Clusters in Nuclear Matter and in Heavy Nuclei

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TECHNISCHE UNIVERSITÄT DARMSTADT

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Nuclear Equation of State and Neutron Stars

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Outline

- **Introduction**
- **Clusters in Nuclear Matter**
  - Theoretical Methods and Applications
  - Descriptions in Different Ranges of Density and Temperature
  - Unified Description with Generalized Relativistic Density Functional
  - Compact Star Matter
- **Clusters in Heavy Nuclei**
  - Application of gRDF to Heavy Nuclei
  - α Particle Correlations at Surface of Sn Nuclei
  - Consequences
- **Conclusions**

Introduction

Correlations and Clusters

- essential in strongly interacting systems
- different types of correlations
  - two-, three-, . . . , many-body correlations
  - short-range/long-range correlations
  - configuration-space/momentum-space correlations (e.g. ^{2}H vs. NN pairing)

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- different systems
    - nuclear matter (different types of pairing, light clusters in dilute matter)
    - light nuclei (clusters in structure and nuclear reactions)
      ⇒ development of structure models with clusters as explicit degrees of freedom (D. M. Brink's α-particle model, resonating group method, . . .)

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    ⇒ development of structure models with clusters as explicit degrees of freedom (D. M. Brink's α-particle model, resonating group method, . . .)
- problems
  - effective theoretical description of cluster formation and dissolution in nuclear matter
  - effects of clusters in heavy nuclei

Clusters in Nuclear Matter

Theoretical Methods and Applications

- **ab-initio approaches**
  - realistic nuclear interaction
    (e.g. fitted to nucleon-nucleon scattering data)
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- **applications**
  - equation of state/phase diagram
    ⇒ description of astrophysical objects
    (neutron stars, their mergers, core-collapse supernovae)
  - simulation of heavy-ion collisions

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- **exact low-density limits**
  - finite temperatures: virial equation of state
  - zero temperature: Lee-Yang type expansions

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  - compact star matter
    (with leptons and Coulomb interaction, charge-neutral system)

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  $\Rightarrow$ different properties and phase diagrams
- **application in astrophysical simulations**
  - wide ranges in density, temperature, isospin asymmetry
  - combination of various effective methods
  - energy density functionals with clusters

  $\Rightarrow$ unified equation of state

Description at Very Low Densities

- **finite temperature, exact limit**
  $\Rightarrow$ **virial equation of state (VEOS)**
  (E. Beth and G. Uhlenbeck, Physica 3(1936) 729, Physica 4 (1937) 915;
  C. J. Horowitz and A. Schwenk, NPA 776 (2006) 55, ...)
  - **expansion** of pressure in powers of fugacities $z_i = \exp(\mu_i/T)$

$$p = TV\left(\sum_i \frac{g_i}{\lambda_i^3} z_i + \sum_{ij} \frac{b_{ij}}{\lambda_i^{3/2}\lambda_j^{3/2}} z_i z_j + \ldots\right) \quad \text{with thermal wavelength} \quad \lambda_i = \left[2\pi/(m_i T)\right]^{1/2}$$

and virial coefficients g_i, b_{ij}, ... $\Rightarrow$ **limitation** $n_i \lambda_i^{-3} \ll 1$

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    and virial coefficients $g_i, b_{ij}, \ldots \Rightarrow$ **limitation** $n_i \lambda_i^{-3} \ll 1$
  - only two-body correlations relevant at lowest densities, encoded in

    $$b_{ij} = \frac{1+\delta_{ij}}{2} \frac{\lambda_i^{3/2}\lambda_j^{3/2}}{\lambda_{ij}^3} \int dE \, \exp\left(-\frac{E}{T}\right) D_{ij}(E) \pm \delta_{ij} \frac{g_i}{2^{5/2}} \qquad \lambda_{ij} = \left\{2\pi/[(m_i+m_j)T]\right\}^{1/2}$$

    with 'density of states' $D_{ij}(E) = \sum_k g_k^{(ij)} \delta(E - E_k^{(ik)}) + \sum_l \frac{g_l^{(ij)}}{\pi} \frac{d\delta_l^{(ij)}}{dE}$

