The role of quark matter in astrophysics

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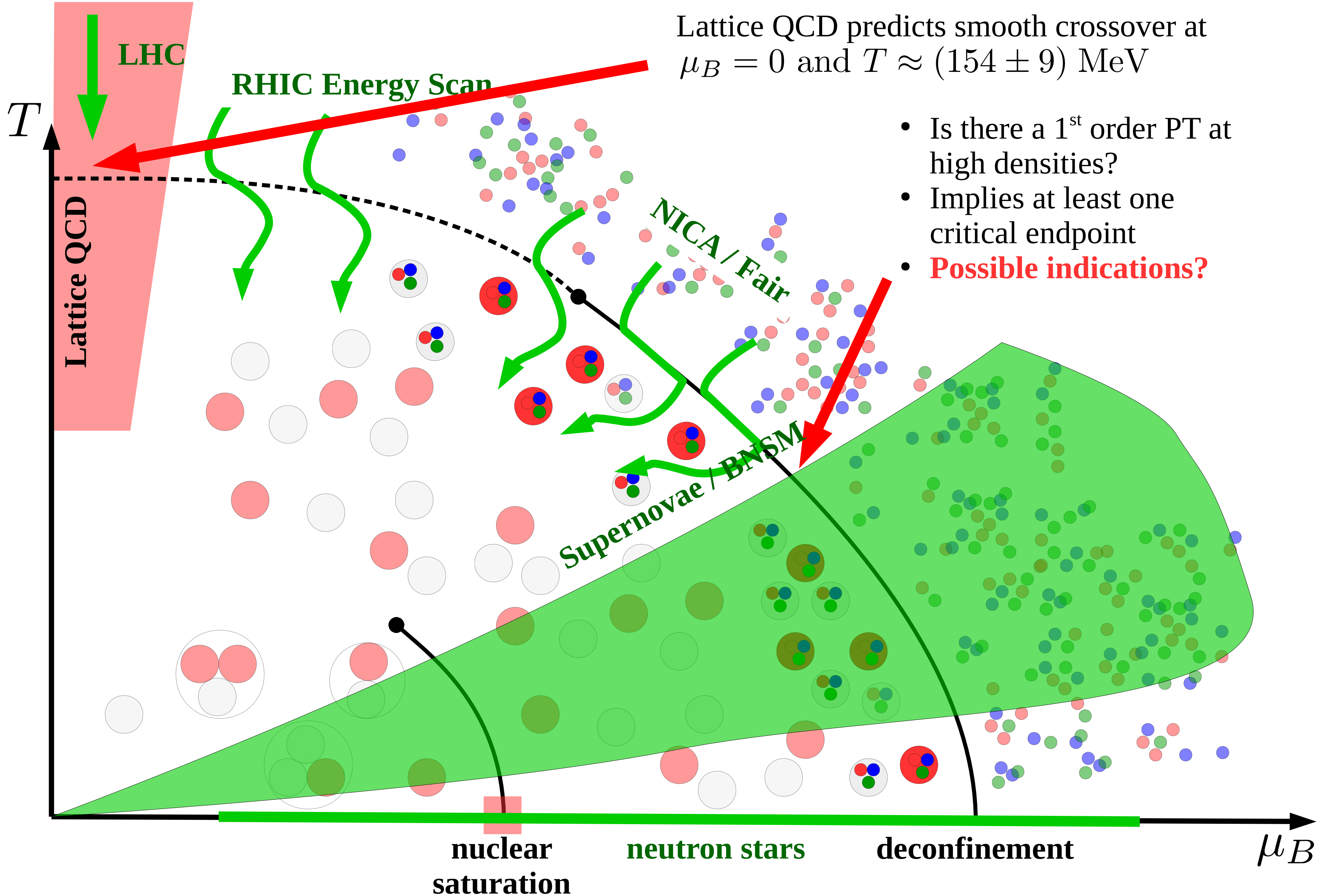
Hirschegg, 16th of January 2020

Possibility of 1st order PT at high densities

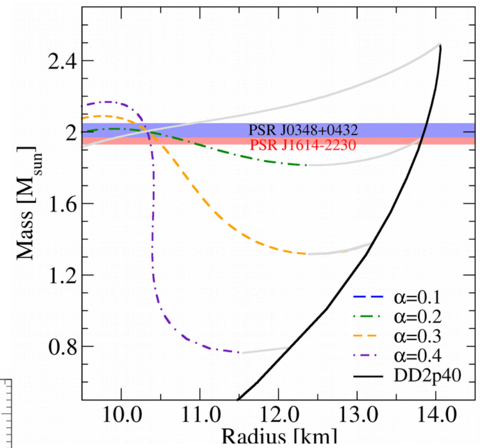


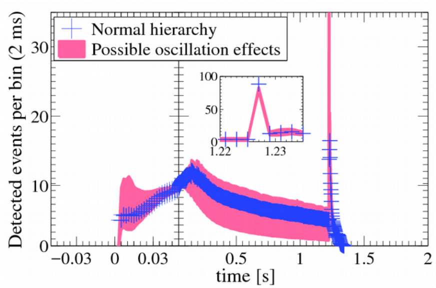
Lattice QCD predicts smooth crossover at $\mu_B = 0$ and $T \approx (154 \pm 9)$ MeV

- Is there a 1st order PT at high densities?
- Implies at least one critical endpoint
- **Possible indications?**

Possible signals of 1st – order phase transitions.

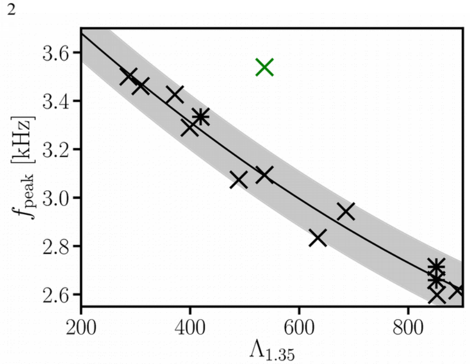
Neutron star configurations

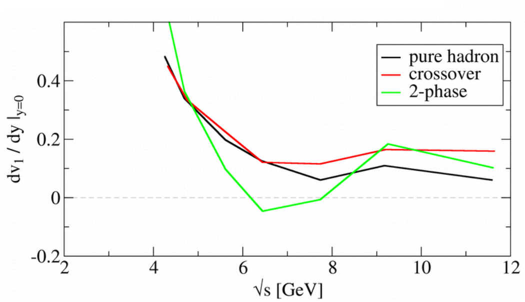




Supernova explosions of 50Ms stars

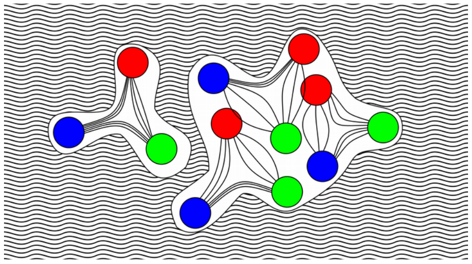
Binary neutron star mergers



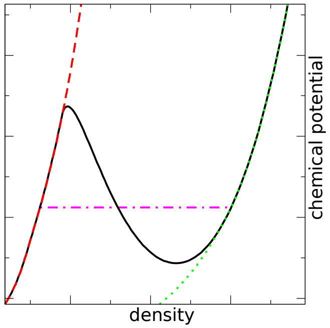


Heavy-Ion Collisions

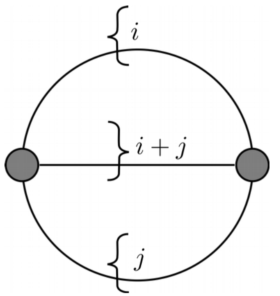
Unified description of the equation of state.



density functional theory

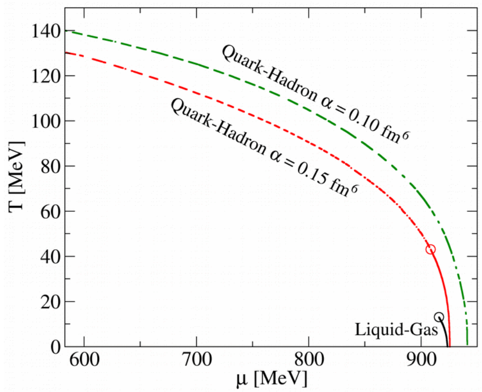


construction of phase transitions

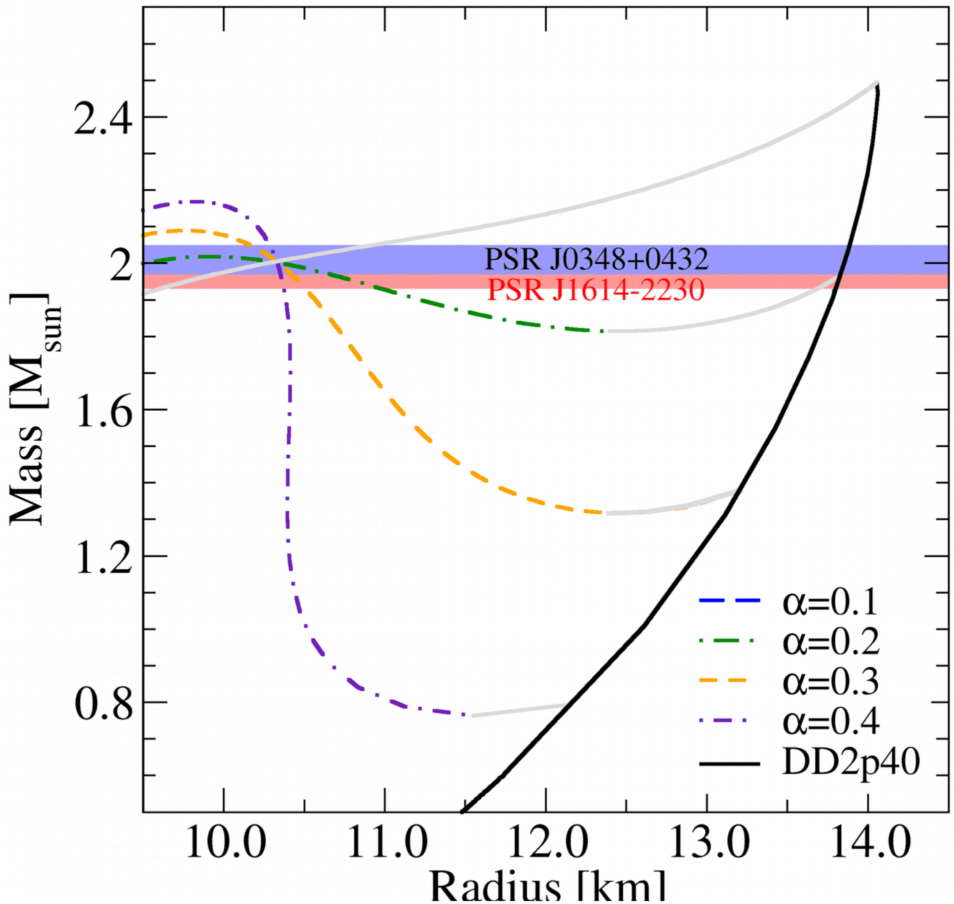


Phi-derivable formalism

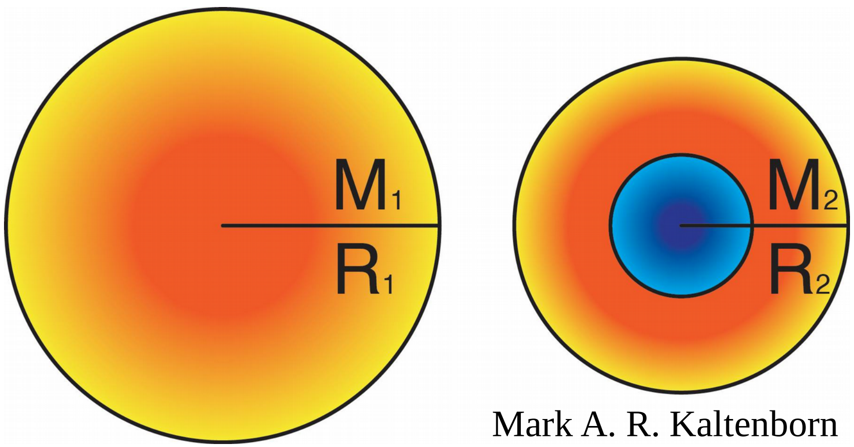
current status



1^{st} order PT – Neutron stars



- Star configurations with same masses, but different radii



Mark A. R. Kaltenborn

- **New class of EOS, that features high mass twins**

- NASA NICER mission: radii measurements ~ 0.5 km

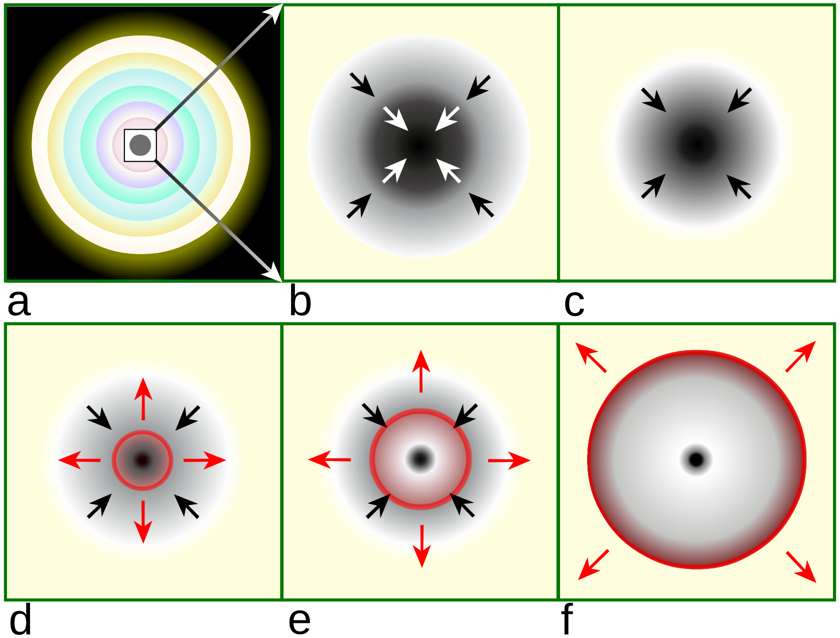
- Existence of twins implies 1st order phase-transition and hence a critical point

