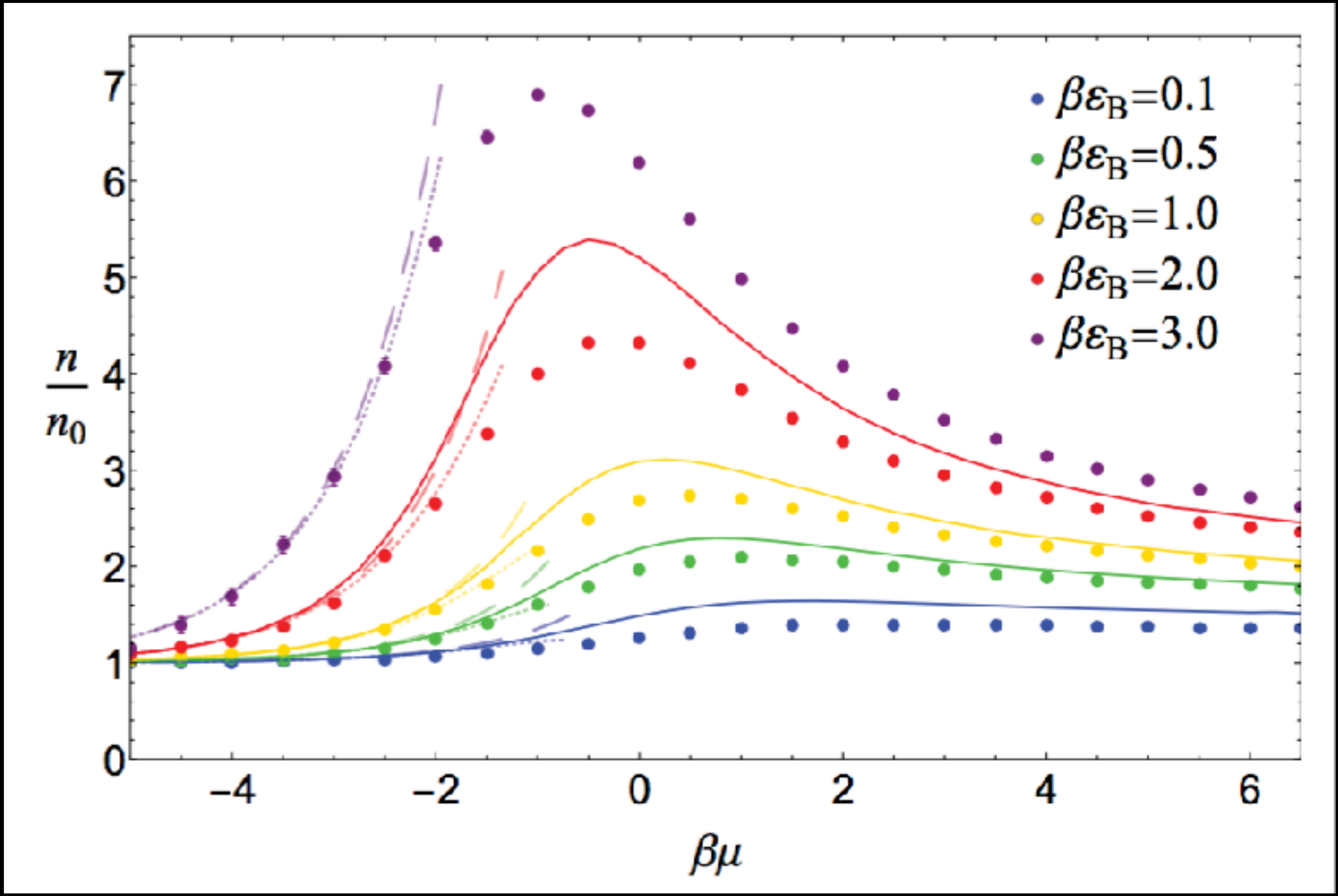
Parameters

$\beta\mu$	$\beta\varepsilon_F$
g	$\beta\varepsilon_B$

$$\varepsilon_F = \frac{k_F^2}{2}$$

$$k_F = \sqrt{2\pi n}$$

Results: Density EoS



E. R. Anderson, J. E. Drut
Phys. Rev. Lett. **115**, 115301 (2015).

Results: Density EoS

Aside: virial expansion in 2D

Virial expansion (relative to noninteracting case)

$$-\beta\Delta\Omega = \ln(\mathcal{Z}/\mathcal{Z}_0) = Q_1 \sum_{n=2}^{\infty} \Delta b_n z^n$$

Determines the thermodynamics at low fugacity $z = e^{\beta\mu}$

$\Delta b_n = b_n - b_n^{(0)}$ are typically computed by solving the n-body problem

change due to interactions

Results: Density EoS

Aside: virial expansion in 2D

Virial expansion (relative to noninteracting case)

$$-\beta\Delta\Omega = \ln(\mathcal{Z}/\mathcal{Z}_0) = Q_1 \sum_{n=2}^{\infty} \Delta b_n z^n$$

Determines the thermodynamics at low fugacity $z = e^{\beta\mu}$

$\Delta b_n = b_n - b_n^{(0)}$ are typically computed by solving the n-body problem

