Thermodynamic Nuclear Equation of State
from
(Chiral) Effective Field Theory

Corbinian Wellenhofer

"Nuclear equation of state and neutron stars"

International Workshop XLVIII on Gross Properties of Nuclei and Nuclear Excitations

Darmstädter Haus, Hirschegg, Austria

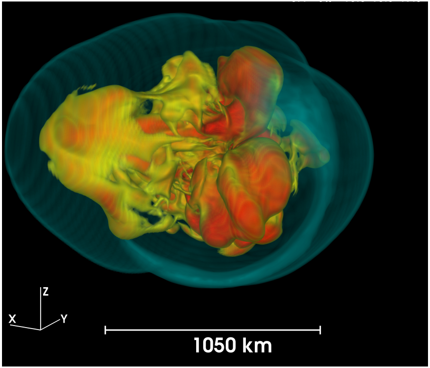
January 16, 2020

TECHNISCHE UNIVERSITÄT DARMSTADT

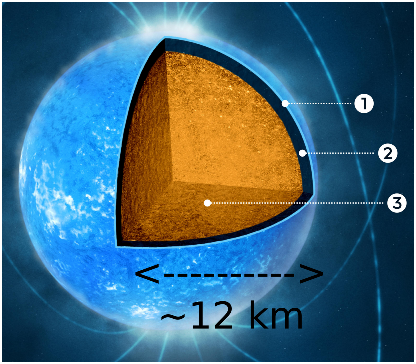
SFB 1245

Matter under Extreme Conditions

Neutron stars & supernova cores: **nuclear matter** at high ρ, T, $\delta = (\rho_n - \rho_p)/\rho$



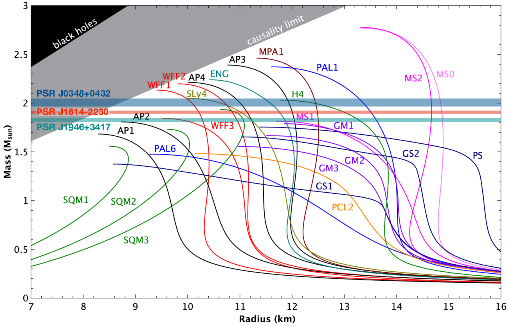
Andresen *et al.*, arXiv:1810.07638



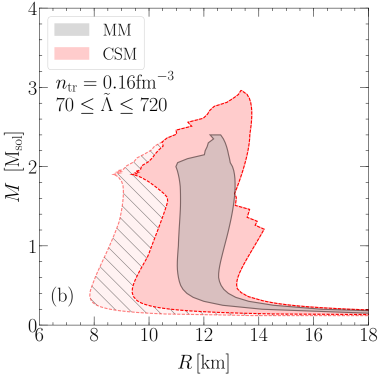
Watts *et al.*, RMP 88 (2016)

- fundamental problem of strong interaction physics: **nuclear many-body problem**
- relevant for: **nucleosynthesis, gravitational waves, heavy-ion collisions**

Nuclear EOS $P(E)$ determines M-R relation of neutron star ($T = 0$)



www.mpifr-bonn.mpg.de



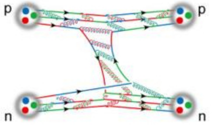
Tews, Margueron, Reddy, EPJ A55 (2019)

- **low densities: → effective field theory (EFT) calculations**
- higher densities: observational constraints, FRG methods

Effective Field Theory of Nuclear Interactions

Low-Energy QCD: $Q < \Lambda_\chi \sim 1$ GeV

- strongly-coupled, confinement
- nonlinear realization of chiral symmetry



by: S. Aoki (Kyoto U)

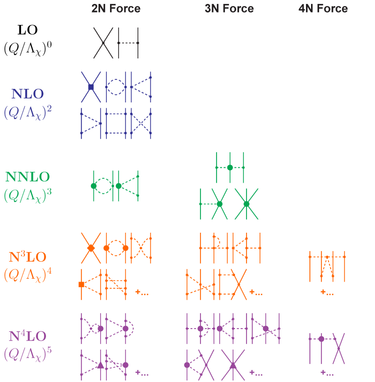
Effective Nuclear Interactions

- General Lagrangian $\mathscr{L}_{\text{EFT}}(N, \pi, \ldots)$
- Low-energy constants $\{c_i\}$
- **Renormalization** (of 'nonrenormalizable' interactions)