M. Kaltenborn, **NUFB**, D. Blaschke, PRD 96, 056024 (2017)

Core-collapse supernova explosions

Massive stars (~ 8 Ms)

- Sequential burning stages of light elements
- Onion structure with iron core (1.4 Ms)
- Gravitational collapse
- Bounce shock through stiffness of EOS
- Mainly neutrino heating drives shock wave

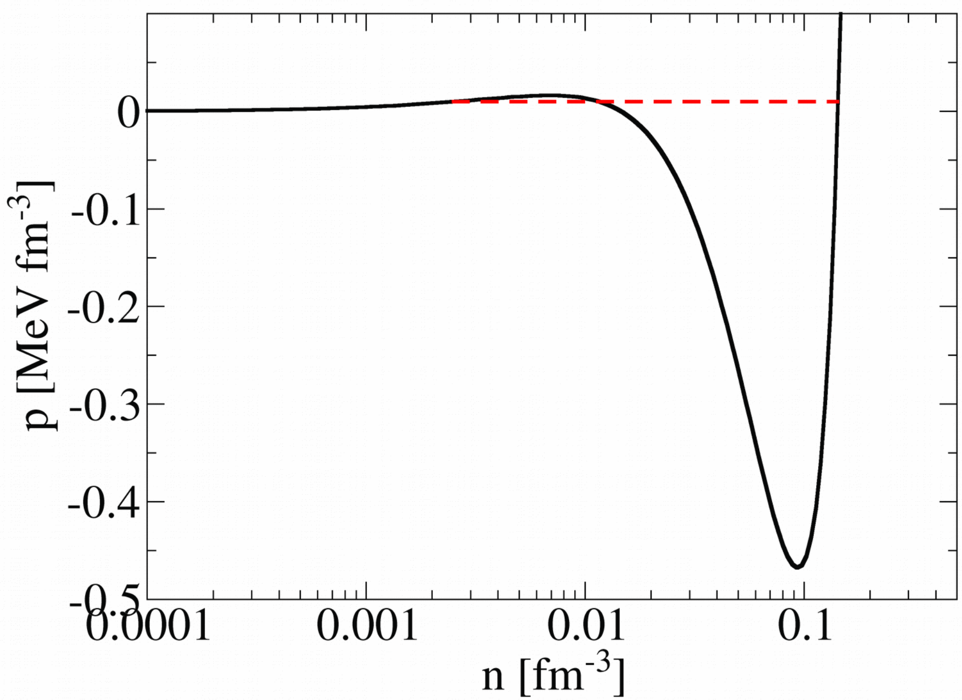


Super-massive stars (~ 50 Ms)

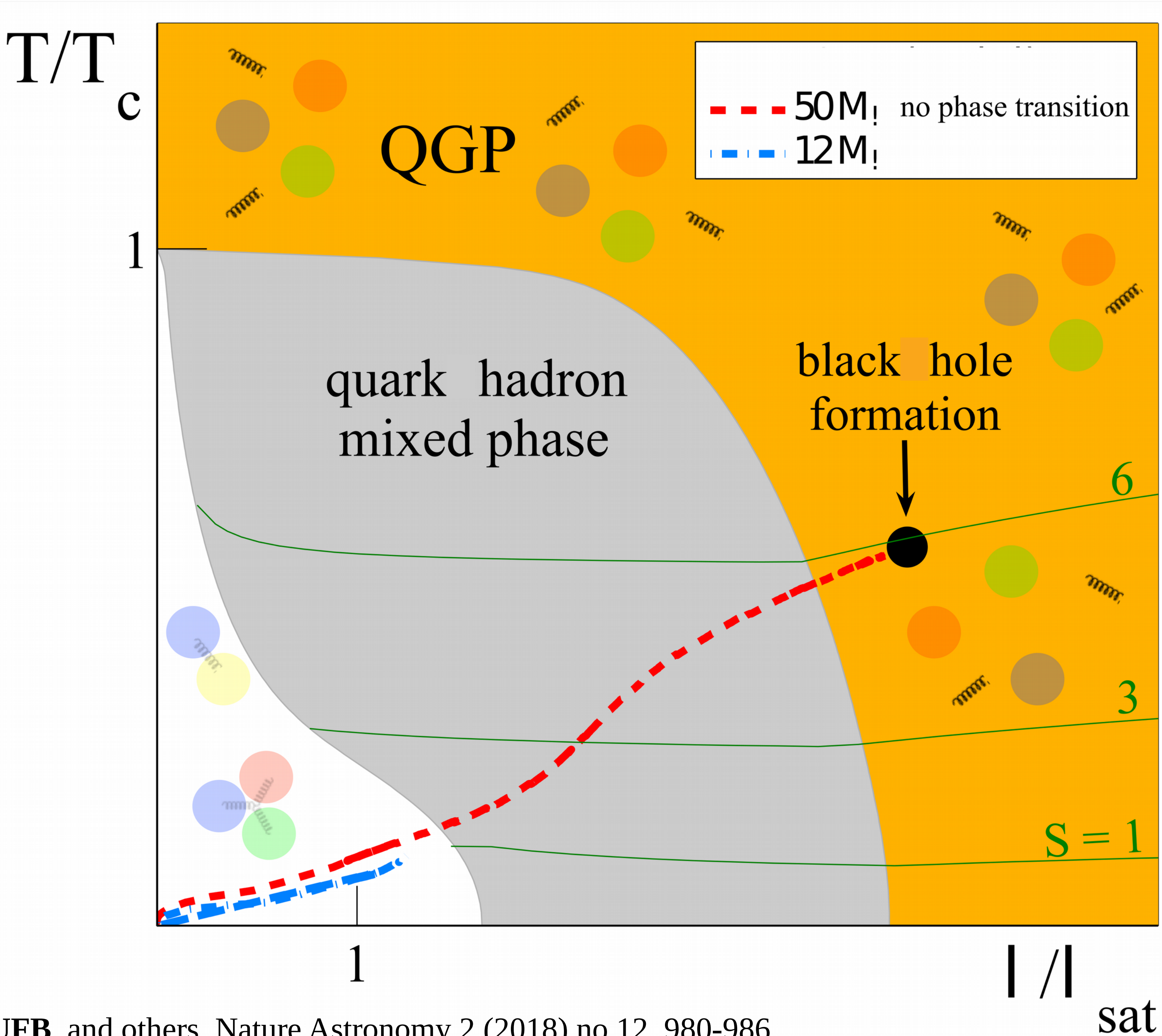
- Can not be explained by canonical models
- Have observational evidences
- One of biggest uncertainties:

high density EOS

how about a quark-hadron PT?

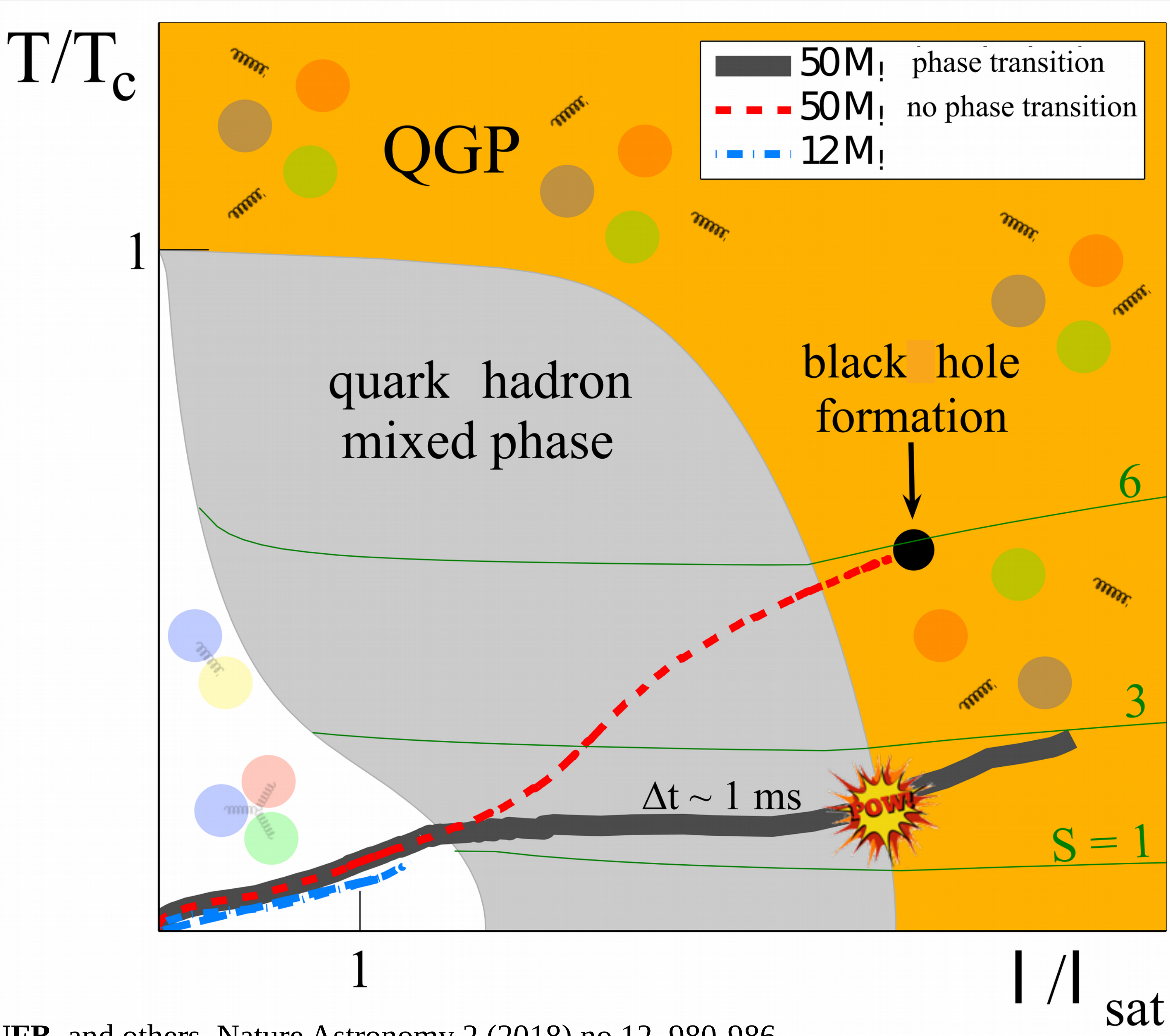


1[st] order PT – Supernovae of massive stars



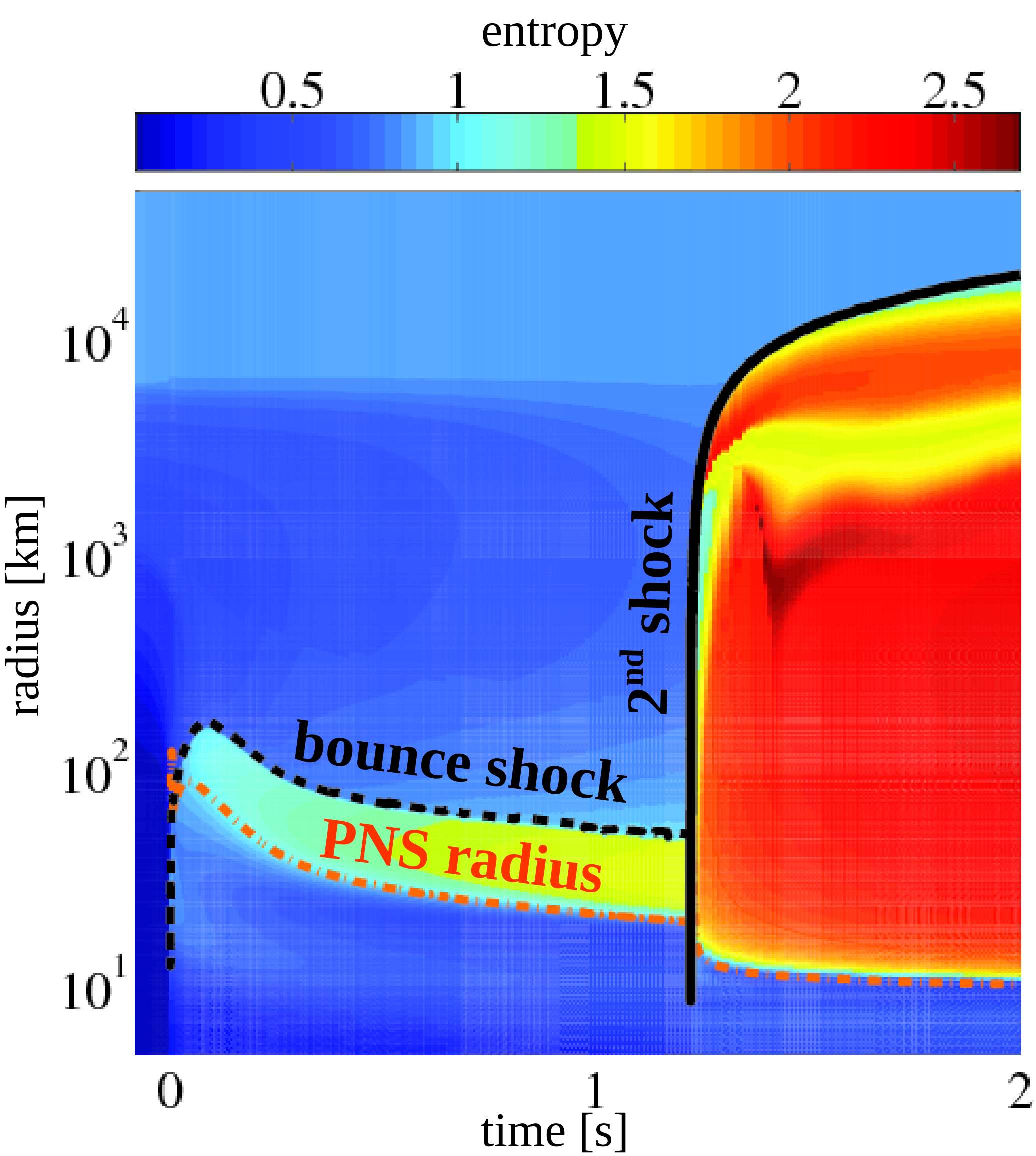
T. Fischer, **NUFB**, and others. Nature Astronomy 2 (2018) no.12, 980-986

