Nuclear Equation of State ($\rho \leq \rho_0$) from Reactions

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Nuclear Equation of State and Neutron Stars

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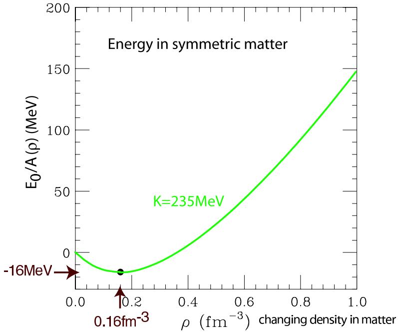
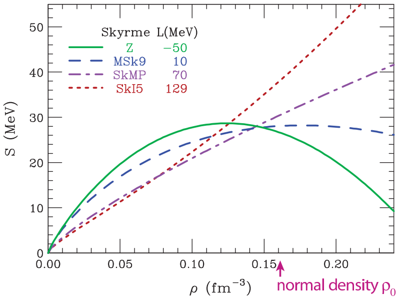
Energy in Uniform Matter

$$\frac{E}{A}(\rho_n, \rho_p) = \frac{E_0}{A}(\rho) + S(\rho) \left(\frac{\rho_n - \rho_p}{\rho}\right)^2 + \mathcal{O}(\ldots^4)$$

symmetric matter (a)symmetry energy $\rho = \rho_n + \rho_p$





$$\frac{E_0}{A}(\rho) = -a_V + \frac{K}{18}\left(\frac{\rho - \rho_0}{\rho_0}\right)^2 + \ldots \qquad S(\rho) = a_a^V + \frac{L}{3}\frac{\rho - \rho_0}{\rho_0} + \ldots$$

Known: $a_a \approx 16\,\text{MeV}$ $K \sim 235\,\text{MeV}$ Unknown: a_a^V ? L ?

Symmetry-Energy Stiffness: *M* & *R* of *n*-Star

$$\frac{E}{A} = \frac{E_0}{A}(\rho) + S(\rho)\left(\frac{\rho_n - \rho_p}{\rho}\right)^2$$

$$S \simeq a_a^V + \frac{L}{3}\frac{\rho - \rho_0}{\rho_0}$$

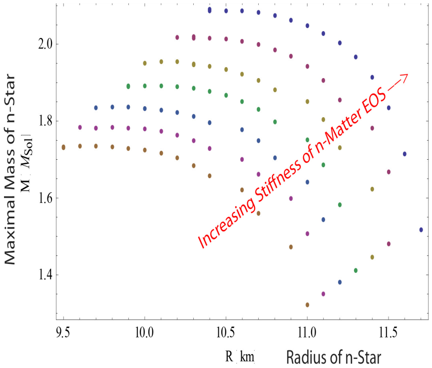
In neutron matter:

$\rho_p \approx 0$ & $\rho_n \approx \rho$.

Then, $\frac{E}{A}(\rho) \approx \frac{E_0}{A}(\rho) + S(\rho)$

Pressure:

$$P = \rho^2 \frac{\mathrm{d}}{\mathrm{d}\rho}\frac{E}{A} \simeq \rho^2 \frac{\mathrm{d}S}{\mathrm{d}\rho} \simeq \frac{L}{3\rho_0}\rho^2$$

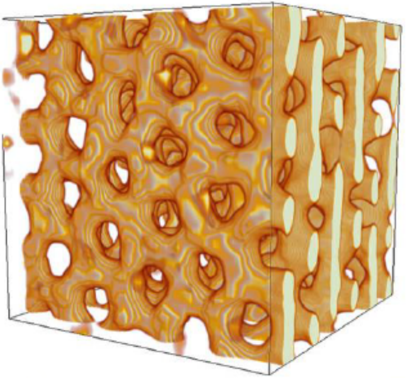


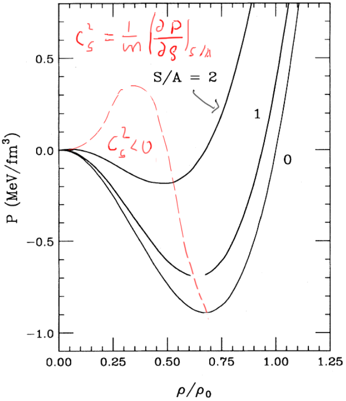
Schematic Calculation by Stephen Portillo (Harvard U)