Goal: systematic expansion for observables

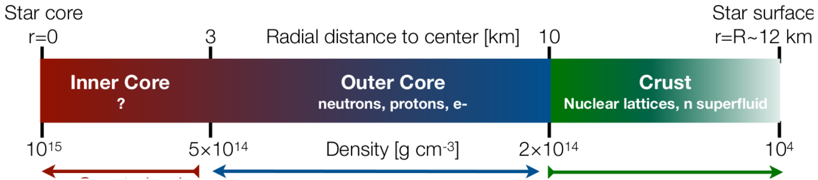
Method 1: perturbative EFT
→ dilute nuclear matter

Few-body systems require resummations!
Method 2: chiral EFT potentials
→ higher densities



Machleidt; Symm. 8 (2016)

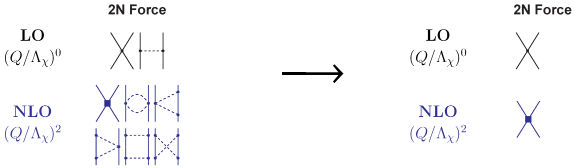
Strategy & Outline



- **Crust:** dilute neutron gas → Part 1
- **Outer Core:** dense nuclear fluid → Part 2
- **Inner core:** constrained extrapolation → talk by Sabrina Schäfer

Simpler Problem: Dilute Fermi Gas

→ (Perturbative) Pionless EFT



Nuclear Many-Body Problem

Chiral EFT Potentials

EFT for Dilute Fermi Gas

EFT Lagrangian

$$\mathscr{L}_{\text{EFT}}[\psi] = \psi^\dagger \left[i\partial_t + \frac{\vec{\nabla}^2}{2M} \right] \psi - \frac{C_0^{\text{(bare)}}}{2} (\psi^\dagger \psi)^2 + \frac{C_2^{\text{(bare)}}}{16} \left[(\psi\psi)^\dagger (\psi \overleftrightarrow{\nabla}^2 \psi) + h.c. \right]$$
$$+ \frac{C_{2'}^{\text{(bare)}}}{8} (\psi \overleftrightarrow{\nabla} \psi)^\dagger \cdot (\psi \overleftrightarrow{\nabla} \psi) - \frac{D_0^{\text{(bare)}}}{6} (\psi^\dagger \psi)^3 + \ldots$$

(Bare) low-energy constants:
- NN interaction: $C_0^{\text{(bare)}}, C_2^{\text{(bare)}}, C_{2'}^{\text{(bare)}}, \ldots$
- 3N interaction: $D_0^{\text{(bare)}}, \ldots$

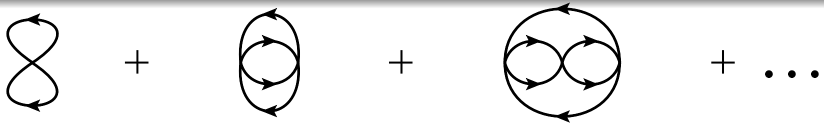
see for example: Hammer & Furnstahl, NPA 678 (2000)

EFT for Dilute Fermi Gas

Perturbative renormalization (sharp cutoff Λ)

Divergent loop integrals ($C_0^{(\text{bare})}$ term):

- NN scattering: $\mathcal{I} = \frac{1}{2\pi}\int_0^{\Lambda} d^3q\, \frac{q^0}{q^2-p^2} = 2\Lambda + \ldots$
- MBPT: $\mathcal{I} = \frac{1}{2\pi}\int_{k_\mathrm{F}}^{\Lambda} d^3q\, \frac{q^0}{q^2-p^2} = 2\Lambda + \ldots$



Perturbative renormalization (sharp cutoff Λ)

Divergent loop integrals ($C_0^{(\text{bare})}$ term):

- NN scattering: $\mathcal{I} = \frac{1}{2\pi} \int_0^{\Lambda} d^3q \, \frac{q^0}{q^2 - p^2} = 2\Lambda + \ldots$
- MBPT: $\mathcal{I} = \frac{1}{2\pi} \int_{k_F}^{\Lambda} d^3q \, \frac{q^0}{q^2 - p^2} = 2\Lambda + \ldots$