1st order PT – Supernovae of massive stars

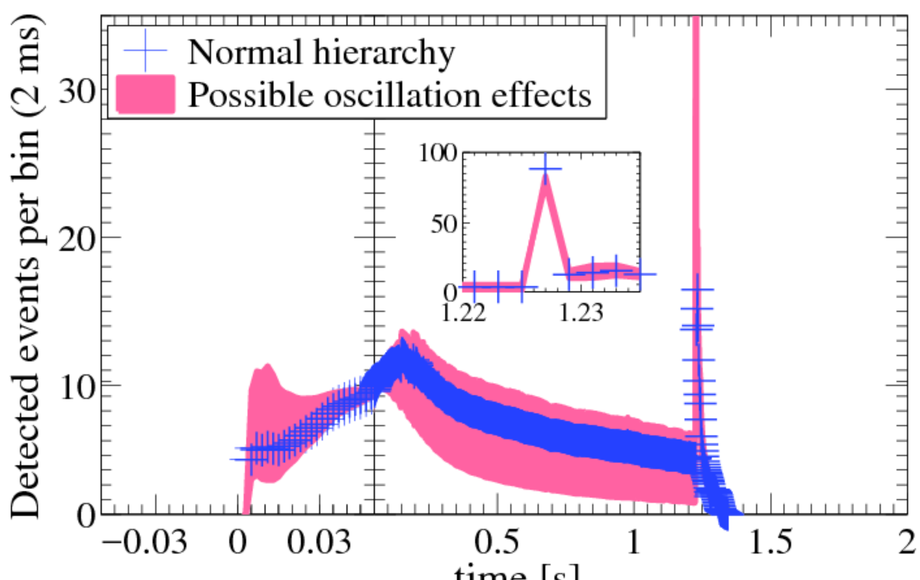


T. Fischer, **NUFB**, and others. Nature Astronomy 2 (2018) no.12, 980-986

1st order PT – Supernovae of massive stars

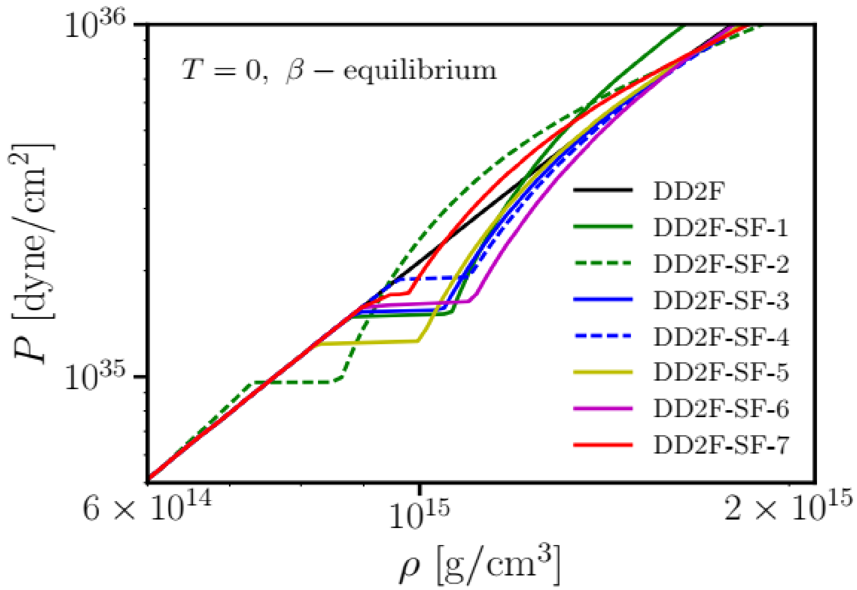


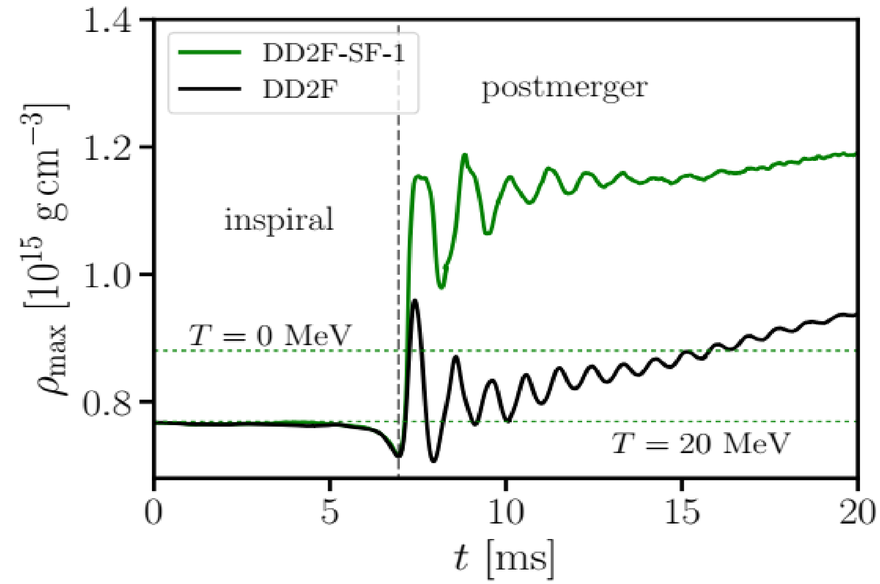
- EoS constructed under consideration of all constraints which are important in astrophysics
- Phase transition releases latent heat to explode “very” massive stars
- Remnant: $2M_\odot$ neutron stars (with quark core) at birth
- Neutrino signal measurable
- Energetic explosion, but almost no nickel

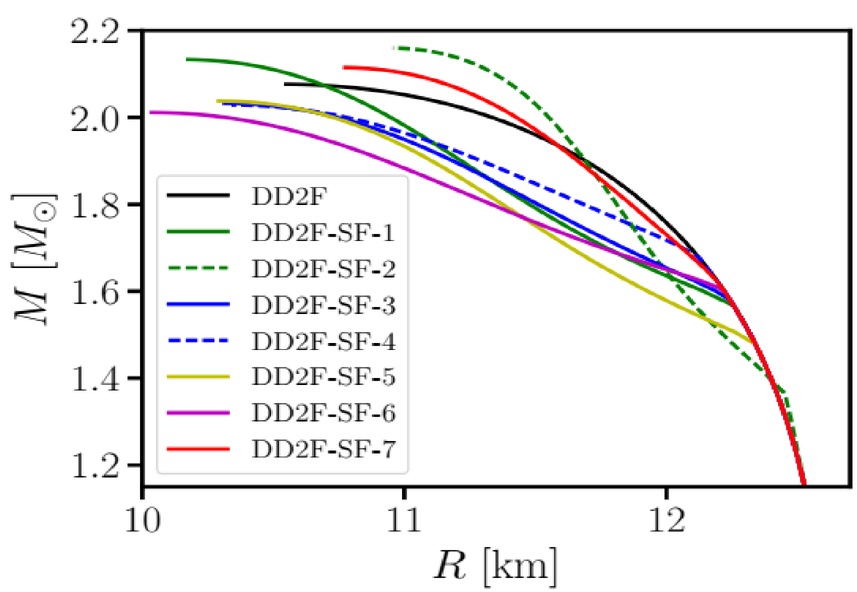


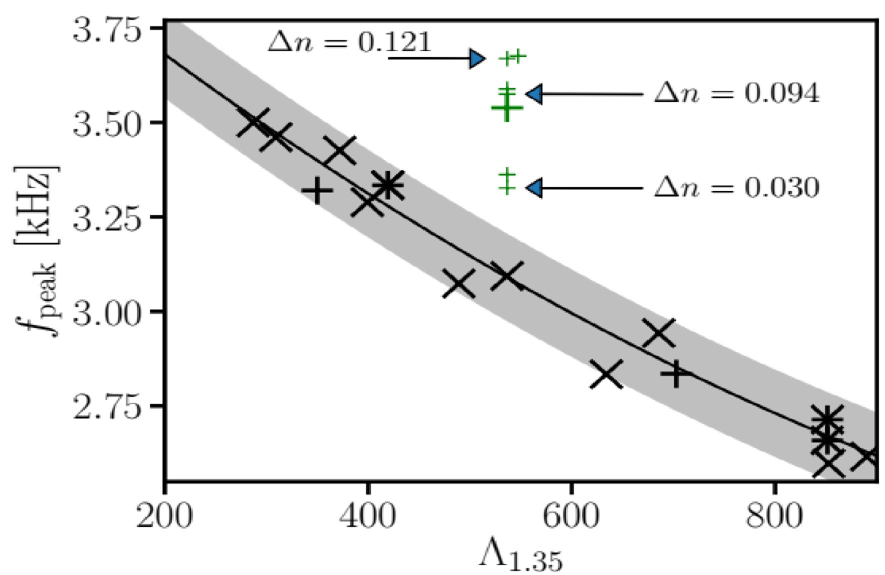
T. Fischer, **NUFB**, and others. Nature Astronomy 2 (2018) no.12, 980-986

1st order PT - Neutron star merger



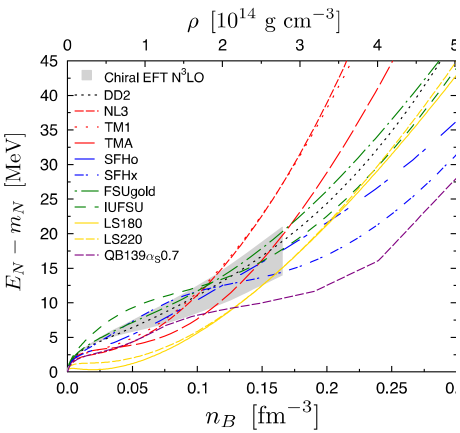






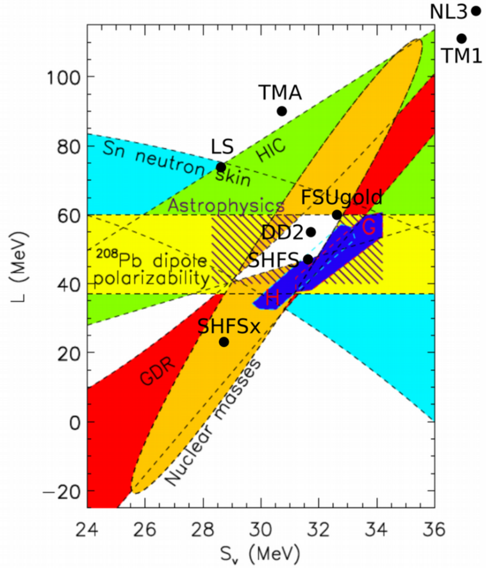
A. Bauswein, **NUFB**, and others, Phys.Rev.Lett. 122 (2019) no.6, 061102

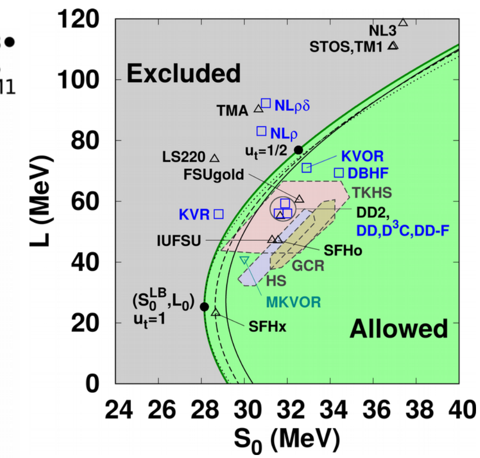
Constraints to consider



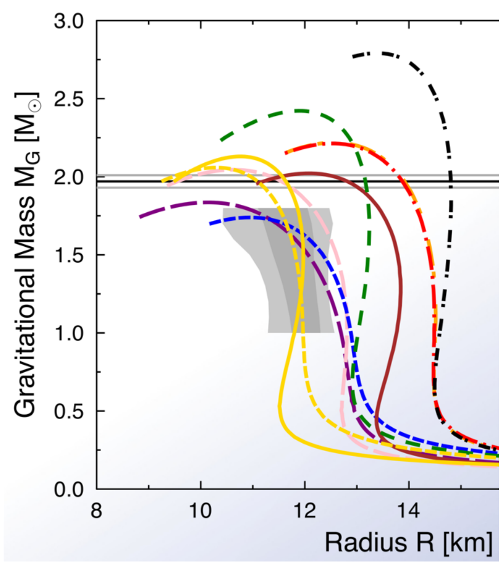
Chiral EFT[1]

Symmetry energy[2]

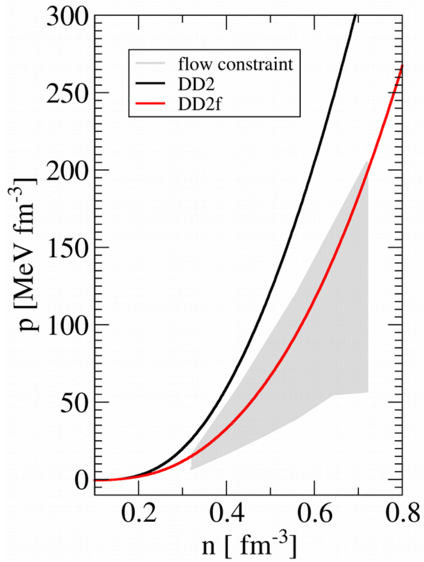




Unitary Constraint[3]