Stiffer symmetry energy correlates with
larger max mass of neutron star & larger radii

Low-ρ Matter Phases: Topological Structures

Caplan&Horowitz RMP89(17)041002





Low-ρ uniform matter unstable

Fig from PD PRC51(95)716

$n \leftrightarrow p$ Symmetry Invariants: Energy & Densities

$$\frac{E}{A}(\rho_n, \rho_p) = \frac{E_0}{A}(\rho) + S(\rho)\left(\frac{\rho_n - \rho_p}{\rho}\right)^2 + \mathcal{O}(\dots^4)$$

symmetric matter (a)symmetry energy $\rho = \rho_n + \rho_p$

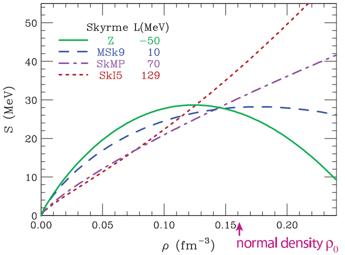
Net $\rho = \rho_n + \rho_p$ isoscalar

Difference $\rho_n - \rho_p$ isovector

$\rho_a = \frac{A}{N-Z}(\rho_n - \rho_p)$ isoscalar

$$\rho_{n,p}(r) = \frac{1}{2}\left[\rho(r) \pm \frac{N-Z}{A}\rho_a(r)\right]$$

ρ & ρ_a universal in isobaric chain!



Energy min in Thomas-Fermi:

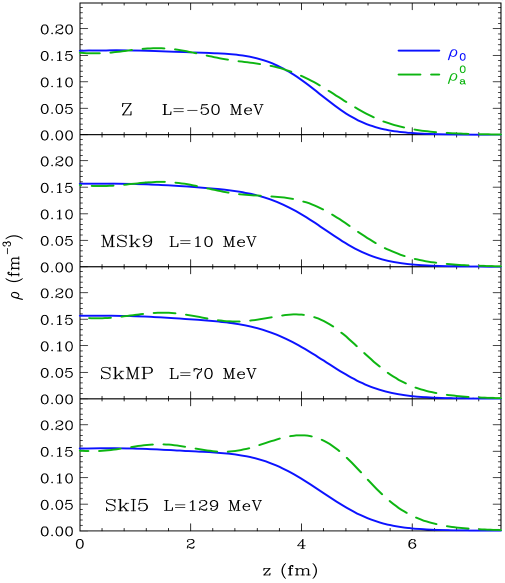
$$\rho_a(r) \propto \frac{\rho(r)}{S(\rho(r))}$$

low $S \Leftrightarrow$ high ρ_a

$$S(\rho) = S(\rho_0) + \frac{L}{3}\frac{\rho - \rho_0}{\rho_0} + \dots$$

Unknown: $S(\rho_0)$? L ?

Relation between ρ, ρ_a & $S(\rho)$

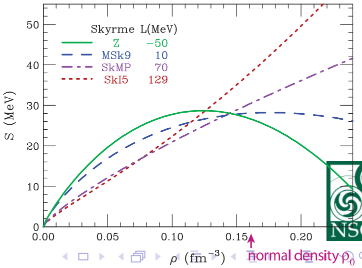


Results f/different Skyrme ints in half-∞ matter

PD&Lee NPA818(09)36

Isoscalar (ρ=ρ_n+ρ_p; blue) & isovector ($\rho_a \propto \rho_n$-ρ_p; green) densities displaced relative to each other.

As $S(\rho)$ changes, $\rho_a(r) \propto \frac{\rho(r)}{S(\rho(r))}$, so does displacement or aura