    $\Rightarrow$ **contributions from bound states and continuum**,
    depends only on bound-state energies $E_k^{(ik)}$ and phase shifts $\delta_l^{(ij)}$ (experiment!)
    (not independent! Levinson theorem)

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- **simplification of VEOS**
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- **extension of VEOS**
  ⇒ **generalized (cluster) Beth-Uhlenbeck approach**
  (G. Röpke, L. Münchow, and H. Schulz, NPA 379 (1982) 536,
  M. Schmidt, G. Röpke, and H. Schulz, Ann. Phys. 202 (1990) 57,
  G. Röpke, N.-U. Bastian et al., NPA 897 (2013) 70,
  N.-U. Bastian et al., arXiv:1804.10178)
  - quantum statistical description with thermodynamic Green's functions
  - part of interaction included in self-energies of quasiparticles
  - modified second virial coefficient
    ⇒ dependence on particle-pair momentum,
    correction factor in continuum contribution

  ⇒ suppression of cluster formation with increasing density

Description at Low Densities – Low-Temperature Limit I

- **pure neutron matter**

  exact limit at $T = 0 \Rightarrow$ Lee-Yang type **expansion**

  (T. D. Lee and C. N. Yang, Phys. Rev. 105 (1957) 1119,

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  $$\frac{E}{N} = \frac{3}{5}\frac{k_n^2}{2m_n}\left[1 + \frac{10}{9\pi}\zeta + \frac{4}{21\pi^2}(11 - 2\ln 2)\zeta^2 + ...\right]$$

  $\zeta = a_{nn}k_n$ with s-wave scattering length a_{nn}
  and Fermi momentum k_n
  $\Rightarrow$ small radius of convergence ($a_{nn} \approx -18.8$ fm)

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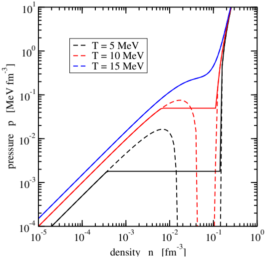
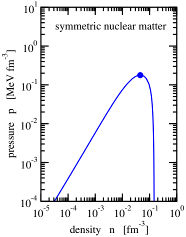
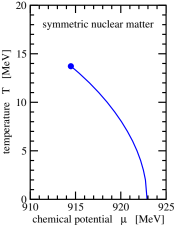
- **nuclear matter**

  condensation of (bosonic) clusters expected,
  does not stop at α condensation
  $\Rightarrow$ increase of cluster size
  (no Coulomb interaction → no size limit)
  $\Rightarrow$ coexistence of low-/high-density phases
  ('liquid-gas phase transition')
  $\Rightarrow$ effect on symmetry energy



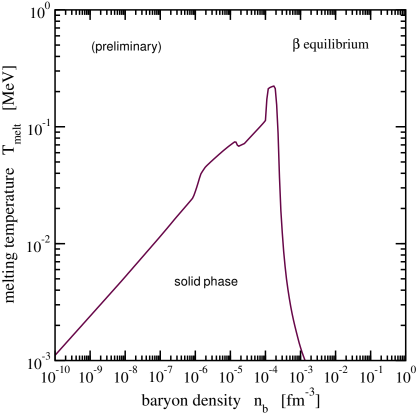




Description at Low Densities – Low-Temperature Limit II

- **compact star matter**
  - charge neutral system (nucleons + leptons) in β equilibrium
  - phase transition to solid crystal (Coulomb correlations essential), driven by plasma parameter $\Gamma = Z_{\text{ion}}^{5/3} e^2/(a_e T) \approx 175$ with $a_e = [3n_e/(4\pi)]^{1/3}$