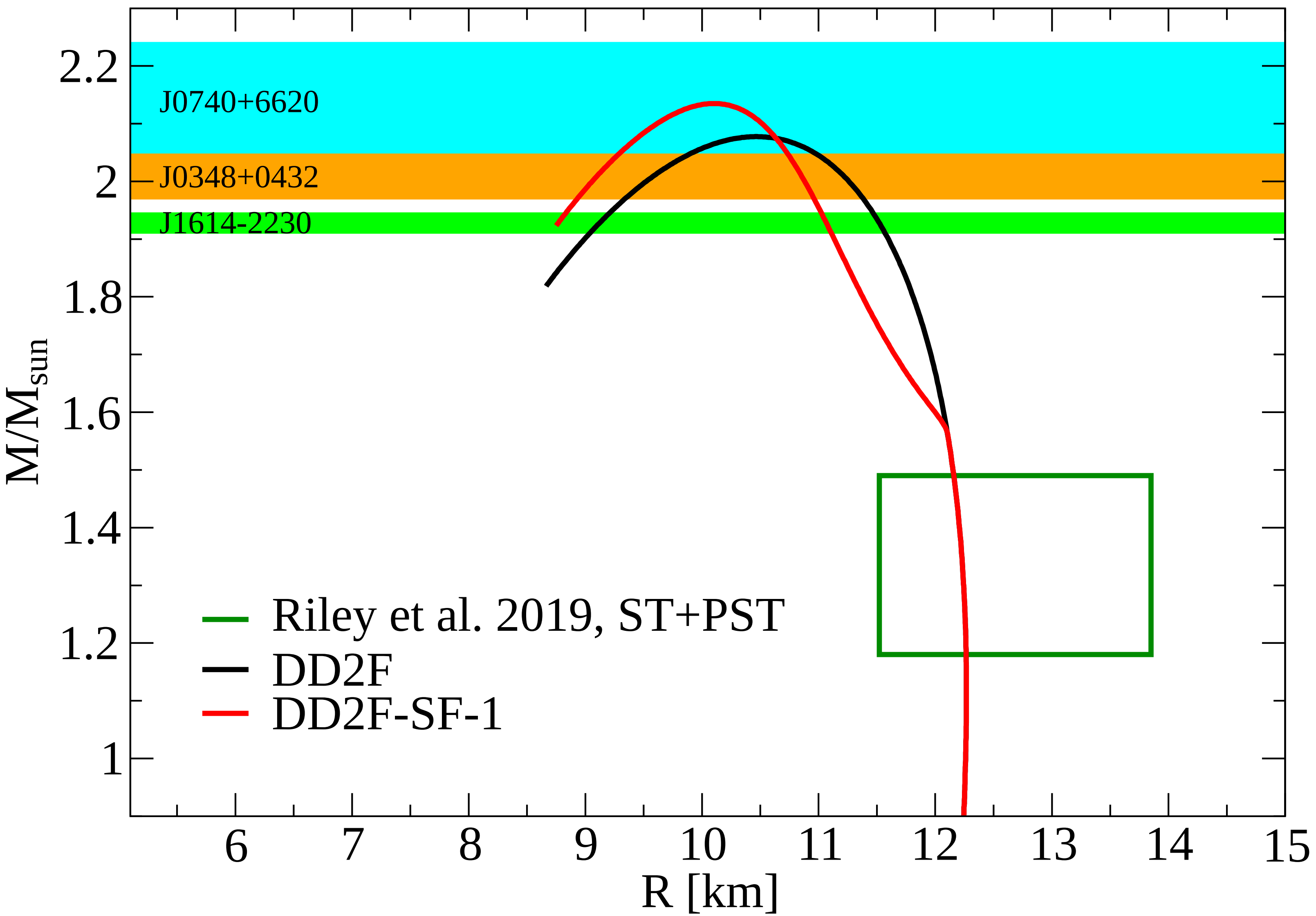
Neutronstar mass[4]



Flow constraint[5]

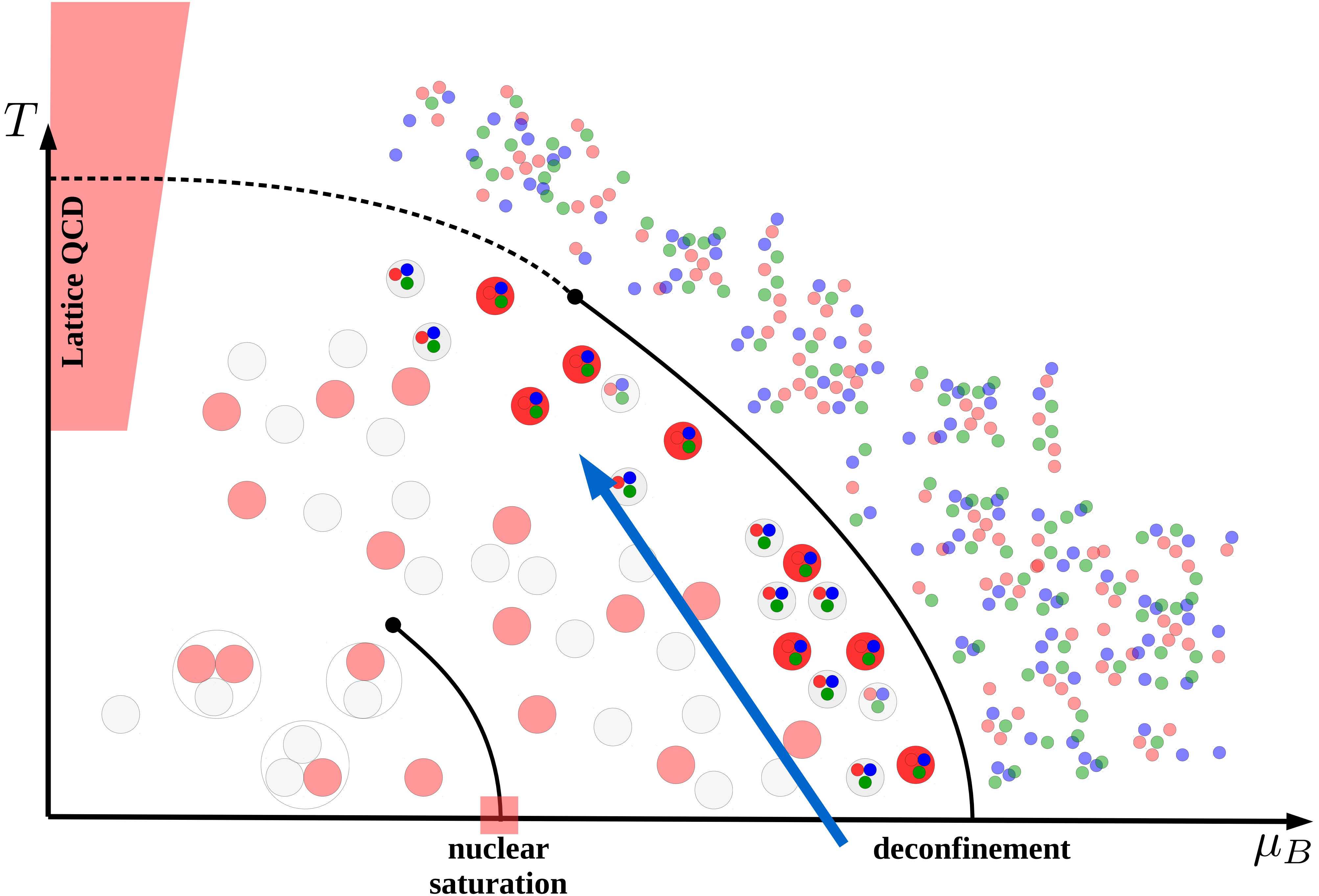
[1] T. Fischer, et. al., (2014) EPJA50, 46
[2] Lattimer & Lim (2013) ApJ 771, 14
[3] Tews, et.al., (2017) ApJ. 848 no.2, 105
[4] J. Antoniadis, et al., (2013) Science 340 6131
[5] P. Danielewicz, et. al., (2002) Science 298 1592-1596

Contraints



2.2
2
1.8
1.6
1.4
1.2
1
J0740+6620
J0348+0432
J1614-2230
M/M$_{sun}$
Riley et al. 2019, ST+PST
DD2F
DD2F-SF-1
6
7
8
9
10
11
12
13
14
15
R [km]

Model for everything?



Relativistic density functionals

Starting with free fermion Lagrangian plus an interaction term, which depends on quark currents

$$\mathcal{L}_{\text{eff}} = \underbrace{\bar{q}\left(\imath\gamma^{\mu}\partial_{\mu} - m\right)q}_{\mathcal{L}_{\text{free}}} - U\left(\bar{q}q, \bar{q}\gamma^{\mu}q\right)$$

Mean field → linear dependence of U on densities is important! → expansion around expectation values

$$U\left(\bar{q}q, \bar{q}\gamma^{\mu}q\right) = U\left(n_{\text{S}}, n_{\text{V}}\right) + \Sigma_{\text{S}}\left(\bar{q}q - n_{\text{S}}\right) + \Sigma_{\text{V}}\left(\bar{q}\gamma^{\mu}q - n_{\text{V}}\right) + \dots$$

derivatives

$$\mathcal{L}_{\text{eff}} \approx \underbrace{\bar{q}\left(\gamma^{\mu}\left(\imath\partial_{\mu} - \Sigma_{\text{V}}\right) - \left(m + \Sigma_{\text{S}}\right)\right)q}_{\mathcal{L}_{\text{quasi}}} - \Theta\left(n_{\text{S}}, n_{\text{V}}\right)$$

$$\Downarrow$$

$$P = g\int \frac{\mathrm{d}^3 p}{(2\pi)^3}\left[\ln\left(1 + e^{-\beta\left(\sqrt{p^2 - M^2} - \tilde{\mu}\right)}\right) + \text{a.p.}\right] - \Theta$$

with

$$n_s = \langle \bar{q}q \rangle \ , \ n_v = \langle \bar{q}\gamma^0 q \rangle \qquad M = m + \Sigma_{\text{S}} \ , \ \tilde{\mu} = \mu - \Sigma_{\text{V}}$$

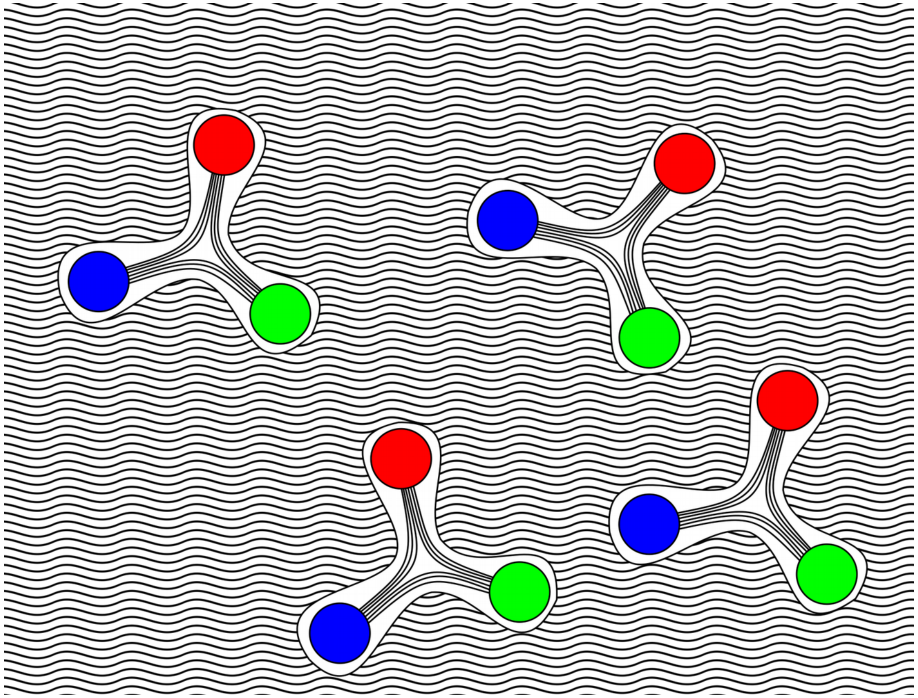
M. Kaltenborn, **NUFB**, D. Blaschke. Phys. Rev. D 2017, 96, 056024

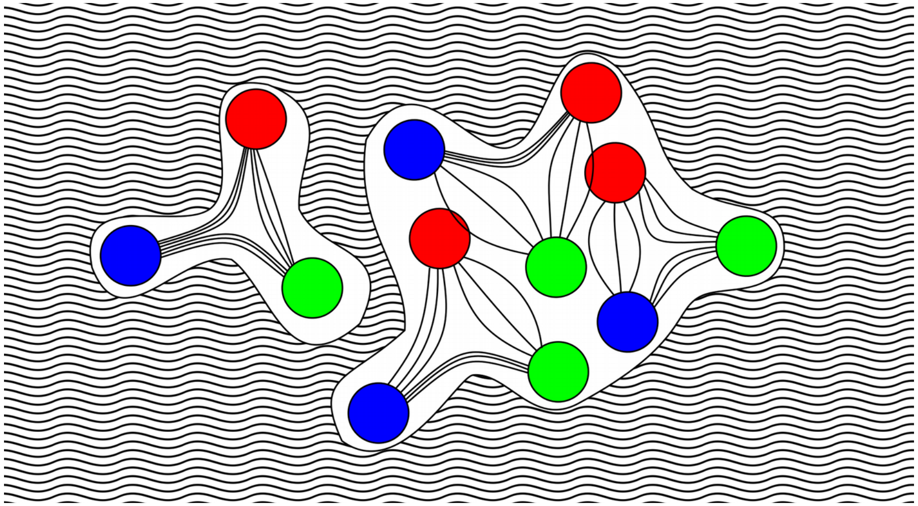
Density functional approach: Stringflip model

Low density

- Color field lines compressed by dual Meissner effect
- String-potential

$$V(r) = \sigma r \sim n^{-1/3}$$





High density

- Dual superconducting vacuum occupied by hadrons
- Pressure on field lines reduced
- Effective string-tension reduced