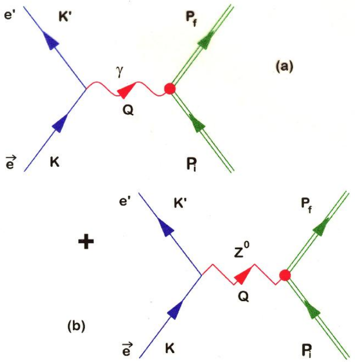


Probing Independently 2 Densities

Jefferson Lab

Direct: $\sim p$

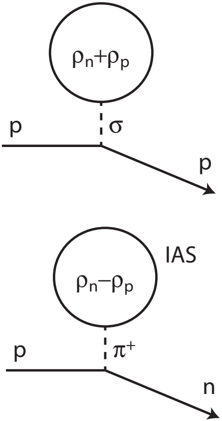
Interference: $\sim n$



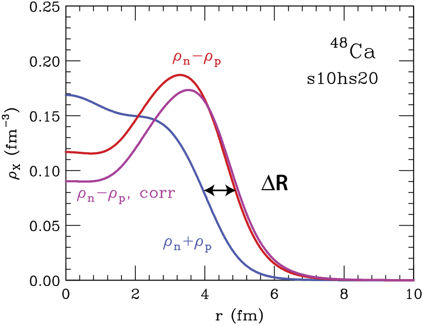
PD, Singh, Lee NPA958(17)147
[after Dao Tien Khoa]

elastic: $\sim p + n$

charge exchange: $\sim n - p$



Expectations on Isovector Aura?



Much Larger Than Neutron Skin!

Surface radius $R \simeq \sqrt{\frac{5}{3}} \langle r^2 \rangle^{1/2}$

rms neutron skin

$$\langle r^2 \rangle^{1/2}_{\rho_n} - \langle r^2 \rangle^{1/2}_{\rho_p}$$

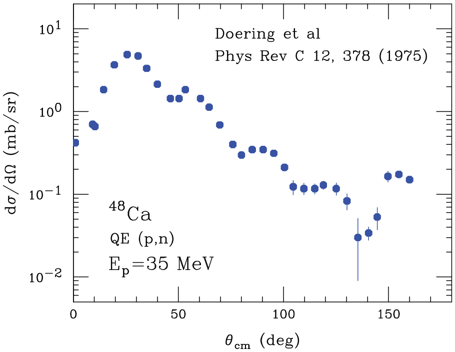
$$\simeq 2 \, \frac{N-Z}{A} \left[\langle r^2 \rangle^{1/2}_{\rho_n - \rho_p} - \langle r^2 \rangle^{1/2}_{\rho_n + \rho_p} \right]$$

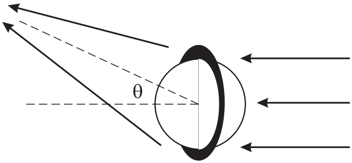
rms isovector aura

Estimated $\Delta R \sim 3 \left(\langle r^2 \rangle^{1/2}_{\rho_n} - \langle r^2 \rangle^{1/2}_{\rho_p} \right)$ for ^{48}Ca/^{208}Pb!

Even before consideration of Coulomb effects that further enhance difference!

Direct Reaction Primer





DWBA:

$$\frac{d\sigma}{d\Omega} \propto \left| \int dr\, \Psi_f^* \, U_1 \, \Psi_i \right|^2$$

Lane Potential
$$U = U_0 + \frac{4\tau T}{A} U_1$$

$U_0 \propto \rho \qquad U_1 \propto \rho_n - \rho_p$

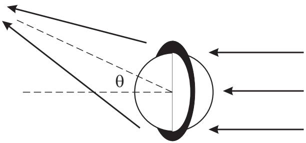
It is common to assume the same geometry for U_0 & U_1, implicitly ρ & ρ_a, e.g. Koning&Delaroche NPA713(03)231

Cross-Section Sensitivity to Isovector Surface

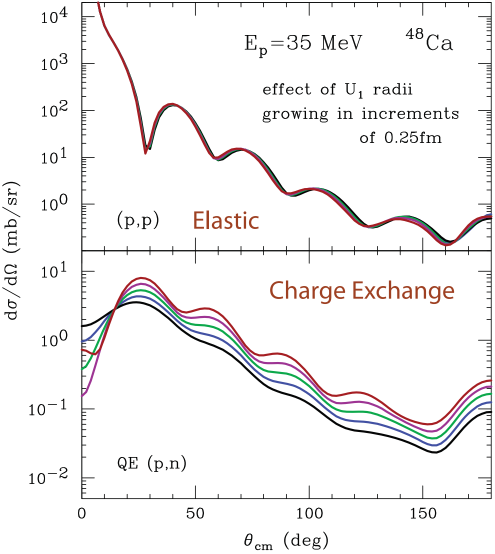
Koning-Delaroche
NPA713(03)231
same radii R for U_0 & U_1!