compact star matter



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  - geometric picture (finite size of particles)
    $\Rightarrow$ **excluded-volume mechanism**
    - applications to compact star matter
      (M. Hempel and J. Schaffner-Bielich, NPA 837 (2010) 210;
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    - generalized formulation, different interpretation
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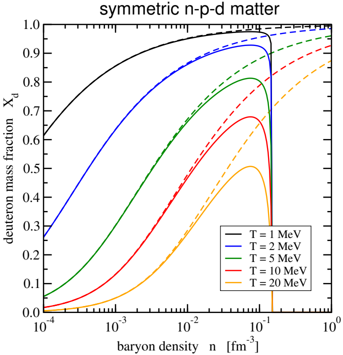

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    - generalized formulation, different interpretation
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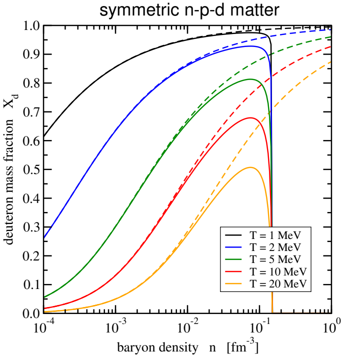
  - medium modification of cluster properties
    $\Rightarrow$ **mass shifts**
    - action of Pauli principle $\Rightarrow$ blocking of states
    - density, temperature, momentum dependence



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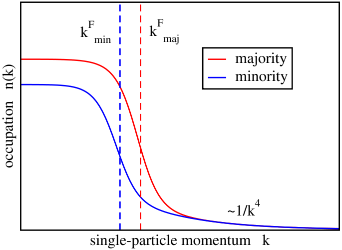

experiments: nucleon knockout from nuclei in inelastic electron scattering

(O. Hen et al. (CLAS Collaboration), Science 346 (2014) 614)

$\Rightarrow$ no sharp cut-off, high-momentum tail

Unified Description at All Densities and Temperatures

- energy density functionals: various types
  - nonrelativistic (e.g. Skyrme, Gogny) or relativistic/covariant
  - often derived from mean-field models in different approximations (Hartree, Hartree-Fock, Hartree-Fock-Bogoliubov)
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- **here: generalized relativistic density functional (gRDF)**
  - nucleons, clusters (= many-body correlations) and mesons as degrees of freedom in grand-canonical ensemble
  - minimal coupling of nucleons (free or bound) to mesons
  - quasiparticles with effective mass $m_i^* = m_i - S_i$ and effective chemical potential $\mu_i^* = \mu_i - V_i$
  - effective interaction by meson exchange with density dependent couplings $\Rightarrow$ vector (V_i) and scalar (S_i) potentials
  - medium dependent masses of clusters

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    - bindung energies from mass tables
      $\Rightarrow$ shell effects included, full distribution, not only average heavy nucleus
  - two-nucleon scattering states
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- **excited states of nuclei**
  temperature dependent degeneracy factors with density of states

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- **medium dependence of particle properties**
  quasiparticles with mass shifts (coupling to mesons, effective Pauli principle)

(S. Typel et al., Phys. Rev. C 81 (2010) 015803; M. D. Voskresenskaya et al., Nucl. Phys. A 887 (2012) 42; M. Hempel et al., Phys. Rev. C 91 (2015) 045805; S. Typel, arXiv:1504.01571; H. Pais et al., arXiv:1612.07022; H. Pais et al. Nuovo Cim. C 39 (2016) 393; S. Typel, J. Phys. G 45 (2018) 114001)