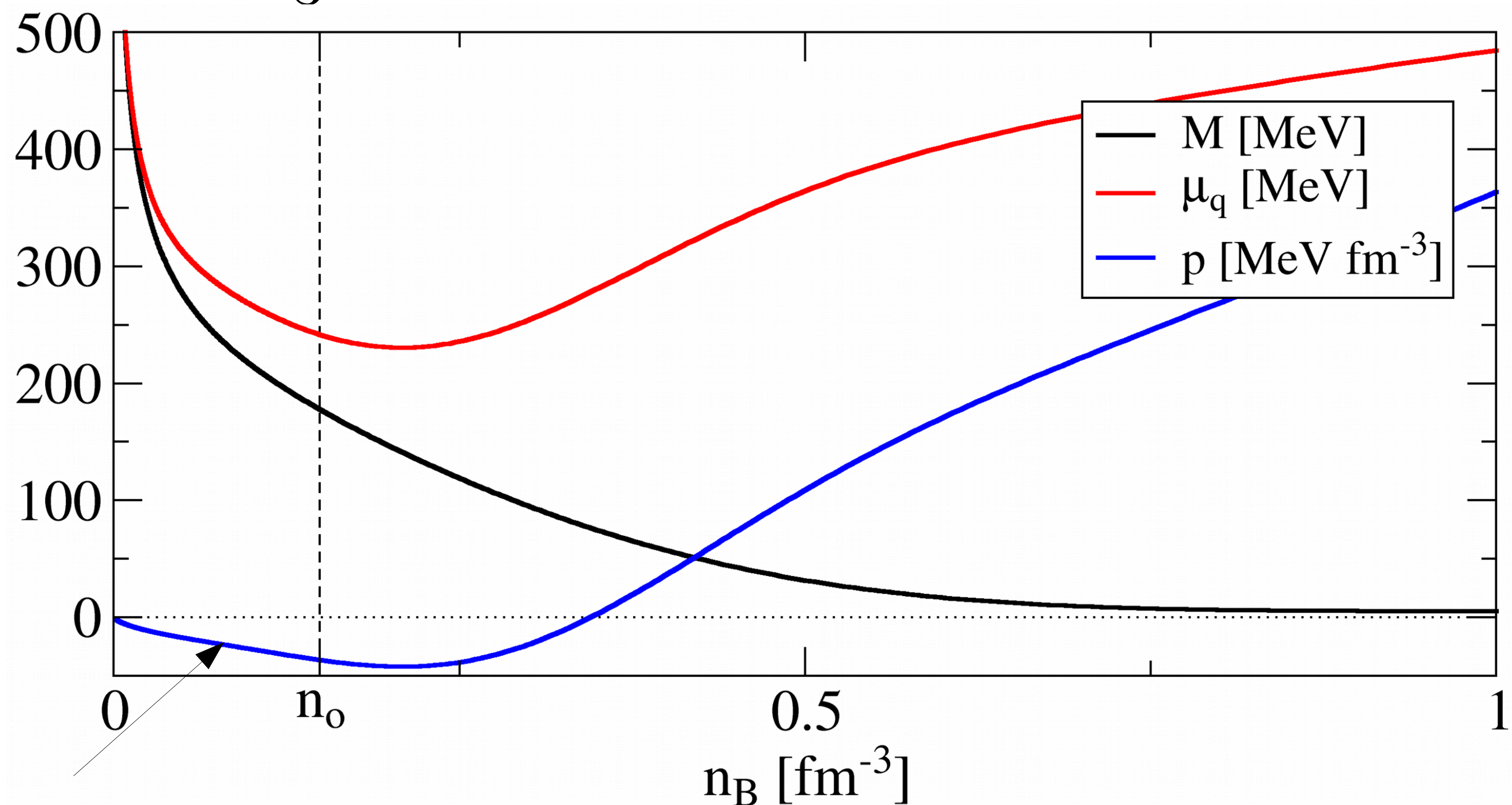
$$\sigma = \Phi \sigma_0$$

G. Ropke, et. al., Phys.Rev. D34 (1986) 3499-3513
M. Kaltenborn, **NUFB**, D. Blaschke, PRD 96, 056024 (2017)

$$U^{\mathrm{SF}}(n_{\mathrm{S}}, n_{\mathrm{V}}) = D(n_{\mathrm{V}}) n_{\mathrm{S}}^{2/3}$$

Mean-field model

$$M_i = m_i + \frac{2}{3} D \cdot (n^{\mathrm{s}})^{-1/3} \qquad D = D_0 \mathrm{e}^{-\alpha(n-n_0)^2}$$



confinement

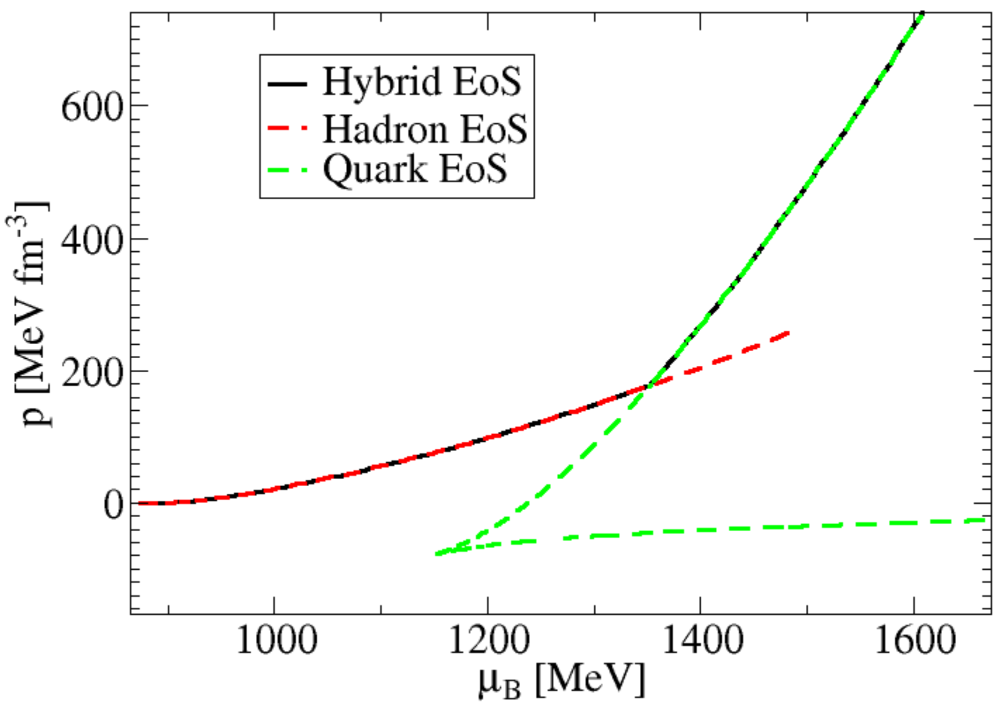
M. Kaltenborn, **NUFB**, D. Blaschke, PRD 96, 056024 (2017)

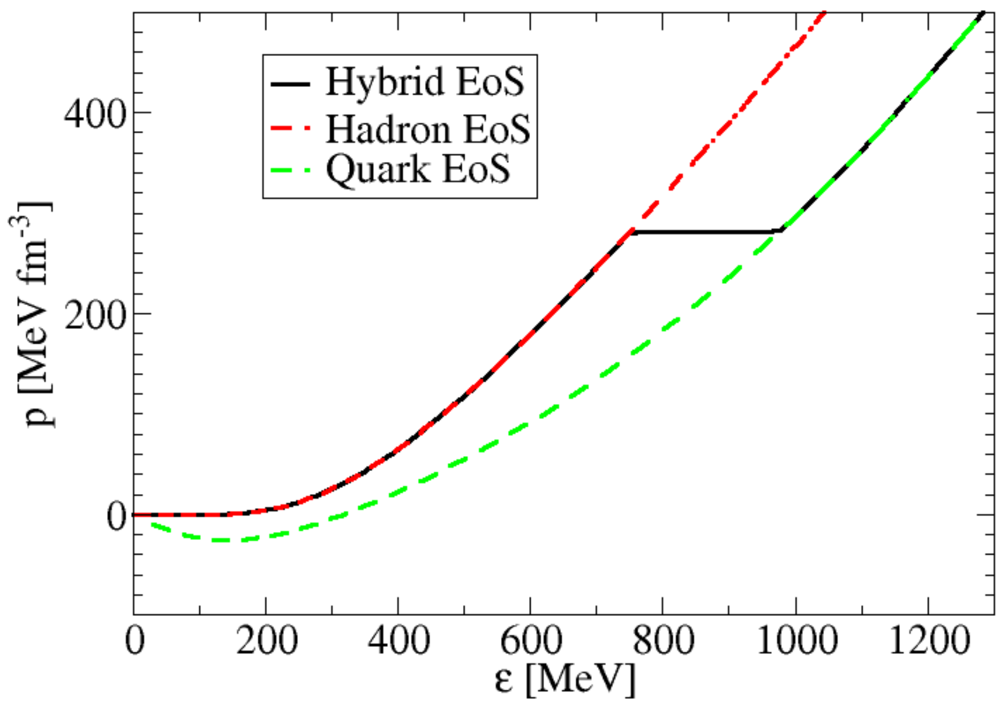
2-phase approach

old

- Two independent models for hadrons and quarks
- Match while fulfilling Gibbs condition for thermal, mechanical and chemical phase equilibrium

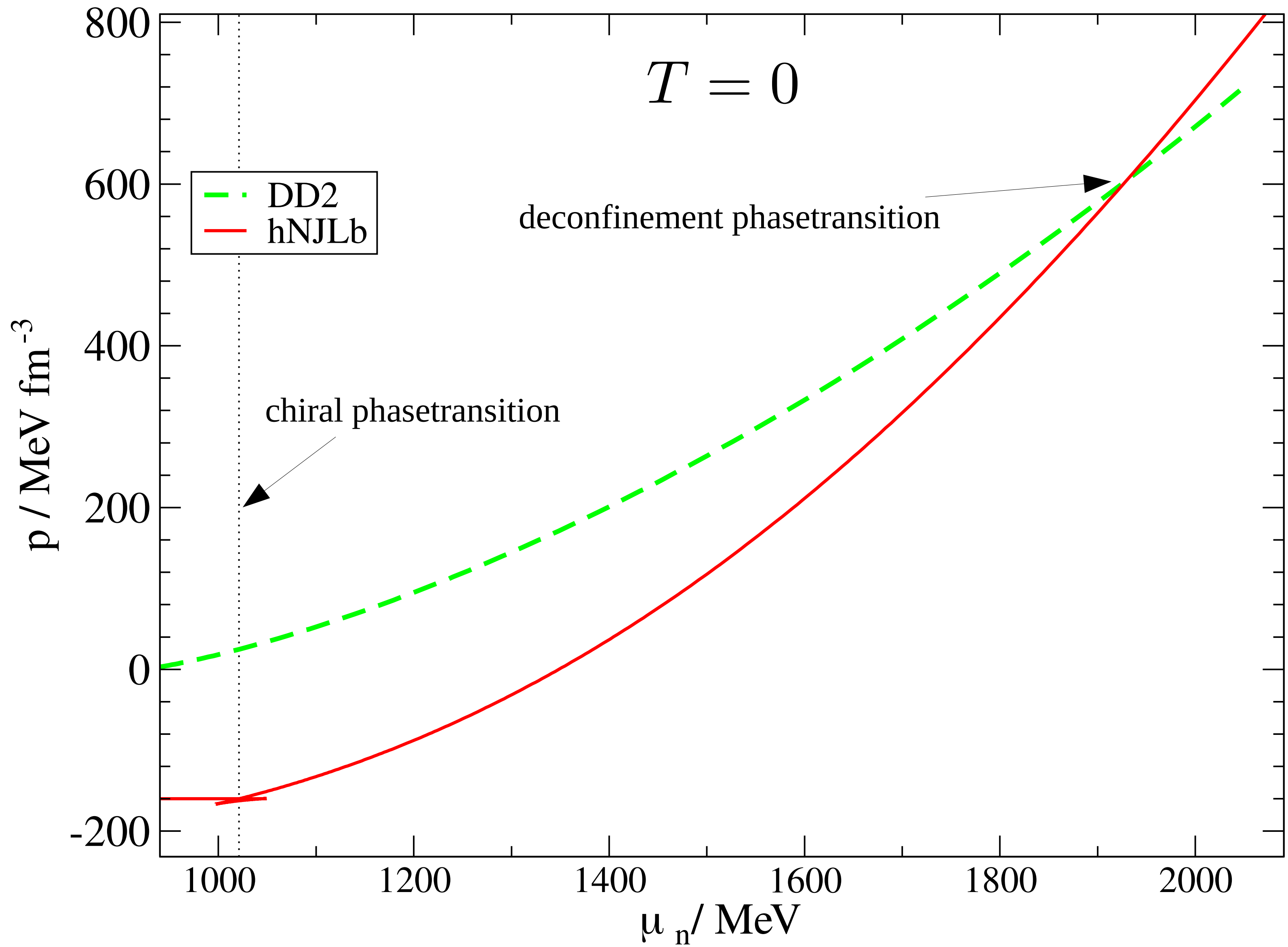
$$T^{\mathrm{H}} = T^{\mathrm{Q}} \quad p^{\mathrm{H}} = p^{\mathrm{Q}} \qquad \mu^{\mathrm{H}} = \mu^{\mathrm{Q}}$$