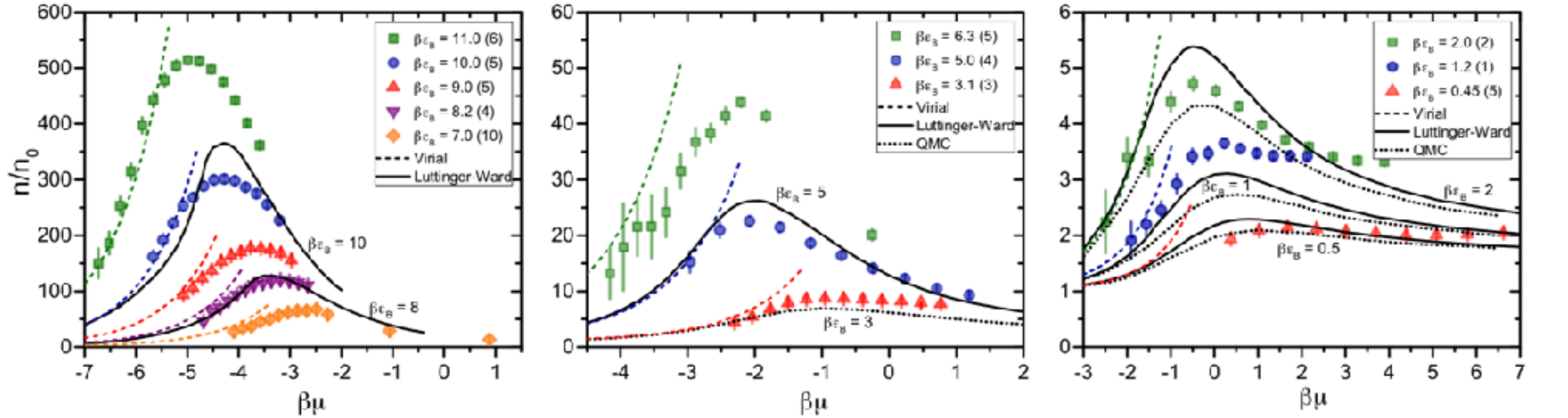
Δb_2 : Known from Beth-Uhlenbeck formula
(also derivable using the anomaly; see Ordóñez et al.)

Δb_3 : Determined numerically with exact methods

V. Ngampruetikorn, J. Levinsen, and M. M. Parish,
Phys. Rev. Lett. **111**, 265301 (2013).

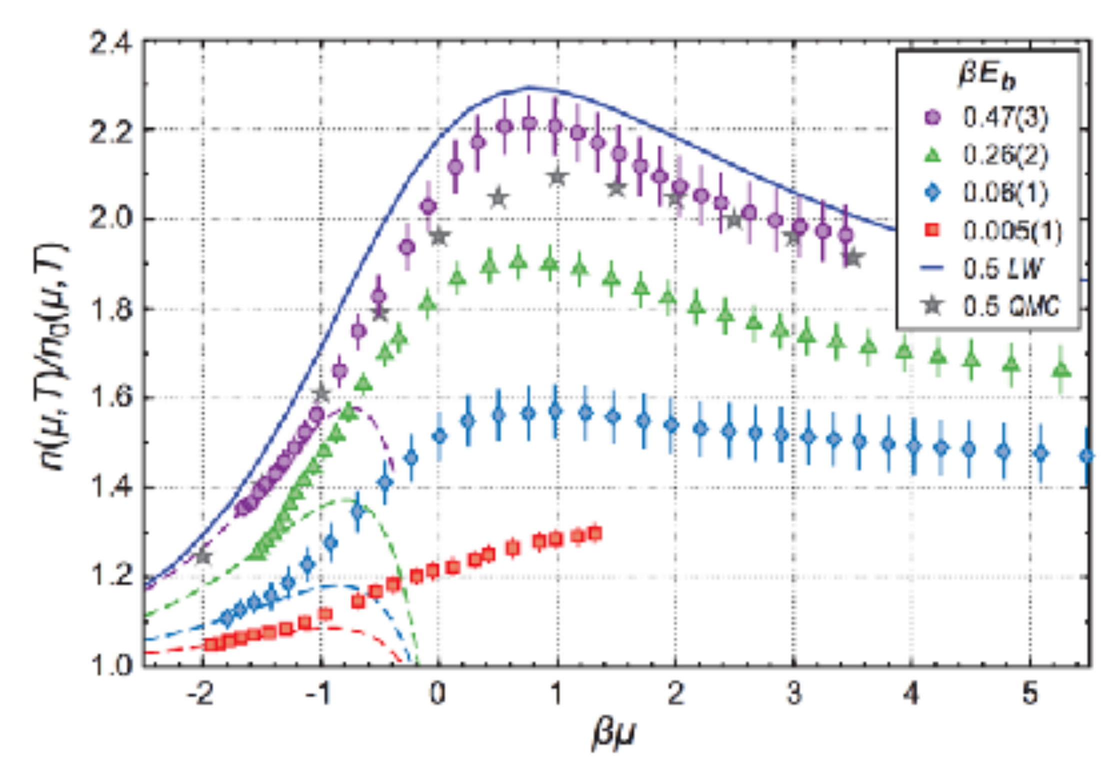
Results: Density EoS

Aside: Experimental results



Phys. Rev. Lett. **116**, 045303 (2016)