Perturbative renormalization (sharp cutoff Λ)

Divergent loop integrals ($C_0^{(\text{bare})}$ term):

- NN scattering: $\mathcal{I} = \frac{1}{2\pi}\int_0^{\Lambda} d^3q\,\frac{q^0}{q^2-p^2} = 2\Lambda + \ldots$
- MBPT: $\mathcal{I} = \frac{1}{2\pi}\int_{k_F}^{\Lambda} d^3q\,\frac{q^0}{q^2-p^2} = 2\Lambda + \ldots$

$\Rightarrow$ **same counterterms renormalize NN scattering and MBPT (ladders)**

EFT for Dilute Fermi Gas

Perturbative renormalization (sharp cutoff Λ)

Divergent loop integrals ($C_0^{(\text{bare})}$ term):

- NN scattering: $\mathcal{I} = \frac{1}{2\pi} \int_0^{\Lambda} d^3q \, \frac{q^0}{q^2 - p^2} = 2\Lambda + \ldots$
- MBPT: $\mathcal{I} = \frac{1}{2\pi} \int_{k_\mathrm{F}}^{\Lambda} d^3q \, \frac{q^0}{q^2 - p^2} = 2\Lambda + \ldots$

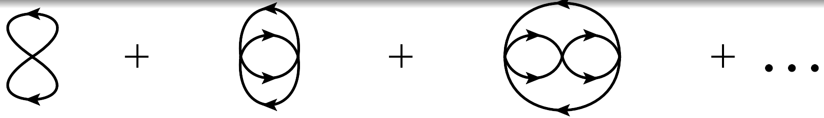
$\Rightarrow$ same counterterms renormalize NN scattering and MBPT (ladders)

$$C_0^{(\text{bare})} = C_0 + C_0 \sum_{\nu=1}^{3} \left(C_0 \frac{M}{2\pi^2} \Lambda \right)^{\nu} + C_2 C_0 \frac{M}{3\pi^2} \Lambda^3 + \ldots$$

$$C_2^{(\text{bare})} = C_2 + C_2 C_0 \frac{M}{\pi^2} \Lambda + \ldots$$

Renormalized perturbation theory:

- power counting w.r.t. physical LECs C_0, C_2, …
- “non-renormalizable” $C_2^{(\text{bare})}$, … require higher-order counterterms at each loop order



Perturbative renormalization (sharp cutoff Λ)

Divergent loop integrals ($C_0^{\text{(bare)}}$ term):

- NN scattering: $\mathcal{I} = \frac{1}{2\pi}\int_0^{\Lambda} d^3q \, \frac{q^0}{q^2-p^2} = 2\Lambda + \ldots$
- MBPT: $\mathcal{I} = \frac{1}{2\pi}\int_{k_\mathrm{F}}^{\Lambda} d^3q \, \frac{q^0}{q^2-p^2} = 2\Lambda + \ldots$

⇒ **same counterterms renormalize NN scattering and MBPT (ladders)**

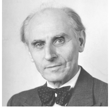
LEC fixing ($\Lambda \to \infty$)

- NN LECs matched to ERE: $C_0 = \frac{4\pi a_s}{M}$, $C_2 = C_0 \frac{a_s r_s}{2}, \ldots$
- $g > 2$: three-body LEC at 4th order, to be fixed in 3N sector

⇒ **predictions for many-body sector**

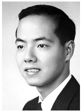
EFT Expansion up to Third Order

Ground-state energy: $E(k_\mathrm{F}) = n\dfrac{k_\mathrm{F}^2}{2M}\left[\dfrac{3}{5} + (g-1)\displaystyle\sum_{\nu=1}^{\infty}\alpha_\nu(k_\mathrm{F})\right]$



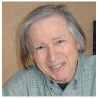
$$\alpha_1(k_\mathrm{F}) = \frac{2}{3\pi}k_\mathrm{F}a_s$$