$$U_1(r) \propto \frac{U_{01}}{1 + \exp\frac{r-R}{a}}$$

$$R \rightarrow R + \Delta R_1$$

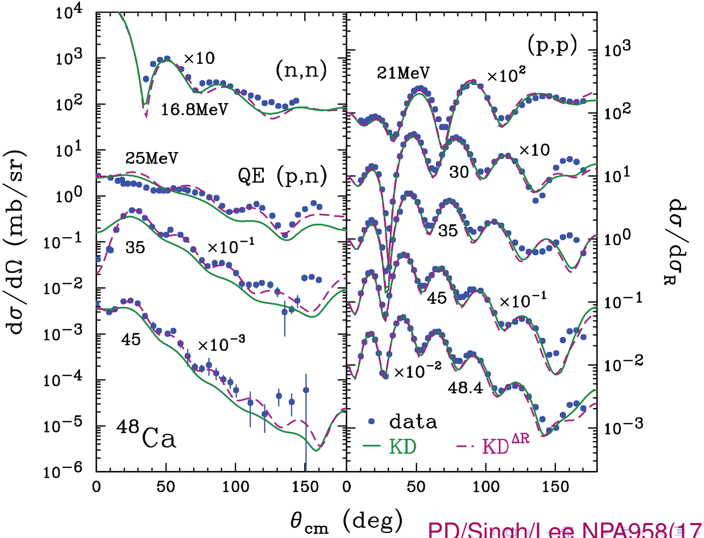


charge-exchange cs
oscillations grow



Simultaneous Fits to Elastic & Charge-Change: ^{48}Ca

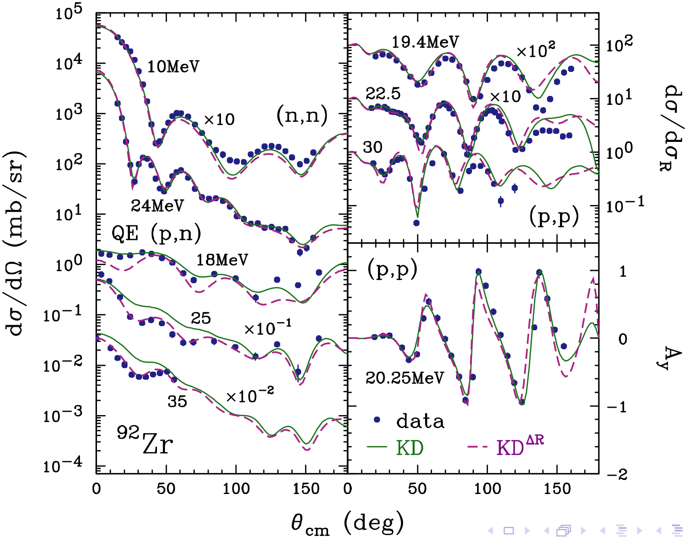
Different radii for densities/potentials: $R_a = R + \Delta R$



PD/Singh/Lee NPA958(17)147

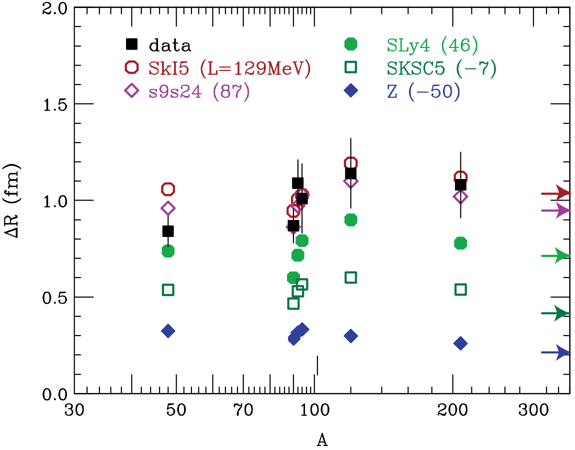
Simultaneous Fits to Elastic & Charge-Change: ^{92}Zr

Different radii for densities/potentials: $R_a = R + \Delta R$



Thickness of Isovector Aura

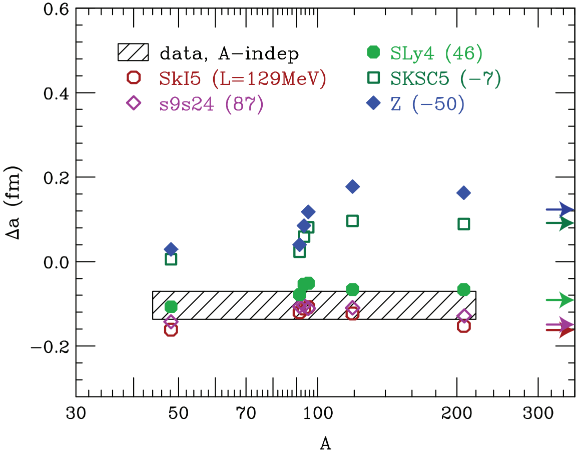
6 targets analyzed, differential cross section + analyzing power



Colored: Skyrme predictions. Arrows: half-infinite matter

Thick ∼ 0.9 fm isovector aura! ∼Independent of *A*...