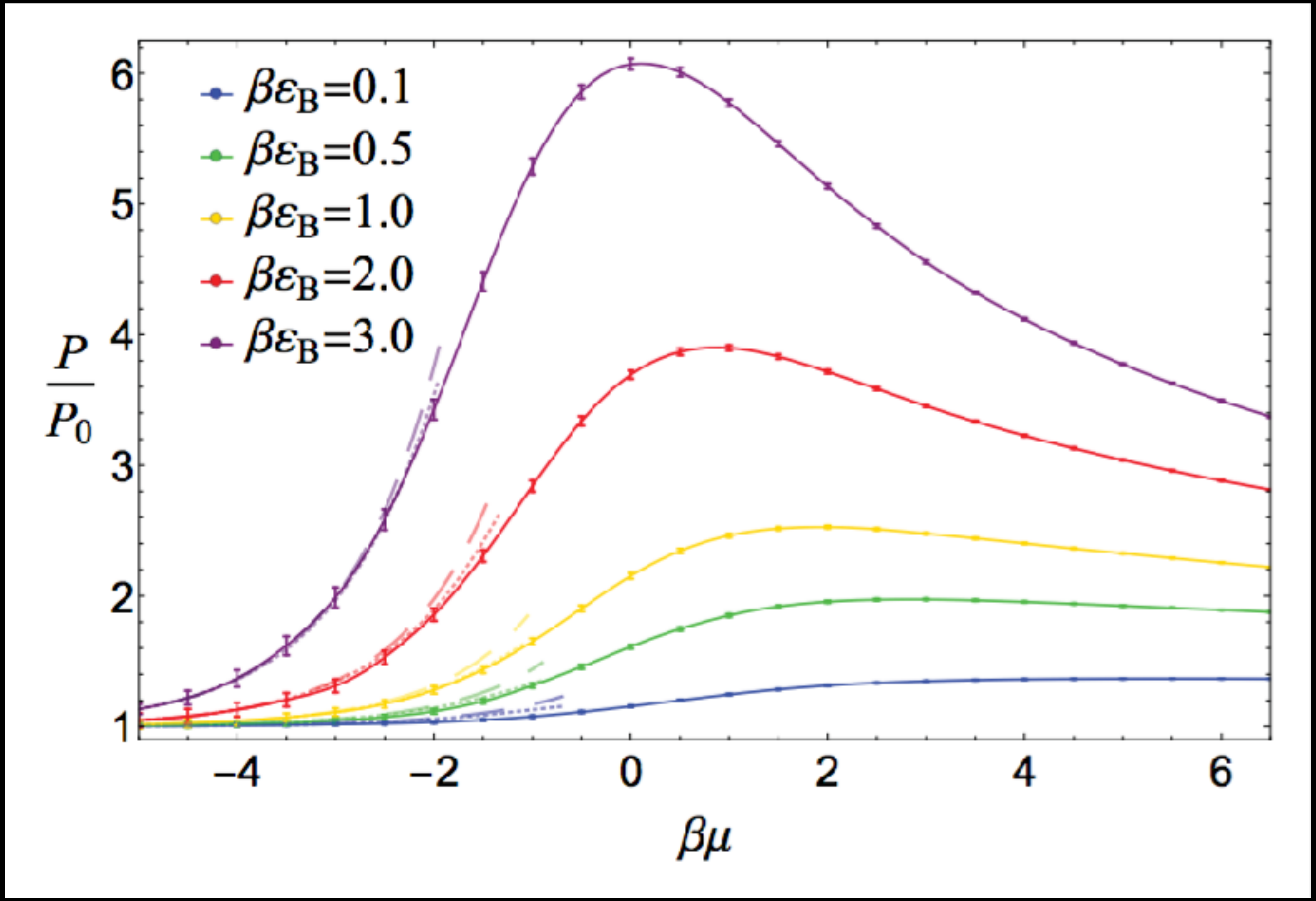
Jochim's group



Vale's group

Phys. Rev. Lett. **116**, 045302 (2016)

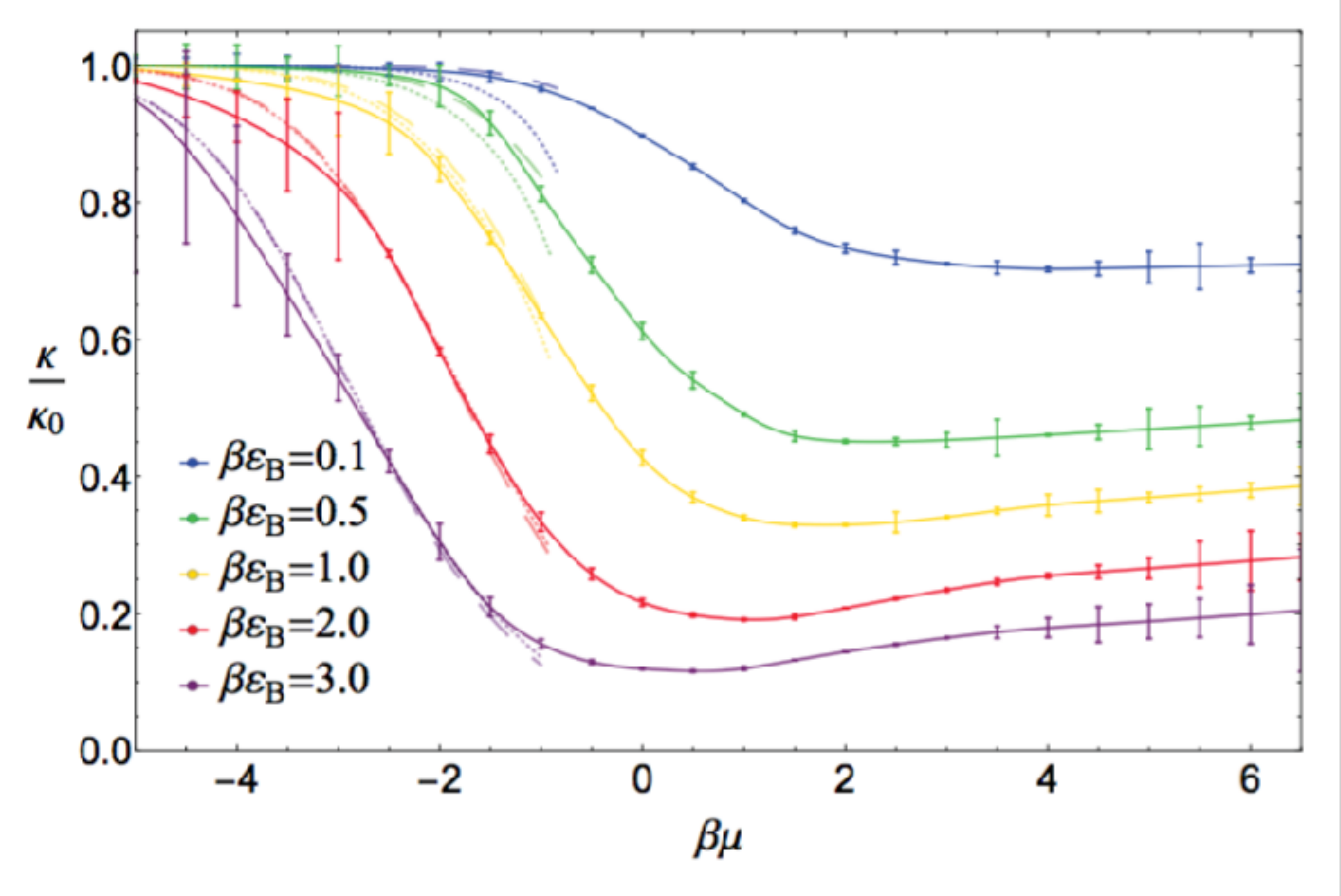
Results: Pressure EoS



E. R. Anderson, J. E. Drut

Phys. Rev. Lett. **115**, 115301 (2015).