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  - light and heavy nuclei
  - nucleon-nucleon correlations in continuum
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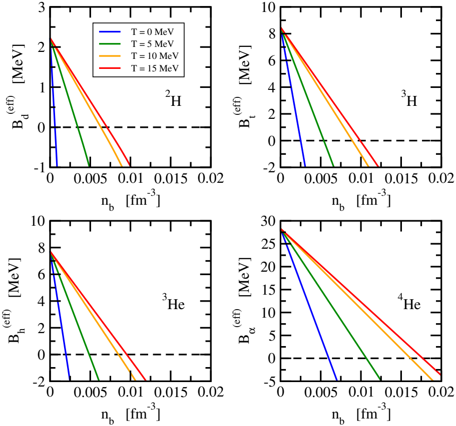
  - electromagnetic shift Δm_i^{Coul} (in compact star matter)
    - electron screening of Coulomb field
      $\Rightarrow$ increase of binding energies

Mass Shifts II

- light nuclei and NN scattering states
  - **parametrization from G. Röpke**
    simplified and modified for high densities and temperatures
  - scattering states:
    mass shifts as for deuteron

effective binding energies $B_i^{(\mathrm{eff})} = B_i^{(0)} - \Delta m_i^{\mathrm{Pauli}}$



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    $n_i^{\text{eff}} = \frac{2}{A_i}\left[Z_i Y_q + N_i(1 - Y_q)\right] n_b$
    $\Rightarrow$ asymmetry of medium
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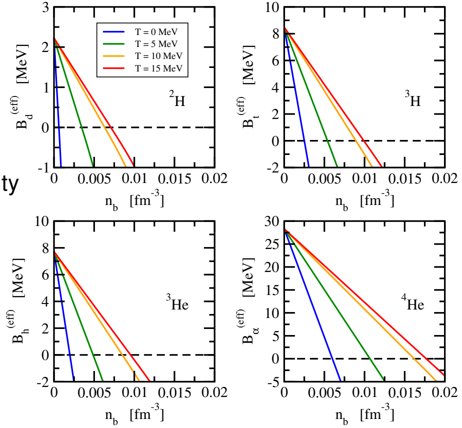

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- heavy nuclei
  - heuristic parametrization

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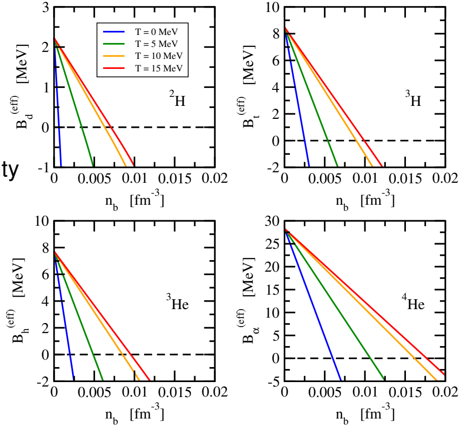

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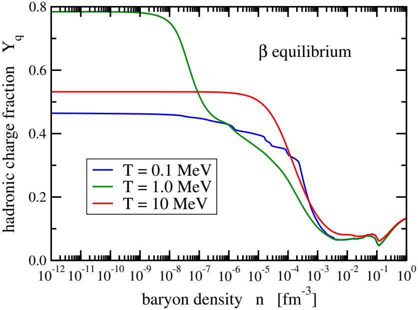
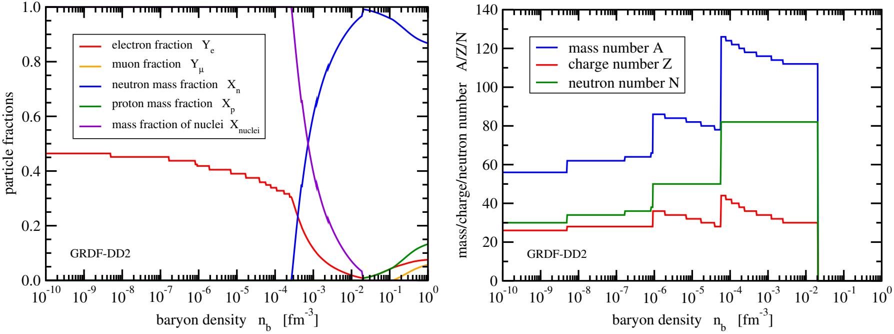