Difference in Surface Diffuseness



Colored: Skyrme predictions. Arrows: half-infinite matter

Sharper isovector surface than isoscalar!

Bayesian Inference

Probability density in parameter space $p(x)$ updated as experimental data on observables E, value $\overline{E}$ with error σ_E, get incorporated

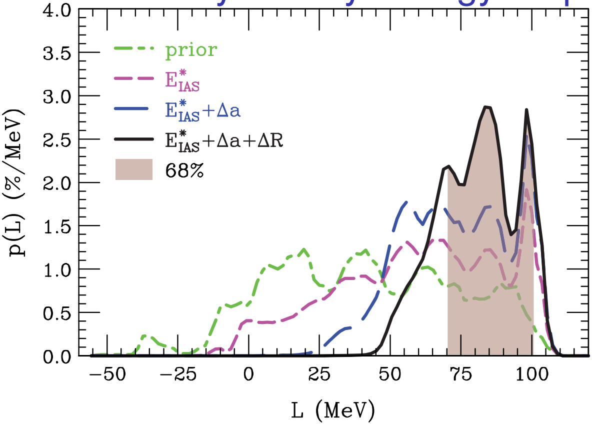
Probability p is updated iteratively, starting with prior p_{prior}
$p(a|b)$ - conditional probability

$$p(x|\overline{E}) \propto p_{\text{prior}}(x) \int \mathrm{d}E \, \mathrm{e}^{-\frac{(E-\overline{E})^2}{2\sigma_E^2}} \, p(E|x)$$

For large number of incorporated data, p becomes independent of p_{prior}

In here, p_{prior} and $p(E|x)$ are constructed from all Skyrme ints in literature, and their linear interpolations. p_{prior} is made uniform in plane of symmetry-energy parameters (L, S_0)

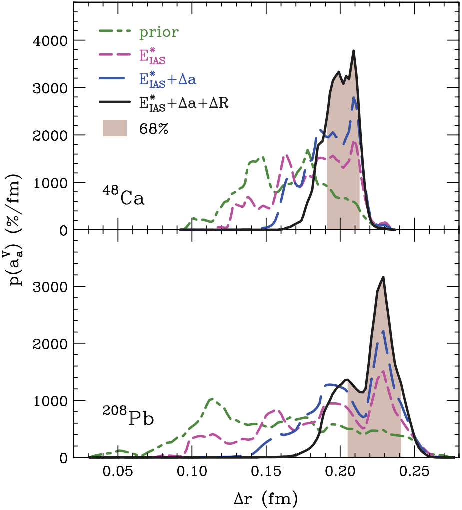
Likelihood f/Symmetry-Energy Slope



E^*_{IAS} - from excitations to isobaric analog states in PD&Lee NPA922(14)1

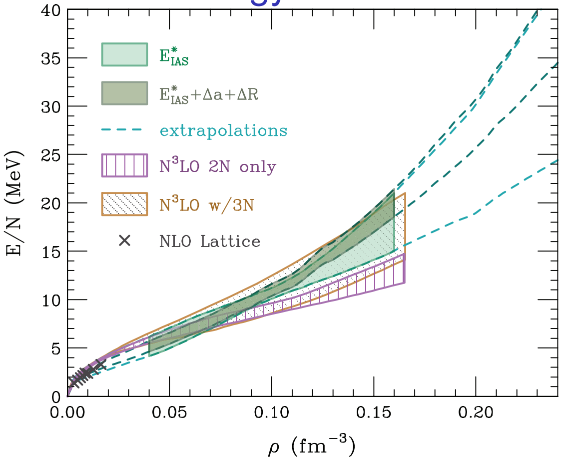
Oscillations in prior of no significance - represent availability of Skyrme parametrizations