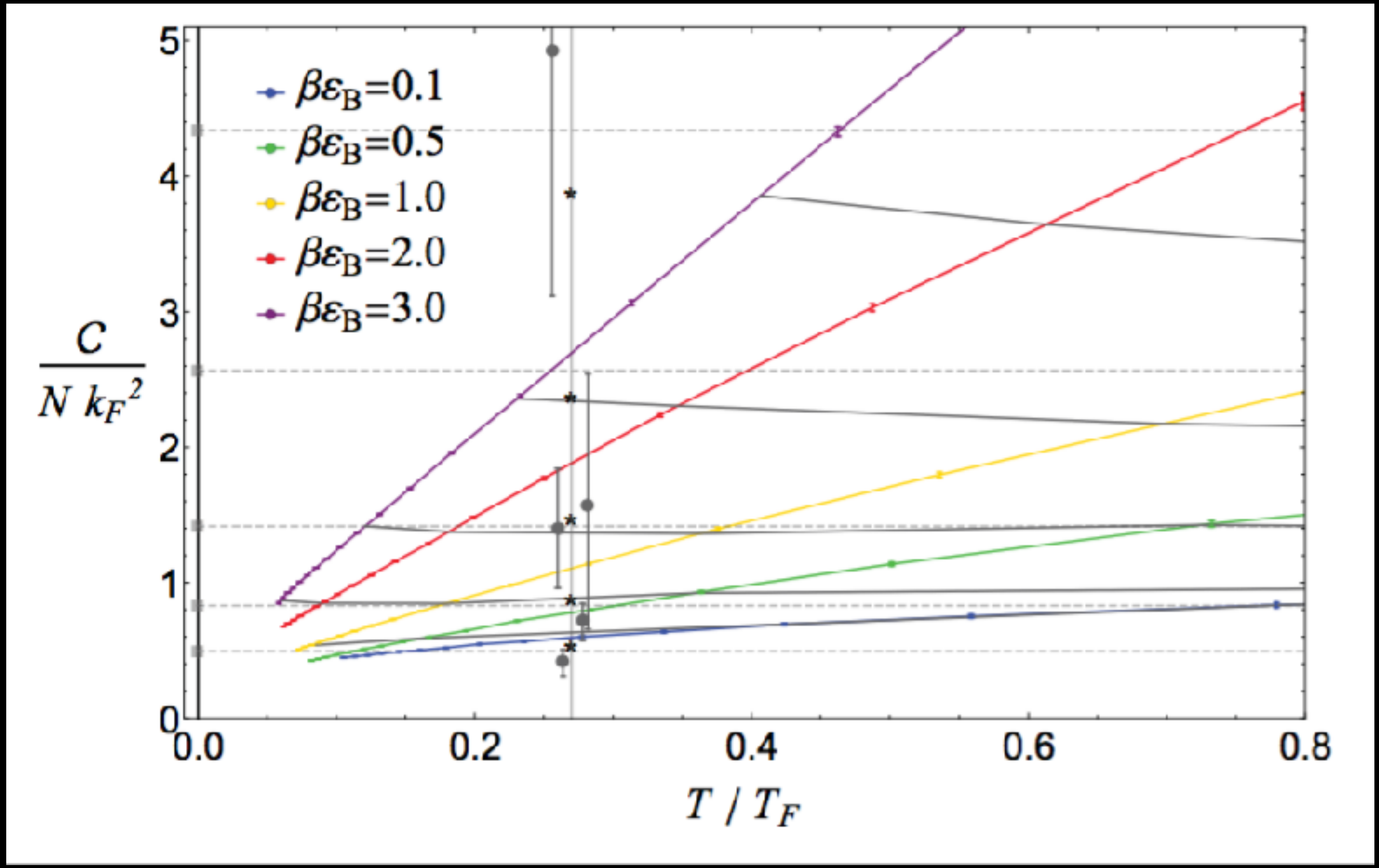
Results: Compressibility



$$\kappa = \frac{\beta}{n^2} \left. \frac{\partial n}{\partial(\beta\mu)} \right|_\beta$$