- neutronisation with increasing baryon density

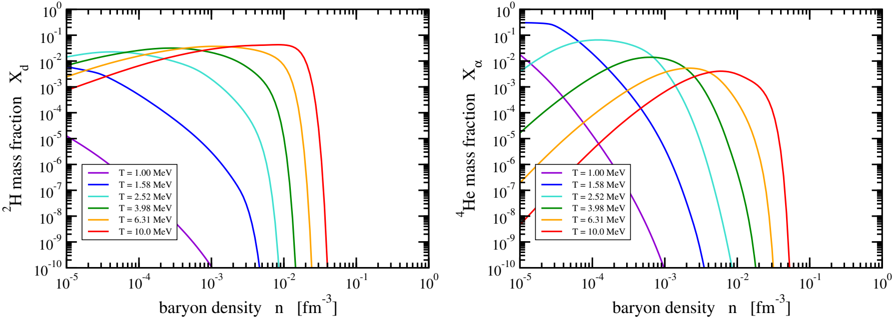
Compact Star Matter
Zero Temperature

- formation of neutron star crust, sequence of ions, phase transitions
- more neutron rich at higher densities, approaching neutron drip density
- 'pasta phases' before transition to uniform matter



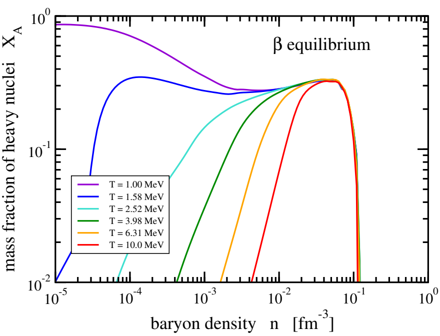
Compact Star Matter
Light Clusters

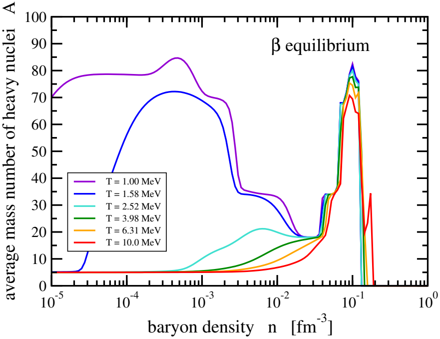
- formation and dissolution with increasing density
- temperature dependence



Compact Star Matter
Heavy Clusters

- full distribution of nuclei with shell effects (ground and excited states)
- single-nucleus approximation (SNA) not sufficient





Light Clusters in Heavy-Ion Collisions

emission of light nuclei

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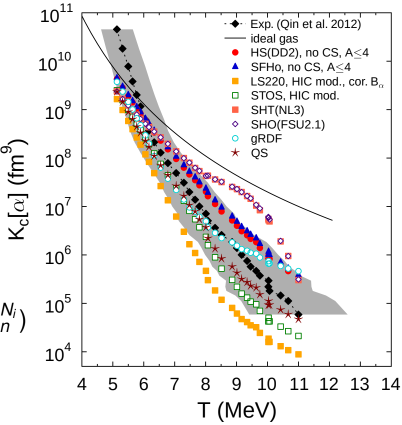
  R. Wada et al. PRC 85 (2012) 064618
- thermodynamic conditions as in neutrinosphere of core-collapse supernovae
- particle yields ⇒ chemical equilibrium constants $K_c[i] = n_i/(n_p^{Z_i} n_n^{N_i})$