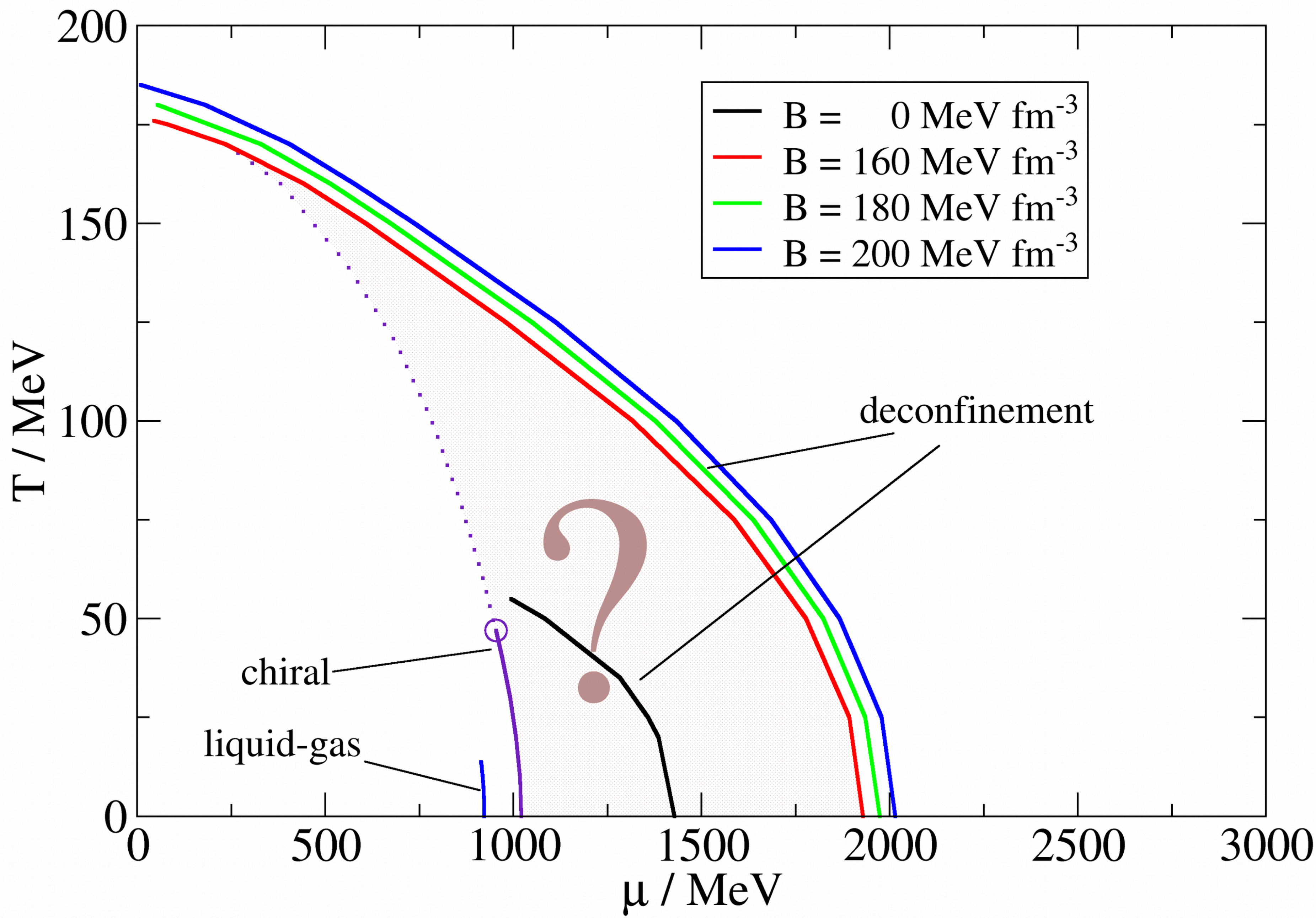
2-phase approach (with NJL)

old



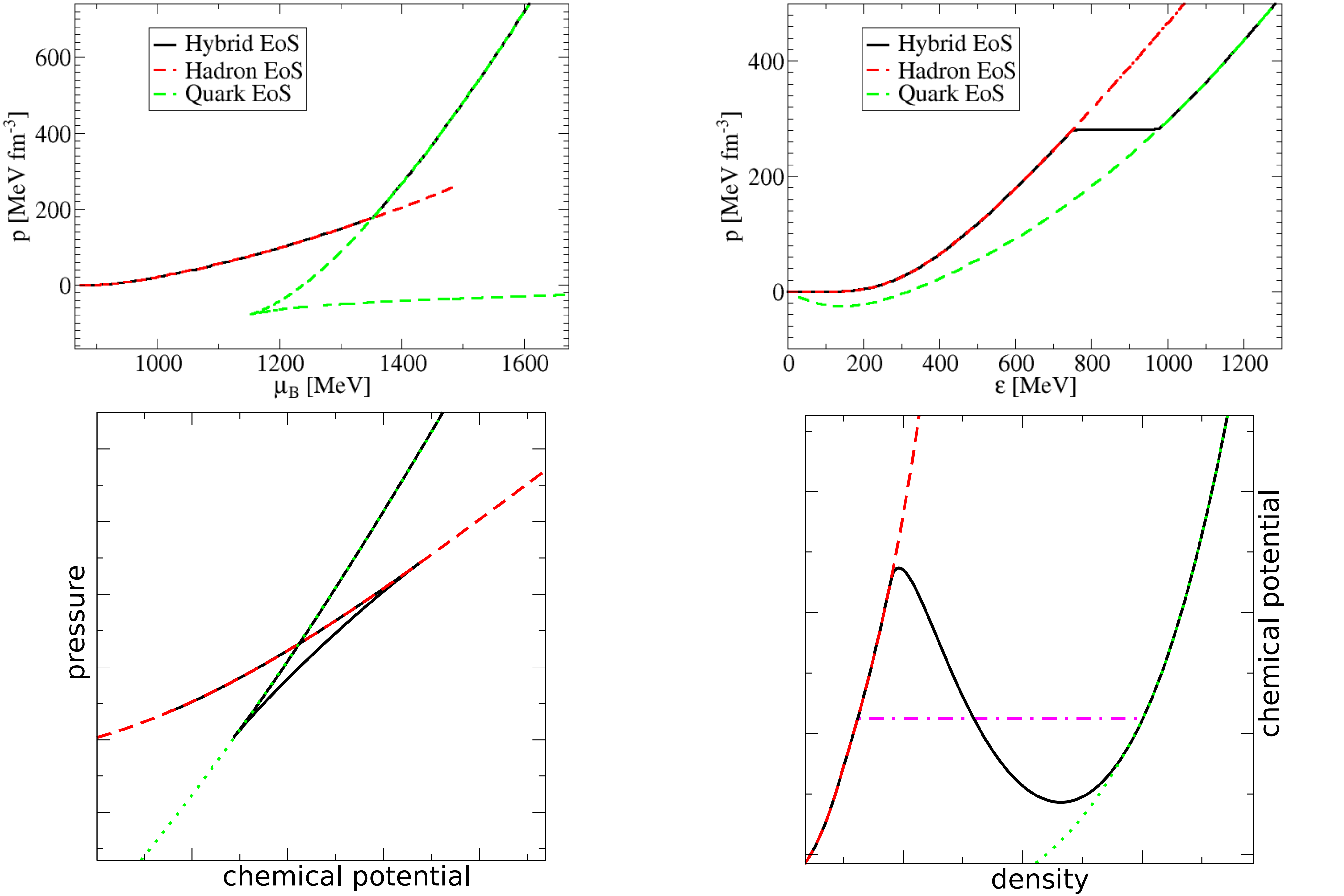
NUFB, D. Blaschke, 2016 J. Phys.: Conf. Ser. 668 012042

2-phase construction with NJL



NUFB, D. Blaschke, 2016 J. Phys.: Conf. Ser. 668 012042

Two-phase approach vs van der Waals wiggle



Cluster expansion

Generating functional formalism by Baym and Kadanoff [1,2]

$$\Omega = -\mathrm{Tr}\ \ln(-G_1^{-1}) - \mathrm{Tr}\Sigma_1 G_1 + \Phi \qquad \text{With} \qquad \Sigma_1(1,1') = \frac{\delta\Phi}{\delta G_1(1,1')}.$$

Can be generalized for a consistent cluster expansion[3]

$$\Omega = \sum_{l=1}^{A}\Omega_l = \sum_{l=1}^{A}\left\{ c_l\left[\mathrm{Tr}\ln\left(-G_l^{-1}\right) + \mathrm{Tr}\left(\Sigma_l\ G_l\right)\right] + \sum_{\substack{i,j\\ i+j=l}} \Phi[G_i, G_j, G_{i+j}]\right\}$$

with

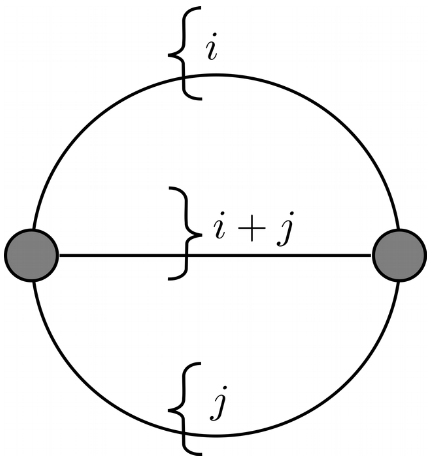
$$\Sigma_A(1\ldots A, 1'\ldots A', z_A) = \frac{\delta\Phi}{\delta G_A(1\ldots A, 1'\ldots A', z_A)}$$

Always sustains full Dyson equation and thermodynamic stability

$$G_A^{-1} = G_A^{0\ -1} - \Sigma_A^{-1} \qquad \frac{\partial\Omega}{\partial G_A} = 0$$

Reduction on generalized sunset diagrams is recommended

$$\Phi[G_i, G_j, G_{i+j}] =$$

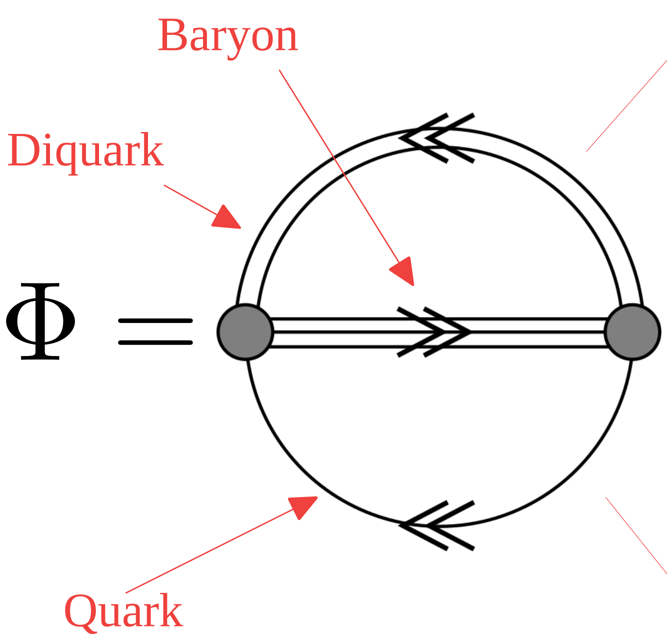


[1]Baym, G.; Kadanoff, L.P. Phys. Rev. 1961, 124, 287–299.
[2]Baym, G. Phys. Rev. 1962, 127, 1391–1401.
[3]**NUFB**, and others, Universe 2018, 4(6), 67

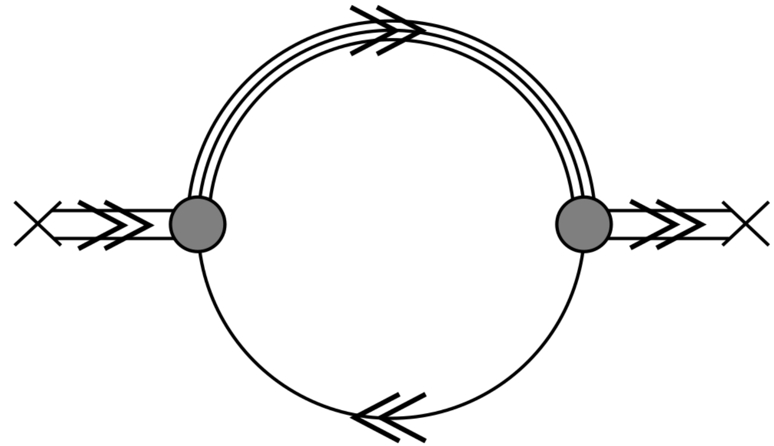
Self energy

$$\Sigma_A = \frac{\delta\Phi}{\delta G_A}$$

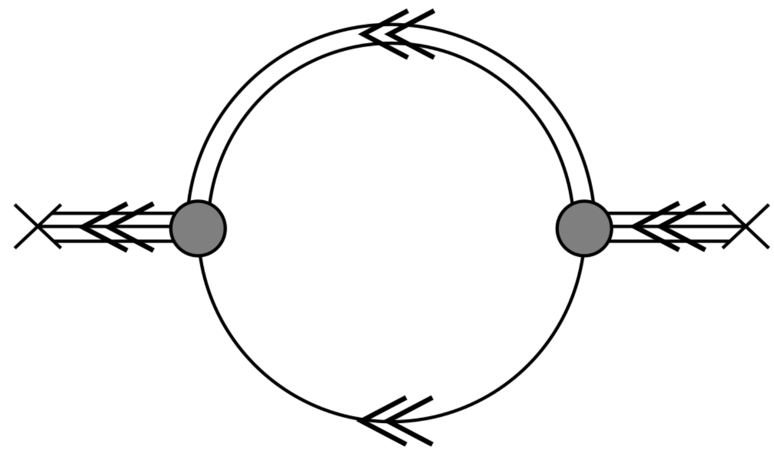
$$\Phi =$$



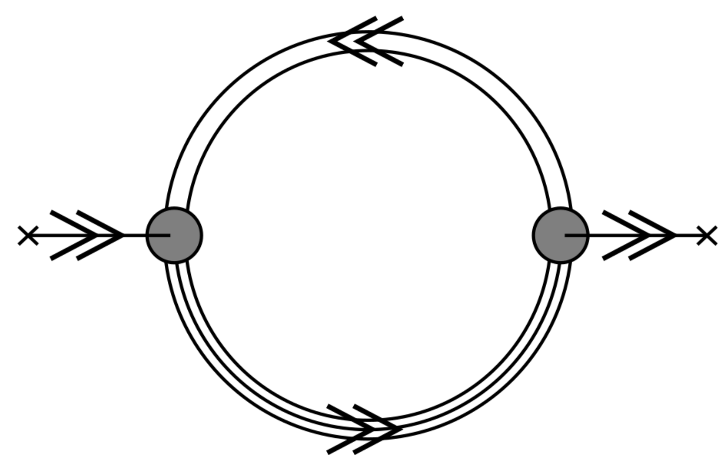
$$\Sigma_{\mathrm{D}} = \frac{\delta\Phi}{\delta G_{\mathrm{D}}} =$$