E. R. Anderson, J. E. Drut
Phys. Rev. Lett. **115**, 115301 (2015).

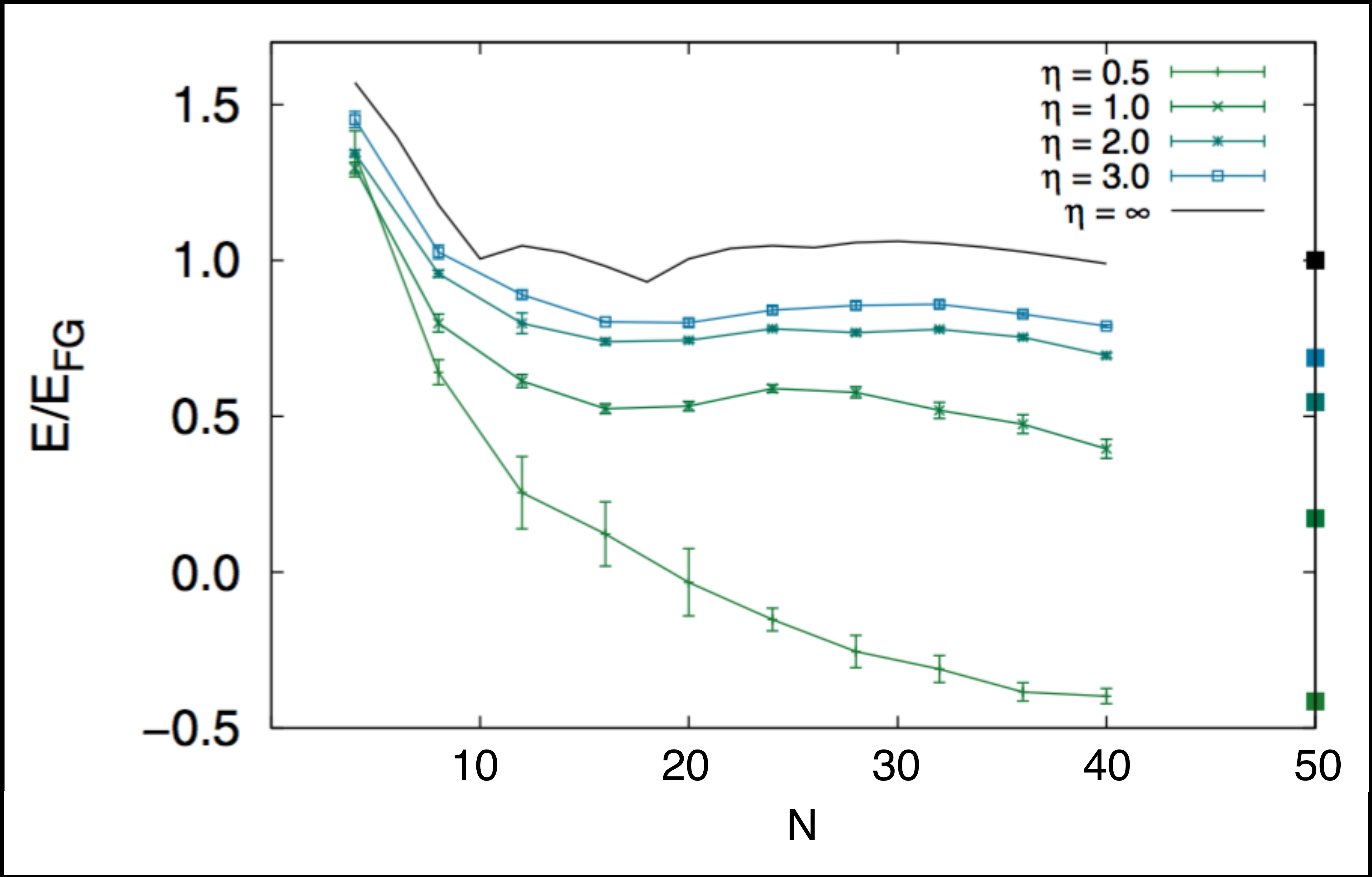
Results: Tan's contact



$$C \equiv \frac{2\pi}{\beta} \left. \frac{\partial(\beta\Omega)}{\partial \ln(a_{2D}/\lambda_T)} \right|_{T,\mu}$$

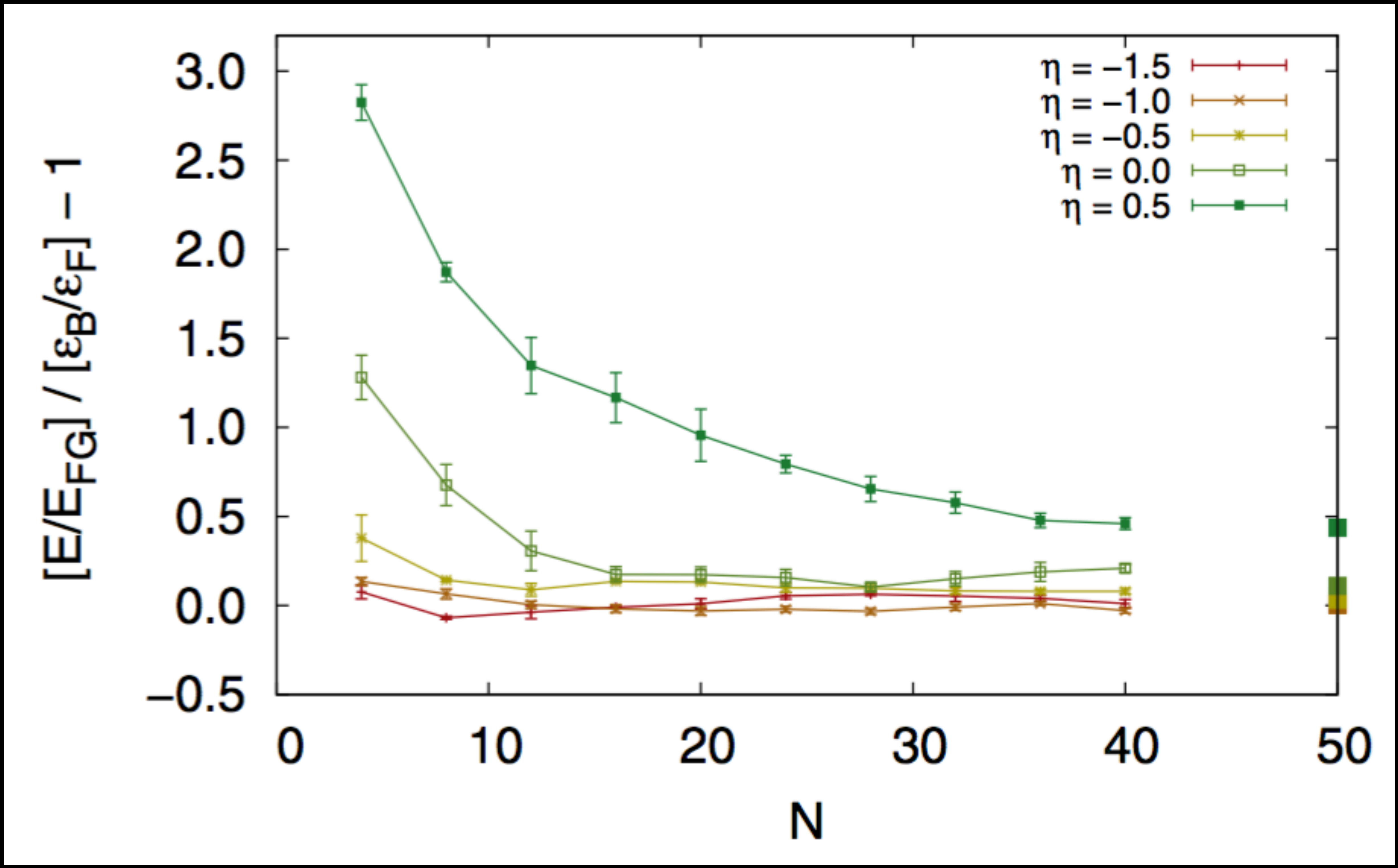
E. R. Anderson, J. E. Drut
Phys. Rev. Lett. **115**, 115301 (2015).

Results: GS Energetics



L. Rammelmüller, W. J. Porter, J. E. Drut
Phys. Rev. A **93,** 033639 (2016).

Results: GS Energetics



L. Rammelmüller, W. J. Porter, J. E. Drut
Phys. Rev. A **93,** 033639 (2016).

A new anomalous system in 1D

Work in collaboration with

Josh McKenney

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W. Daza

C. Lin

C. Ordóñez

UNIVERSITY of
HOUSTON

A new anomalous system in 1D

Three species of fermions with a contact three-body force

$$\hat{H} = \int dx \left[\sum_{s=1,2,3} \hat{\psi}_s^\dagger(x) \left(-\frac{\hbar^2 \nabla^2}{2m} \right) \hat{\psi}_s(x) - g \hat{n}_1(x) \hat{n}_2(x) \hat{n}_3(x) \right]$$

Three species: 1, 2, 3

Only a contact three-body force (nothing else!)