  L. Qin et al., PRL 108 (2012) 172701
- mixture of ideal gases not sufficient
- new data from INDRA collaboration

  H. Pais et al., arXiv:1911.10849



M. Hempel, K. Hagel, J. Natowitz, G. Röpke, S. Typel, PRC C 91 (2015) 045805

Further Properties of GRDF-DD2 Model

- very reasonable nuclear matter parameters
  $n_{\text{sat}} = 0.149\ \text{fm}^{-3}$, $a_V = 16.02$ MeV, $K = 242.7$ MeV,
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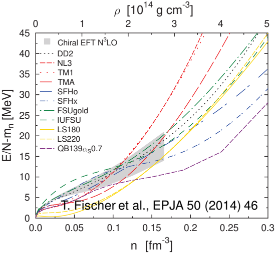

T. Fischer et al., EPJA 50 (2014) 46

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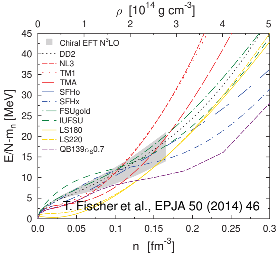
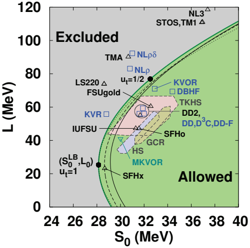


T. Fischer et al., EPJA 50 (2014) 46



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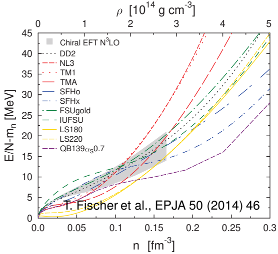

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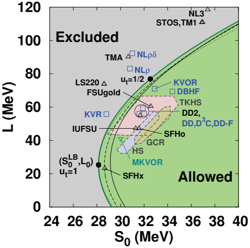
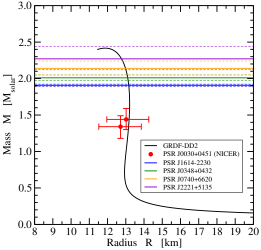
- neutron star mass-radius relation consistent with maximum mass and recent mass-radius constraints

  $M_{\text{max}} = 2.42\ M_{\odot}$, $R_{1.4} = 13.2$ km





Clusters in Heavy Nuclei

Application of Generalized Relativistic Density Functional to Heavy Nuclei

- extension to zero temperature
- simplified nuclear structure calculation

  (S. Typel, PRC 89 (2014) 064321)
  - nucleons and α particles as degrees of freedom
  - Thomas-Fermi approximation for nucleons
  - explicit α-particle wave function in WKB approximation
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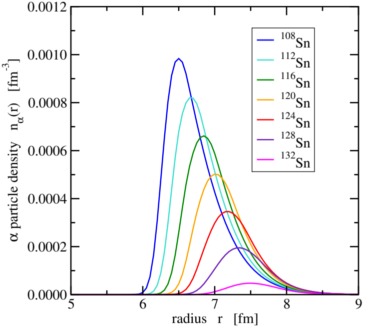
  - α particle distribution at surface of nuclei



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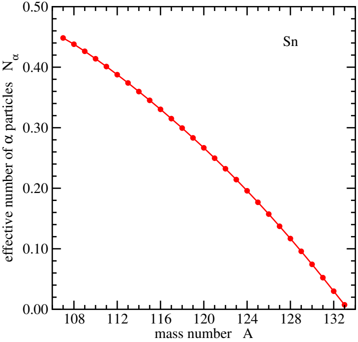
- application to chain of Sn isotopes
  - α particle distribution at surface of nuclei
  - reduced probability of α occurrence with increasing neutron excess

    (consistent with trend of α particle reduced widths in (d,^{6}Li) pickup reactions on Sn nuclei, A. A. Cowley, Phys. Rev. C 93 (2016) 054329)



Study of α Particle Correlations at Surface of Sn Nuclei I

- **quasifree (p,pα) knockout reactions on Sn nuclei**
  - experimental signatures:
    - dependence of cross sections on neutron excess
    - localisation of α particles at surface
      $\Rightarrow$ broad momentum distribution