Likelihood f/Neutron-Skin Values



Sizeable *n*-Skins

Likelihood f/Energy of Neutron Matter



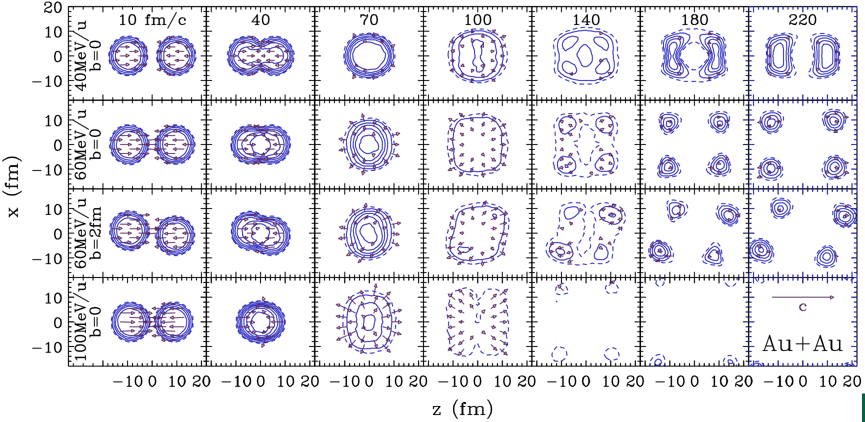
E^*_{IAS} - from excitations to isobaric analog states in PD&Lee NPA922(14)1

Some oscillations due to prior

Modest-E Collisions Probe Low-ρ EOS

Boltzmann eq simulations of central $Au + Au$ collisions: ρ-contours

PD *et al* arXiv:1910.10500 different energies & centralities



Late equilibration in expanding low-ρ matter that cannot stay uniform

$\Rightarrow$ Ring formation: Threshold phenomenon at $E/A \gtrsim 60$ MeV/u

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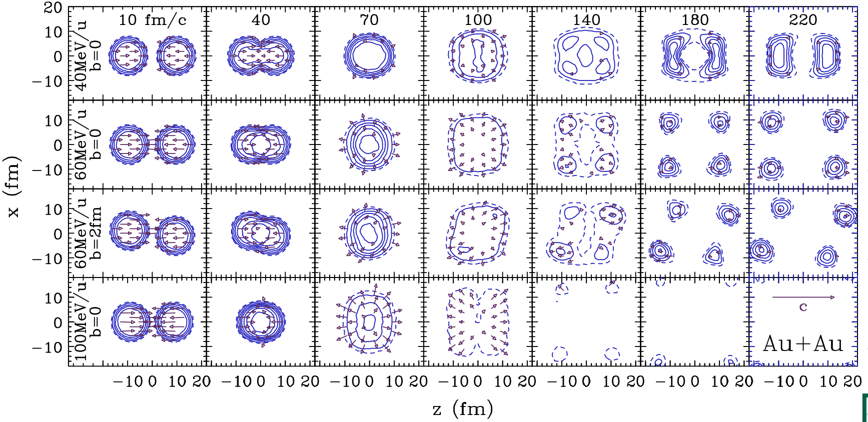

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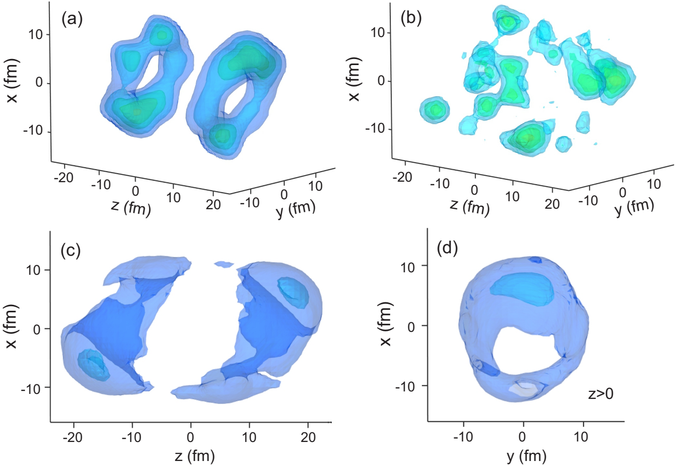
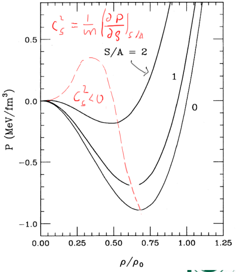
Boltzmann vs Fluctuating Dynamics