Coupling is dimensionless!

$$[g] = 1$$

Does a 3-body bound state form?

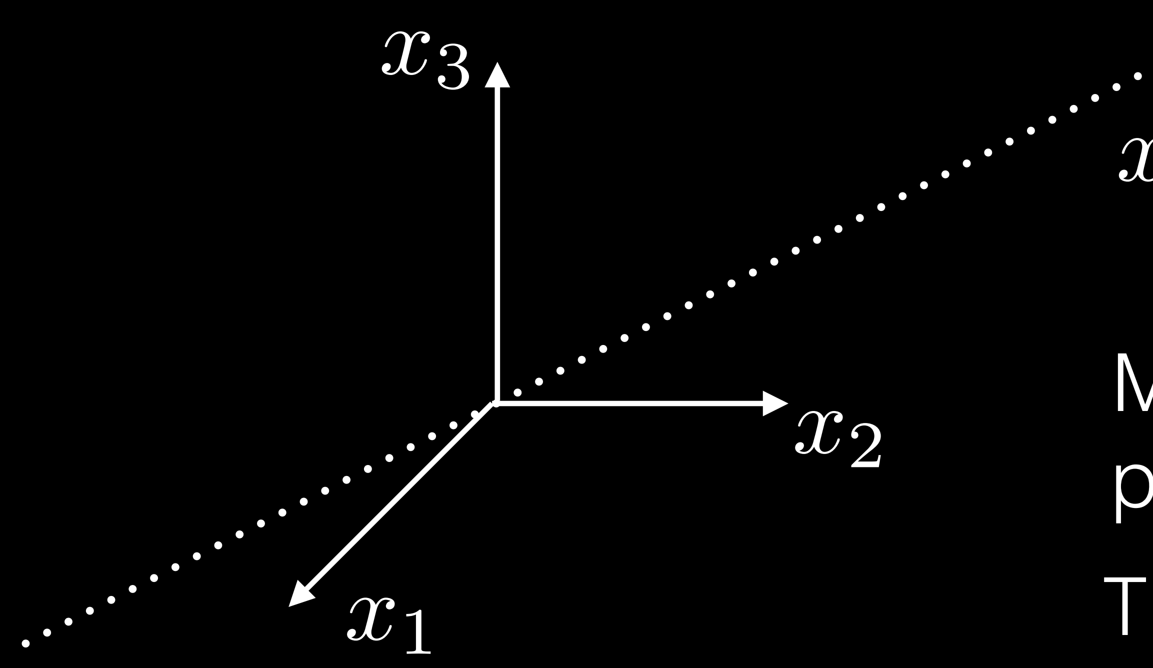
Is there an anomaly?

Three-body problem & anomaly

Factoring motion. Think of this as a 3D problem.

$$\left[-\frac{\nabla_X^2}{2m} + g\delta(x_2 - x_1)\delta(x_3 - x_2)\right]\psi(X) = E\psi(X)$$

$$X = (x_1, x_2, x_3)$$



$$x_1 = x_2 = x_3$$

Motion factors into parallel and perpendicular to that line.

The latter is effectively in 2D.

Three-body problem & anomaly

Mapping to two-dimensional one-body problem

$$\left[-\frac{\nabla_X^2}{2m} + g\delta(x_2 - x_1)\delta(x_3 - x_2)\right]\psi(X) = E\psi(X)$$

$$X = (x_1, x_2, x_3)$$

Three-body problem & anomaly

Mapping to two-dimensional one-body problem

$$\left[-\frac{\nabla_X^2}{2m} + g\delta(x_2 - x_1)\delta(x_3 - x_2)\right]\psi(X) = E\psi(X)$$

$$X = (x_1, x_2, x_3)$$

Separating center-of-mass and relative motion…

$$\left[\frac{-\nabla_q^2}{2\bar{m}} + \tilde{g}\delta(q_1)\delta(q_2)\right]\phi(q_1, q_2) = E_r\phi(q_1, q_2)$$

$$\tilde{g} = (2/\sqrt{3})g$$

$$Q = \frac{1}{3}(x_1 + x_2 + x_3)$$

$$q_1 = x_2 - x_1$$

$$q_2 = \frac{1}{\sqrt{3}}(x_1 + x_2 - 2x_3)$$

Scale-anomalous 2D problem!

It is 1D, but not amenable to Bethe Ansatz.

Three-body problem & anomaly

Other cases?

Coupling units (contact n-body, d-dimensions)

$$[g] = L^{(2-d(n-1))}$$

When is it dimensionless?

$$d(n-1) = 2 \longrightarrow \begin{array}{l} d=1 \;\; \& \;\; n=3 \\ d=2 \;\; \& \;\; n=2 \end{array}$$

$$\frac{\partial g}{\partial \ln(\beta \epsilon_B)} \propto g^2$$

Only two cases!

Thermodynamics and contact

Exact properties

Virial coefficients

No interaction unless 3 or more particles present…

$$\longrightarrow \quad \Delta b_2 = 0$$

Equivalence of the 1D 3-body and 2D 2-body problems…
(in relative coordinates)

$$\longrightarrow \quad \boxed{\Delta b_3 = \frac{1}{\sqrt{3}} \Delta b_2^{(2D)}}$$

Known from Beth-Uhlenbeck formula

Thermodynamics and contact

Exact properties

Truly scale invariant

$$P = \beta^\alpha f(\beta\mu)$$

$$\alpha = -d/2 - 1$$

$$P = \frac{2}{d}\frac{E}{V}$$

Anomalous

$$P = \beta^\alpha f(\beta\mu, \beta\epsilon_B)$$

$$P - \frac{2}{d}\frac{E}{V} = \frac{2}{d}\beta^\alpha \frac{\partial f}{\partial \ln(\beta\epsilon_B)}$$

$$\mathcal{C}_3 = -\frac{\partial g}{\partial \ln(\beta\epsilon_B)}\langle \hat{n}_1 \hat{n}_2 \hat{n}_3 \rangle$$