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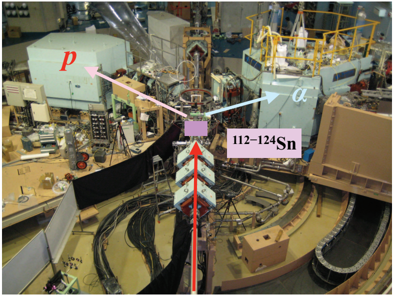
- **experiments at RCNP, Osaka (E461)**
  - targets: stable $^{112-124}$Sn nuclei
  - beam: 392 MeV protons, 100 pnA
  - proton detection: Grand Raiden
  - α detection: LAS
  - first experiment (June 2015): failure of some detectors
  - second experiment (February 2018): successful



Study of α Particle Correlations at Surface of Sn Nuclei II

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    - spectrometer setting: $\theta_{\mathrm{lab}}(p)$ = 45.3 deg, $\theta_{\mathrm{lab}}(\alpha)$ = 60 deg
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  - theory
    - distorted-wave eikonal model in impulse approximation $\Rightarrow$ factorization of cross section
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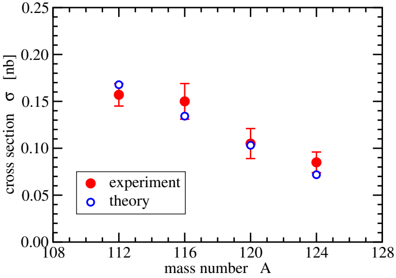
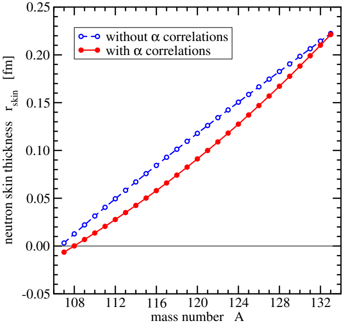


Consequences of α Particle Correlations at Surface of Heavy Nuclei

- Sn nuclei
  - reduction of neutron skin thickness
  - no effect for np-symmetric nuclei and very neutron-rich nuclei
  - strong effect

mass number	112	116	120	124
rel. change	-44%	-31%	-23%	-15%



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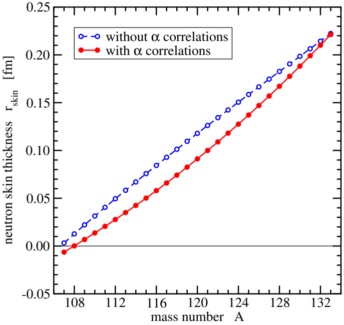

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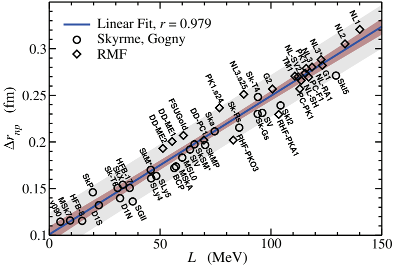
$\Rightarrow$ affects correlation of neutron skin thickness with slope parameter L of symmetry energy (no α cluster effects in conventional energy density functional calculations)

$\Rightarrow$ systematic uncertainty

X. Viñas et al., Eur. Phys. J. A 50 (2014) 27



Conclusions

Summary and Outlook

- cluster formation essential feature in nuclei and nuclear matter
- effective theoretical description with generalized relativistic density functional
- applications
  - equation of state of nuclear and compact star matter
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- new parametrisation of nucleon-meson couplings
  - scalar density dependence to avoid problems at zero baryon density
  - inclusion of tensor couplings
(project with Diana Alvear, visitor in Erasmus+ program)
- clusters as effective means to describe correlations above saturation density
(project with Stefano Burrello, Alexander-von-Humboldt fellowship)
- improvement of mass shifts for light and heavy nuclei,
effects of momentum dependence?