$$\Sigma_{\mathrm{B}} = \frac{\delta\Phi}{\delta G_{\mathrm{B}}} =$$



$$\Sigma_{\mathrm{Q}} = \frac{\delta\Phi}{\delta G_{\mathrm{Q}}} =$$



$$G_A^{-1} = {G_A^0}^{-1} - \Sigma_A^{-1}$$

Analogy to density functional approach

Phi-derivable approach

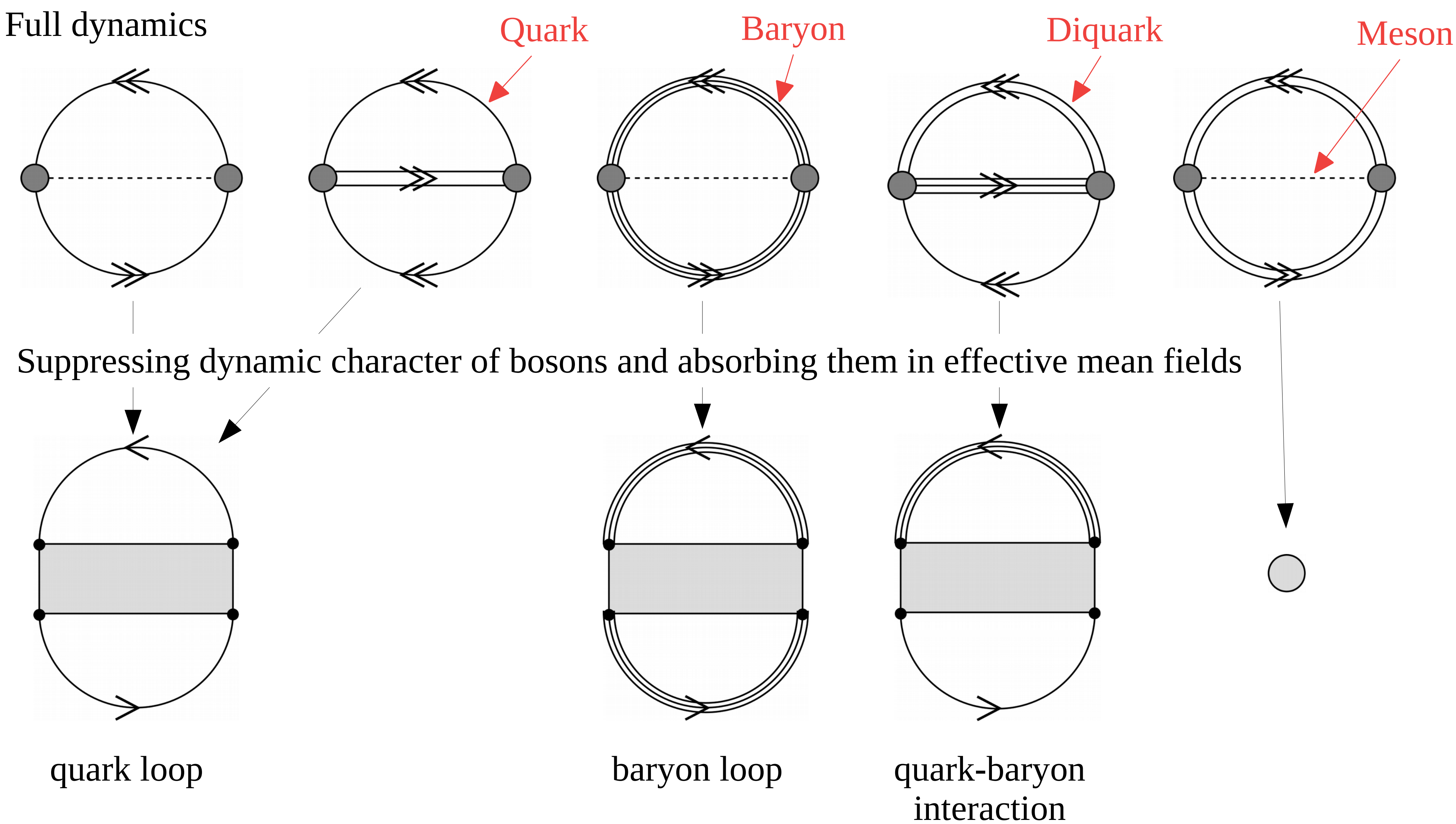
$$\Omega = -\mathrm{Tr}\ \ln(-G_1) - \mathrm{Tr}\Sigma_1 G_1 + \Phi[G_1]$$

Density functional approach

$$\Omega = \Omega^{\mathrm{quasi}} - n_{\mathrm{s}}\Sigma_{\mathrm{s}} - n_{\mathrm{v}}\Sigma_{\mathrm{v}} + U(n_{\mathrm{s}}, n_{\mathrm{v}})$$

$$\Downarrow$$

First step: cluster expansion on basis of densities instead of Green functions (local limit)



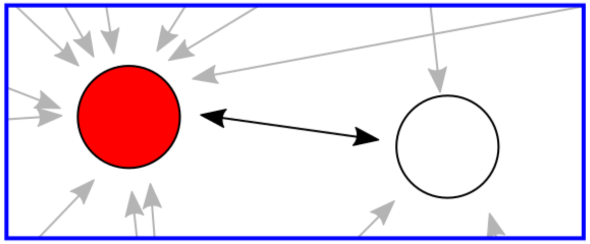
→ **Real self energies → quasi particles**

$$\delta := \arctan \frac{\operatorname{Im} \Sigma}{\operatorname{Re} \Sigma} = n\pi$$

Generalized Beth-Uhlenbeck

Cluster expansion

$$n_{\mathrm{u}} = n_{\mathrm{u}}^{\mathrm{free}} + 2n_{\mathrm{p}}^{\mathrm{free}} + 1n_{\mathrm{n}}^{\mathrm{free}}$$
$$n_{\mathrm{d}} = n_{\mathrm{d}}^{\mathrm{free}} + 1n_{\mathrm{p}}^{\mathrm{free}} + 2n_{\mathrm{n}}^{\mathrm{free}}$$



Chemical equilibrium

$$\mu_i = B_i \mu_{\mathrm{B}} + C_i \mu_{\mathrm{C}}$$

Generalized Beth-Uhlenbeck formula

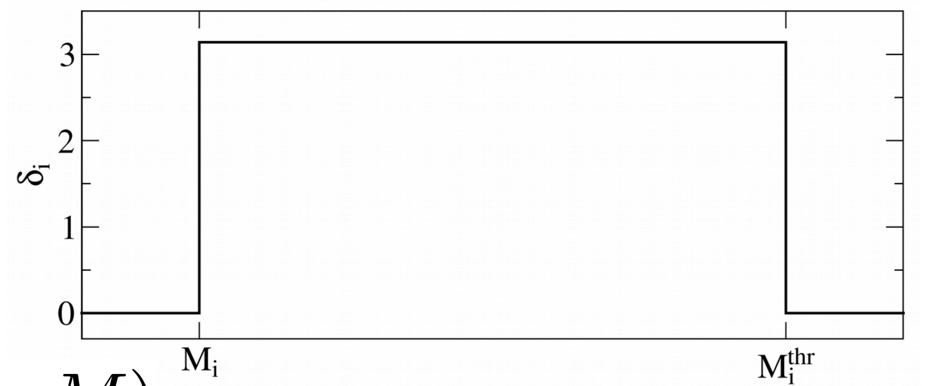
$$n_i^{\mathrm{free}} = g_i \int \frac{\mathrm{d}^3 p}{(2\pi)^3} \int \frac{\mathrm{d}E}{2\pi} f_i(E_i) 2\sin^2\delta_i(E) \frac{\mathrm{d}\delta_i(E)}{\mathrm{d}E}$$

Substitution: $E_{\mathrm{i}} = \sqrt{p^2 + (m_i + S_i)^2} + V_i$

$$n_i^{\mathrm{free}} = g_i \int \frac{\mathrm{d}^3 p}{(2\pi)^3} \int \frac{\mathrm{d}M}{2\pi} f_i\left(\sqrt{p^2 + M^2} + V_i\right) 2\sin^2\delta_i(M) \frac{\mathrm{d}\delta_i(M)}{\mathrm{d}M}$$

Analogy to density functional approach

$$n_i^{\mathrm{free}} = g_i \int \frac{\mathrm{d}^3p}{(2\pi)^3} \int \frac{\mathrm{d}M}{2\pi} f_i\left(\sqrt{p^2+M^2}+V_i\right) 2\sin^2\delta_i(M)\frac{\mathrm{d}\delta_i(M)}{\mathrm{d}M}$$



$$\delta_{i=\mathrm{u,d}}(M) = \pi\,\Theta(M-M_i)$$

$$\delta_{i=\mathrm{p,n}}(M) = \pi\,\Theta(M-M_i)\,\Theta(M_i^{\mathrm{th}}-M)$$

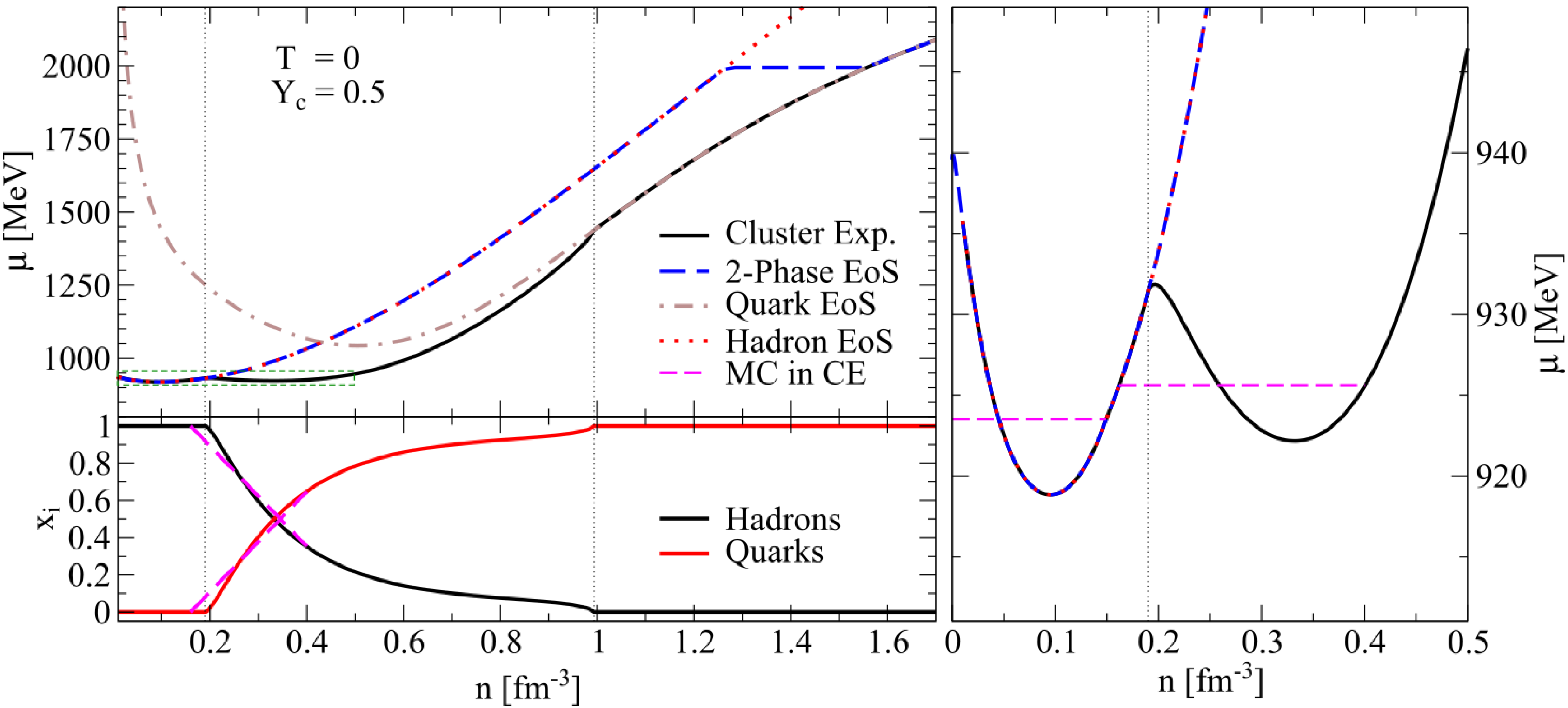
$$n_{i=\mathrm{p,n}} = g_i \int \frac{\mathrm{d}^3p}{(2\pi)^3}\left[f_i(\sqrt{p^2+M_i^2}+V_i) - f_i(\sqrt{p^2+(M_i^{\mathrm{th}})^2}+V_i)\right]\Theta(M_i^{\mathrm{thr}}-M_i)$$

$$= \left(n_{\mathrm{N}}^{\mathrm{qu}} - n_{\mathrm{q}}^{\mathrm{thr}}\right)\Theta(M_i^{\mathrm{thr}}-M_i)$$

$$M_{\mathrm{p}}^{\mathrm{thr}} = 2M_{\mathrm{u}} + 1M_{\mathrm{d}}$$

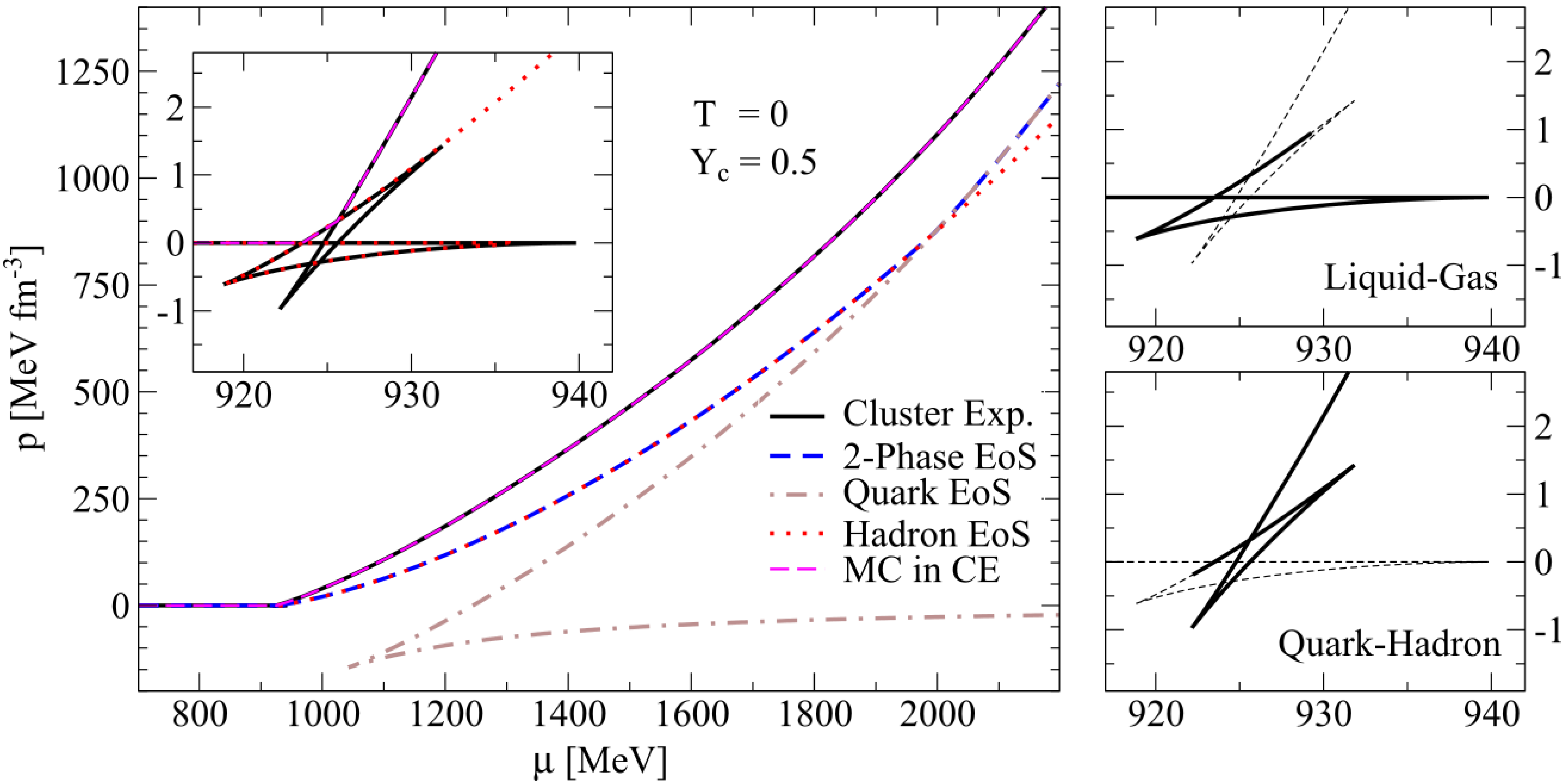
$$M_{\mathrm{n}}^{\mathrm{thr}} = 1M_{\mathrm{u}} + 2M_{\mathrm{d}}$$