“Contact density”

of our 1D problem

Thermodynamics and contact

Exact properties in a harmonic trap

Truly scale invariant

$$\Omega = \omega f(\beta\mu, \beta\omega)$$

$$E = 2V_{\text{ext}}$$

Anomalous

$$\Omega = \omega f(\beta\mu, \beta\omega, \beta\epsilon_B)$$

$$E - 2V_{\text{ext}} = \omega \underline{\frac{\partial f}{\partial \ln(\beta\epsilon_B)}}$$

$$C_3 = \frac{\partial g}{\partial \ln(\beta\epsilon_B)} \int dx \langle \hat{n}_1 \hat{n}_2 \hat{n}_3 \rangle$$

"Contact" of our 1D problem

Thermodynamics and contact

Toward the many-body problem

The path-integral representation of the partition function requires a Hubbard-Stratonovich transformation

For the usual two-body force case:

$$e^{\tau g_2 \hat{n}_1(\mathbf{x}) \hat{n}_2(\mathbf{x})} = \frac{1}{2\pi} \int_{-\pi}^{\pi} d\sigma (1 + A\hat{n}_1(\mathbf{x}) \sin\sigma)(1 + A\hat{n}_2(\mathbf{x}) \sin\sigma),$$

interaction

auxiliary field

$$A = \sqrt{2(e^{\tau g_2} - 1)}$$

$$\mathcal{Z} = \mathrm{Tr} e^{-\beta(\hat{H} - \mu \hat{N})} = \int \mathcal{D}\sigma \det{}^2 M[\sigma]$$

Thermodynamics and contact

Toward the many-body problem

The path-integral representation of the partition function requires a Hubbard-Stratonovich transformation

For the three-body force case:

$$e^{\tau g_3 \hat{n}_1(\mathbf{x})\hat{n}_2(\mathbf{x})\hat{n}_3(\mathbf{x})} = \frac{1}{3\pi}\int d\sigma (1 + B\hat{n}_1(\mathbf{x})F(\sigma))(1 + B\hat{n}_2(\mathbf{x})F(\sigma))(1 + B\hat{n}_3(\mathbf{x})F(\sigma))$$

interaction

auxiliary field

$$F(\sigma) = e^{i2\sigma/3}\cos^2\sigma$$

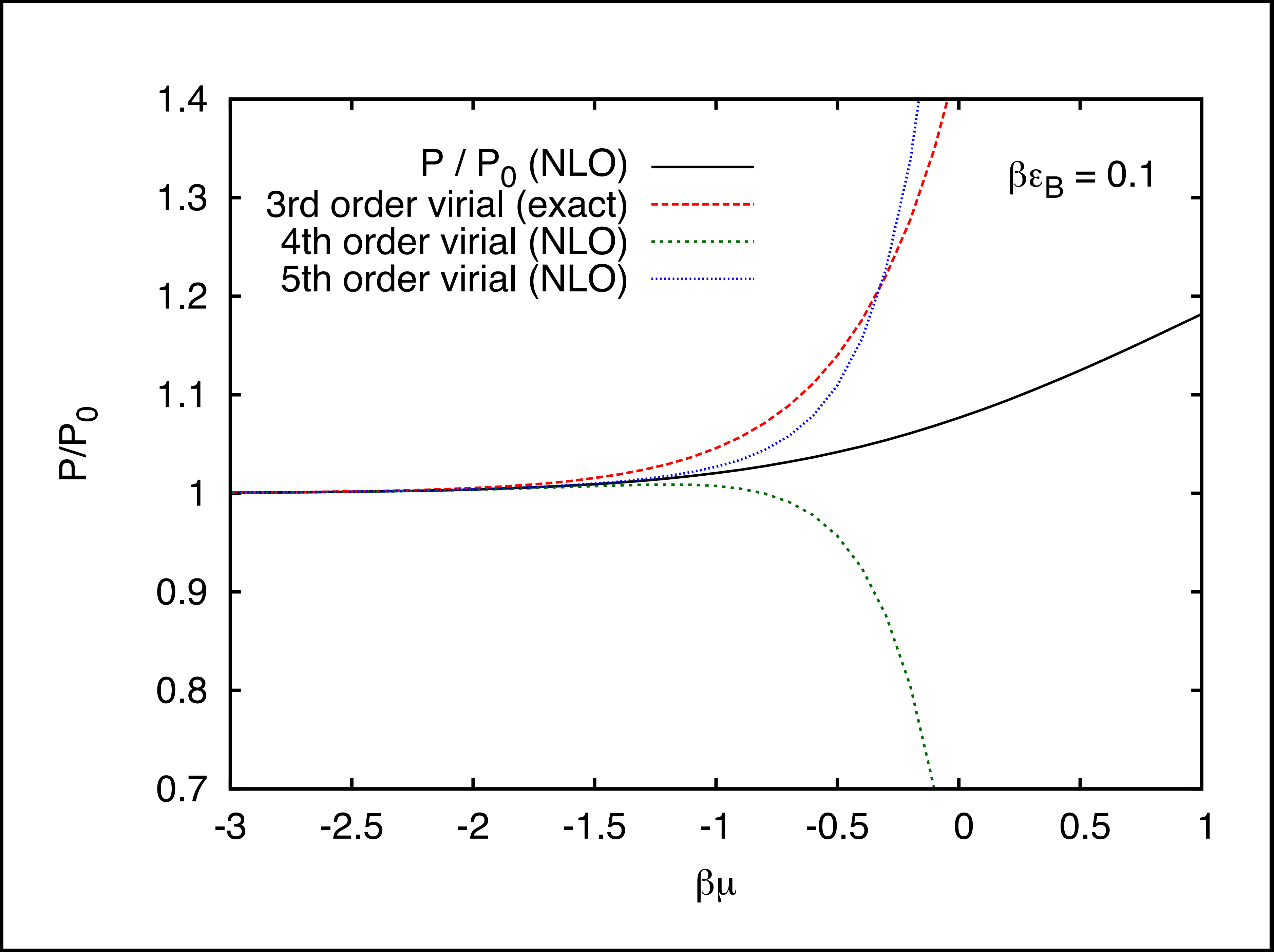
$$B = 1.63...\left(e^{\tau g_3} - 1\right)^{1/3}$$

$$\mathcal{Z} = \mathrm{Tr} e^{-\beta(\hat{H} - \mu\hat{N})} = \int \mathcal{D}\sigma \det{}^3 M[\sigma]$$