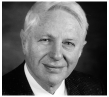
W. Lenz (1929)



$$\alpha_2(k_\mathrm{F}) = \frac{4}{35\pi^2}(11 - 2\ln 2)(k_\mathrm{F}a_s)^2$$

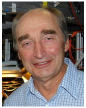
T.D. Lee & C.N. Yang (1957)





C. de Dominicis & P.C. Martin (1957)

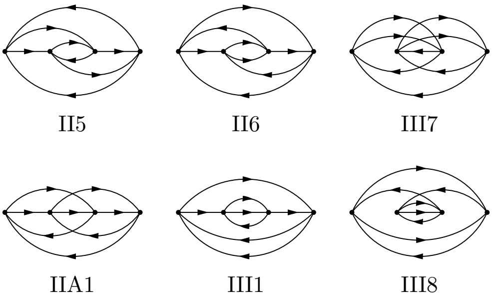
$$\alpha_3(k_\mathrm{F}) = \Big[0.0755732(0) + 0.0573879(0)\,(g-3)\Big](k_\mathrm{F}a_s)^3 + \frac{1}{10\pi}(k_\mathrm{F}a_s)^2 k_\mathrm{F}r_s + \frac{1}{5\pi}\frac{g+1}{g-1}(k_\mathrm{F}a_p)^3$$



V.N. Efimov (1966)

Calculation of $E_4(k_\mathrm{F})$

C_0 contributions: MBPT(1,2): 1 diag, MBPT(3): 3 diags, MBPT(4): 33 diags, MBPT(5): 668



- I(1,2,4,5), II(1,2,6), III(1,8): UV power divergences: subladders, NN counterterms
- II(5,6), IIA1, III1: logarithmic UV divergences: 3N counterterm $g > 2$, cancel $g = 2$

  $\rightsquigarrow$ logarithmic term at fourth order $(g-2)(k_\mathrm{F} a_s)^4 \ln k_\mathrm{F}/\Lambda_0$

  V. N. Efimov, Phys. Lett. 15, (1965)
- III(1,2,8,10): infrared divergences: cancel in III(1+8) and III(2+10)

EFT Expansion at Fourth Order

$$\alpha_4(k_\mathrm{F}) = -0.0425(1)\,(k_\mathrm{F} a_s)^4 \; + 0.0644872(0)\,(k_\mathrm{F} a_s)^3 k_\mathrm{F} r_s + \gamma_4\,(g-2)\,(k_\mathrm{F} a_s)^4$$

with: $\gamma_4(k_\mathrm{F}) = \dfrac{M\,D_0(\Lambda_0)}{108\pi^4 a_s^4} + 0.2707(4) - 0.00864(2)\,(g-2) + \dfrac{16}{27\pi^3}\left(4\pi - 3\sqrt{3}\right)\ln(k_\mathrm{F}/\Lambda_0)$

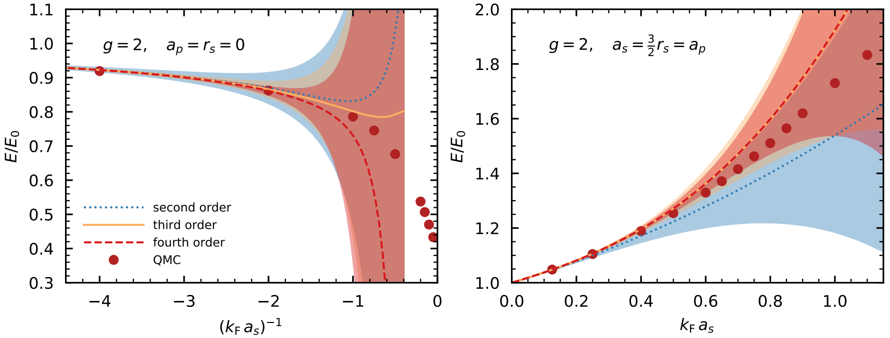
Efimov, Phys. Lett. 15, (1965)
Braaten & Nieto, PRB 55 (1997)
Hammer & Furnstahl, NPA 678 (2000)
Kaiser, EPJA 48 (2012)
Wellenhofer, Drischler, Schwenk, arXiv:1812.08444