Cluster-expansion of Quarks



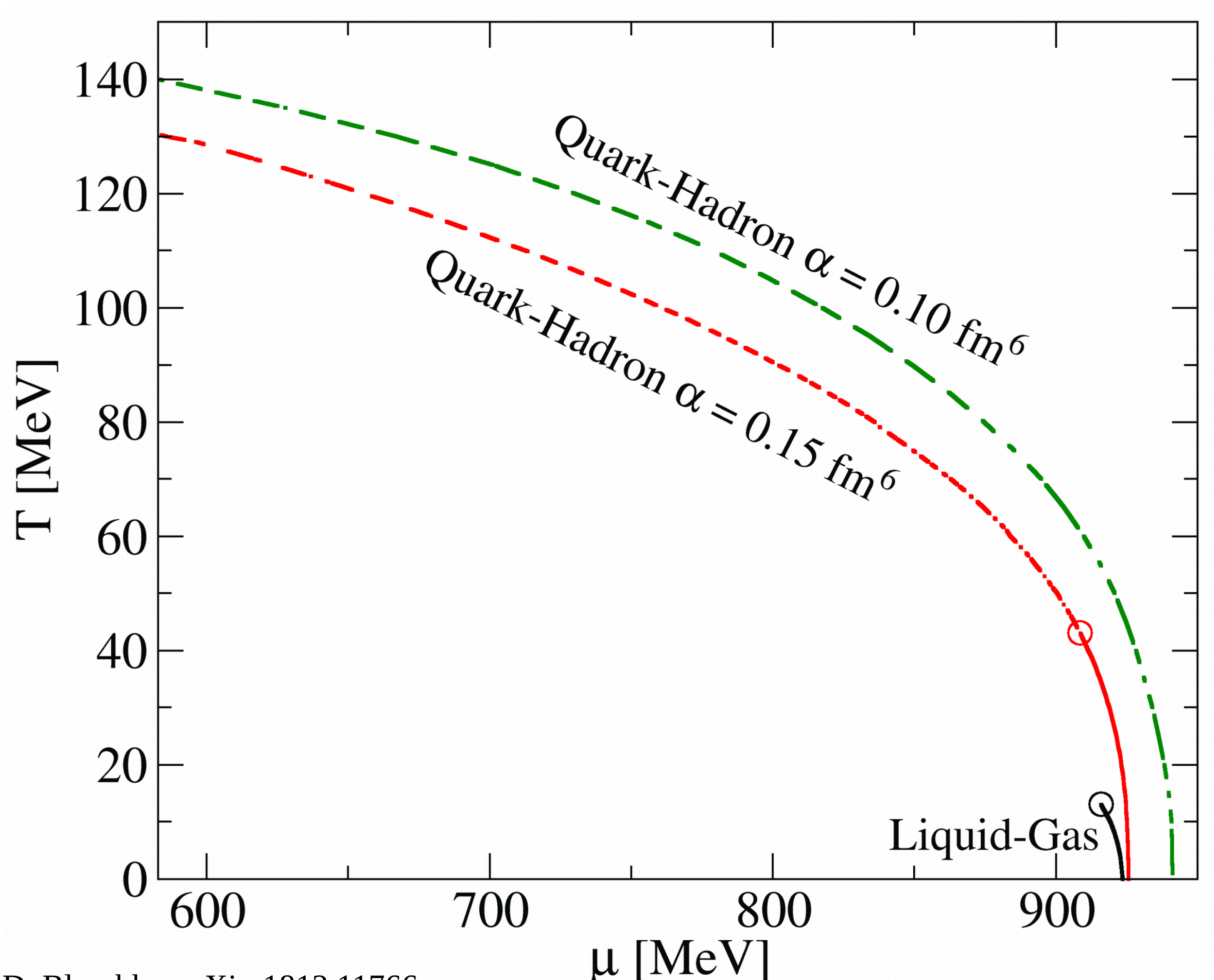
NUFB, D. Blaschke, arXiv:1812.11766

Cluster-expansion of Quarks



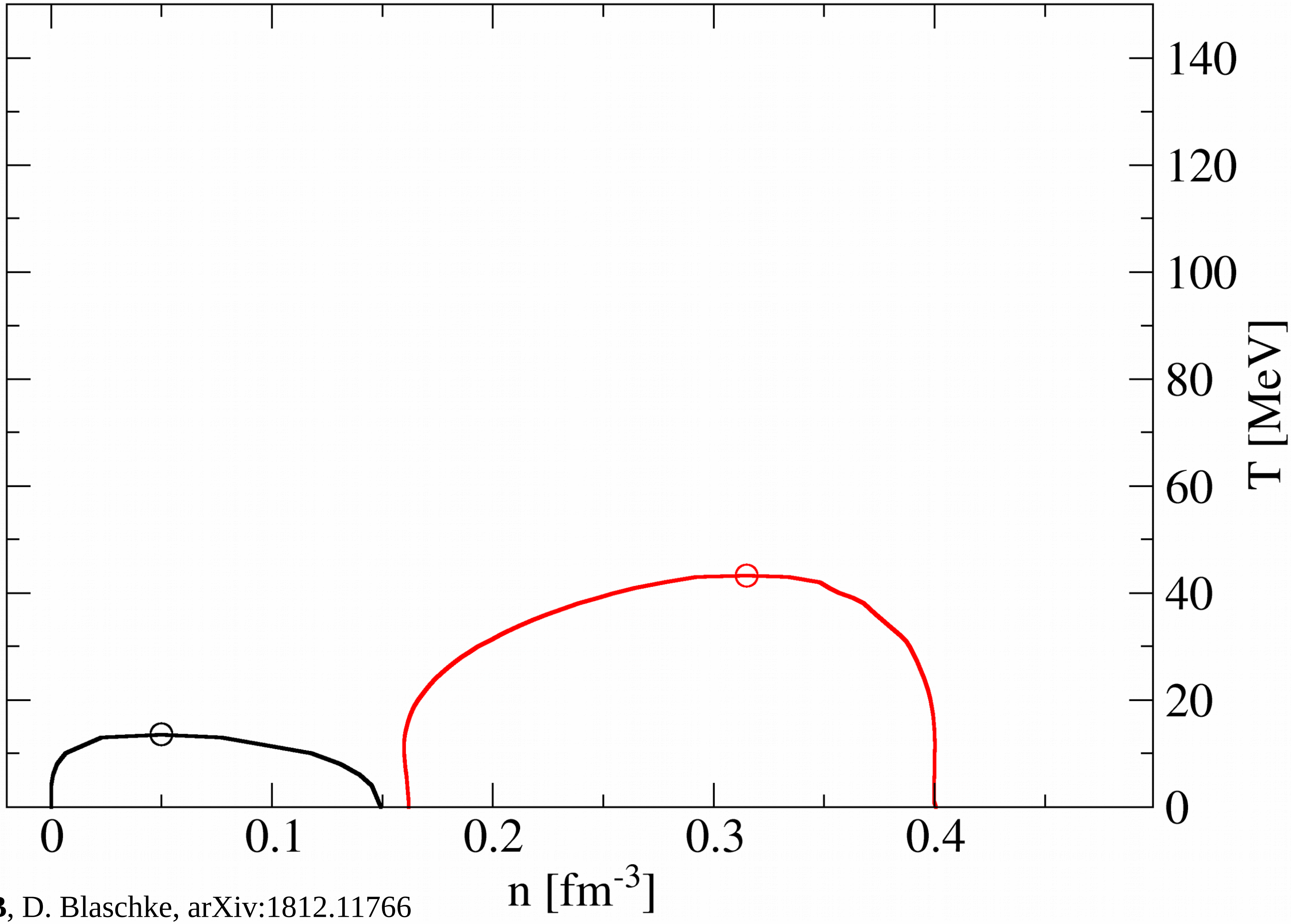
NUFB, D. Blaschke, arXiv:1812.11766

Cluster-expansion



NUFB, D. Blaschke, arXiv:1812.11766

Cluster-expansion

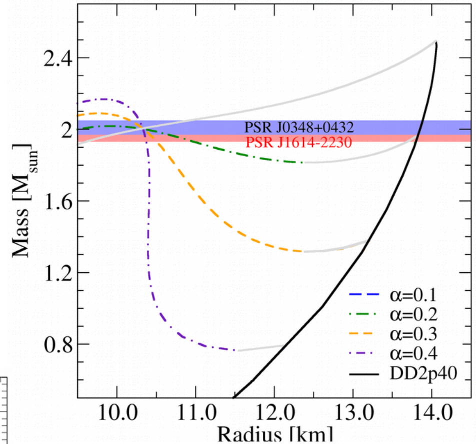


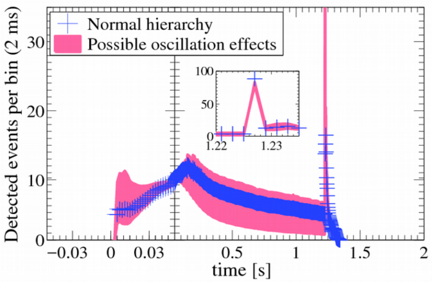
NUFB, D. Blaschke, arXiv:1812.11766

~~Outline~~ Summary

Possible signals of 1st – order phase transitions.

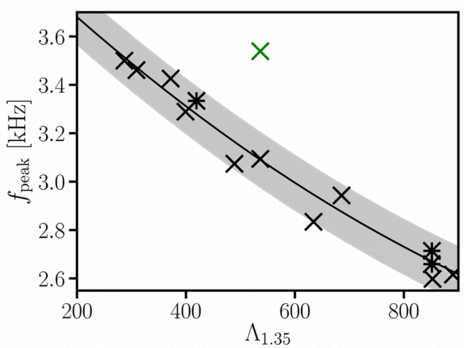
Neutron star configurations

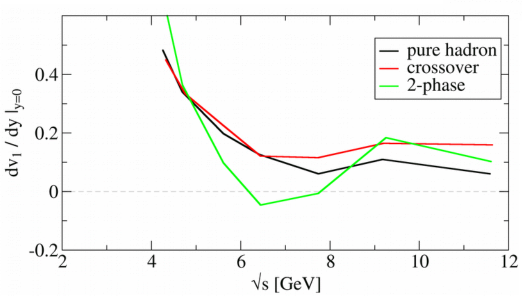




Supernova explosions of 50Ms stars

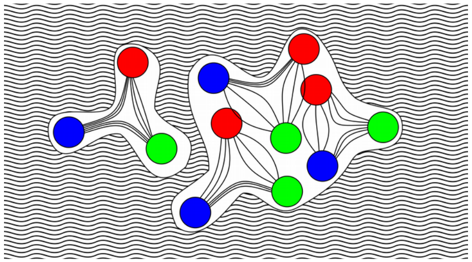
Binary neutron star mergers





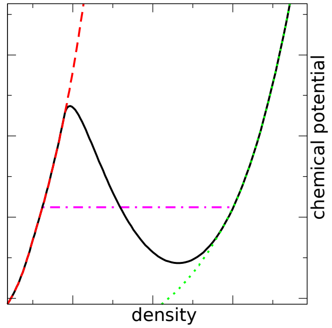
Heavy-Ion Collisions

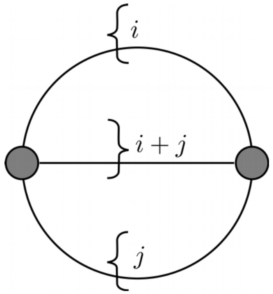
Unified description of the equation of state.



density functional theory

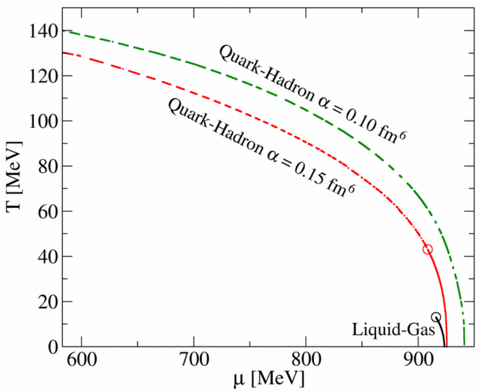
construction of phase transitions





Phi-derivable formalism

current status



Last Slide

Conclusions

- Possible scenarios are explored in which a 1st order phase transition is detectable in
    - neutron star configurations
    - neutrino signals of supernova explosions
    - Gravitational wave signal of binary neutron star mergers
    - Flow data of heavy-ion collision experiments
- Astrophysical objects and HIC collisions are based on the same physics of strongly interacting many-particle systems
- Hadrons are bound states of quarks and should be treated as such
- A cluster virial expansion within the Beth-Uhlenbeck formalism can be derived from the PHI-derivable approach
- Initial reduction to mean field already results in a consistent description of Quark-Hadron phase transition

Outlook

- Density functional with chiral physics
- Reproduction of Lattice results
- Continuum contributions and substructure effects
- Cluster mean field

Thank you!

Collaboration

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