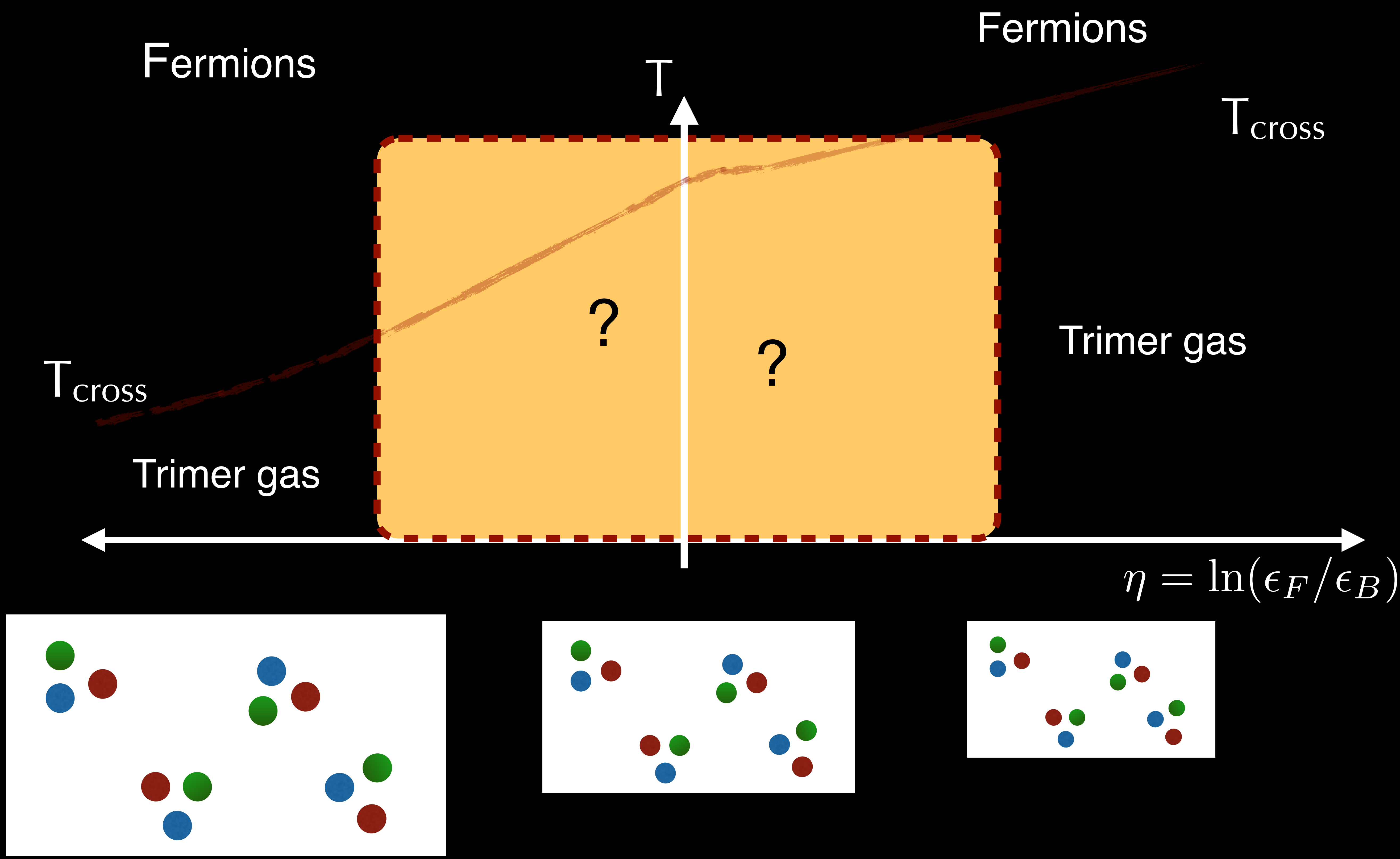
Straightforwardly generalized to n-body forces. But: sign problem.

Thermodynamics and contact

Toward the many-body problem: pressure EoS



The 1D trimer crossover



Thermodynamics and contact

Open questions

How can we realize this system experimentally?

What is the effect of asymmetries?

Can we induce superfluid correlations?

How to deal with sign problem?

Can we use diagrammatic self-consistent methods (Luttinger-Ward)?

What are the transport properties?

Summary & Conclusions

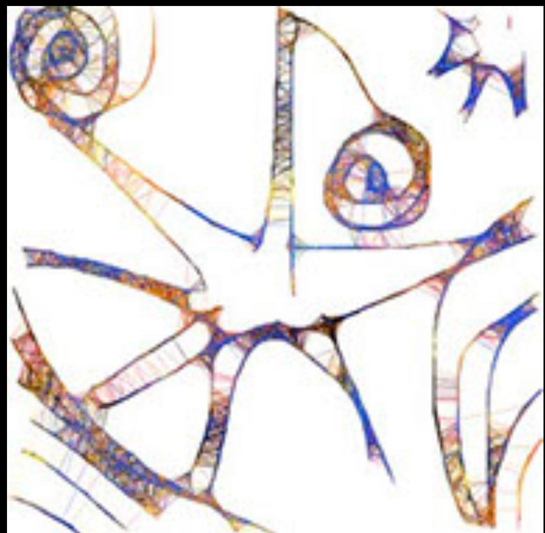
- There are two possible scale-anomalous non-relativistic systems with contact interactions: 2D with 2-body forces
  1D with 3-body forces

- We have the beginnings of a characterization of the thermodynamics and contact of these systems, both in the ground state and at finite temperature, complementing other approaches and comparing with experiments.

- There is room for improvement in the 2D case at finite temperature.

- Treating the 1D case with 3-body forces in a non-perturbative way presents a sign problem that remains open.

- However, we have derived universal relations and virial theorems involving the 3-body contact, and determined b_3.



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Thank you!