(a) - Boltzmann, (b)-(d) - Brownian Motion model - extra ρ fluctuations





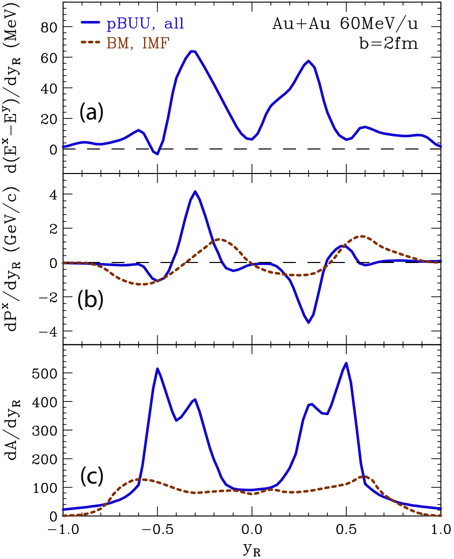
(c) & (d) - average over events, similar to Boltzmann!

Experimental Detection: Flow

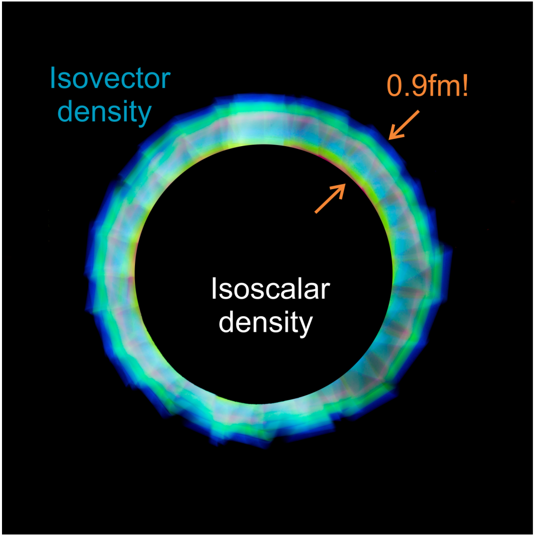
Expansion in the final state maps structures from configuration onto velocity space:

(a)2nd & (b)1st order moments vs rapidity &

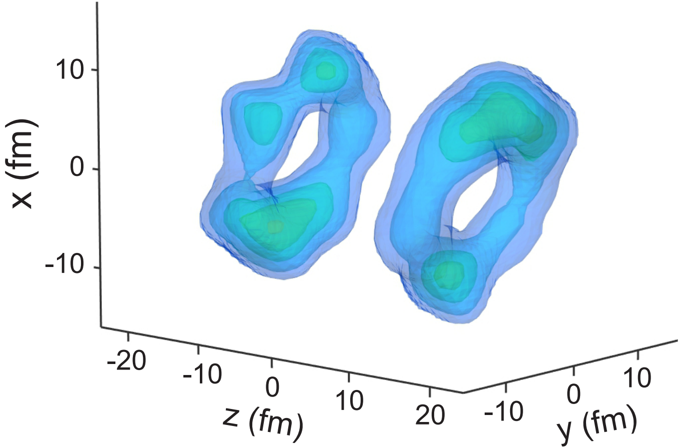
(c) rapidity distribution



Isovector Aura



Rings at Onset of Collective Expansion



Conclusions

- Symmetry-energy polarizes densities, pushing isovector density out to low isoscalar density
- For large *A*, isovector-isoscalar surface displacement expected roughly independent of nucleus and dependent on *L*
- Surface displacement studied in comparative analysis of data on elastic scattering and quasielastic charge-exchange reactions
- Analysis produces thick isovector aura $\Delta R \sim 0.9\,\text{fm}!$
- Symmetry & neutron energies are stiff!
  $L = (70 - 100)\,\text{MeV}$, $S(\rho_0) = (33.5 - 36.5)\,\text{MeV}$ at 68% level
- Matter diving into low-ρ region undergoes *detectable* spinodal decomposition into rings w/stones

PD, Lee & Singh NPA818(09)36, 922(14)1, 958(17)147 + in progress
PD, Lin, Stone & Iwata arXiv:1910.10500
DOE DE-SC0019209

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