EFT Expansion at Fourth Order

$$\alpha_4(k_F) = -0.0425(1)\,(k_F a_s)^4 + 0.0644872(0)\,(k_F a_s)^3 k_F r_s + \gamma_4\,(g-2)\,(k_F a_s)^4$$

with: $\gamma_4(k_F) = \dfrac{M\,D_0(\Lambda_0)}{108\pi^4 a_s^4} + 0.2707(4) - 0.00864(2)\,(g-2) + \dfrac{16}{27\pi^3}\left(4\pi - 3\sqrt{3}\right)\ln(k_F/\Lambda_0)$

Efimov, Phys. Lett. 15, (1965)
Braaten & Nieto, PRB 55 (1997)
Hammer & Furnstahl, NPA 678 (2000)
Kaiser, EPJA 48 (2012)
Wellenhofer, Drischler, Schwenk, arXiv:1812.08444



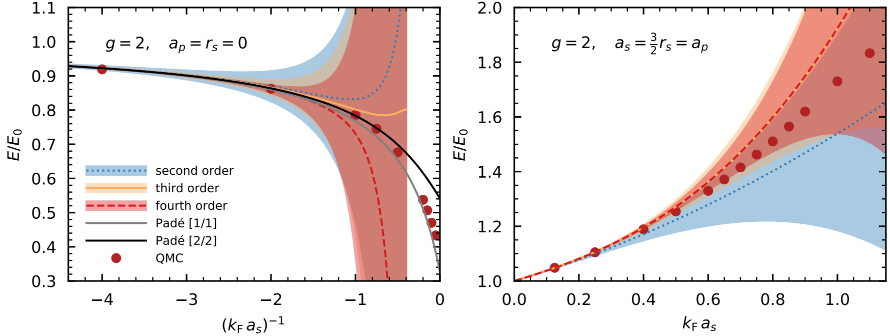
Systematic uncertainty bands by estimating next-order term as $C_{n+1} < \mathrm{Max}[C_n]$

EFT Expansion at Fourth Order

$$\alpha_4(k_F) = -0.0425(1)\,(k_F a_s)^4 + 0.0644872(0)\,(k_F a_s)^3 k_F r_s + \gamma_4\,(g-2)\,(k_F a_s)^4$$

with: $\gamma_4(k_F) = \dfrac{M\,D_0(\Lambda_0)}{108\pi^4 a_s^4} + 0.2707(4) - 0.00864(2)\,(g-2) + \dfrac{16}{27\pi^3}\left(4\pi - 3\sqrt{3}\right)\ln(k_F/\Lambda_0)$

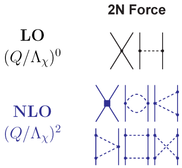
Efimov, Phys. Lett. 15, (1965)
Braaten & Nieto, PRB 55 (1997)
Hammer & Furnstahl, NPA 678 (2000)
Kaiser, EPJA 48 (2012)
Wellenhofer, Drischler, Schwenk, arXiv:1812.08444



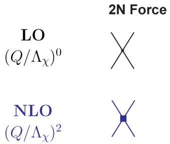
Extrapolation/resummation methods (e.g., Pade Approximants), unitary limit $\xi = 0.37$

Simpler Problem: Dilute Fermi Gas

→ (Perturbative) Pionless EFT



←



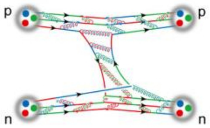
Nuclear Many-Body Problem

Chiral EFT Potentials

Effective Field Theory of Nuclear Interactions

Low-Energy QCD: $Q < \Lambda_\chi \sim 1$ GeV

- strongly-coupled, confinement
- nonlinear realization of chiral symmetry



by: S. Aoki (Kyoto U)

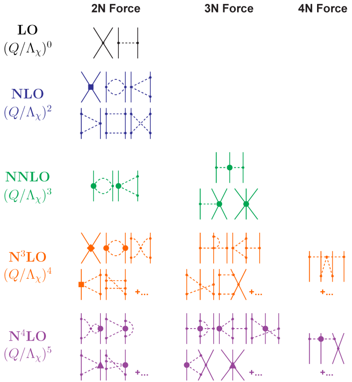
Effective Nuclear Interactions

- General Lagrangian $\mathscr{L}_{\text{EFT}}(N, \pi, \ldots)$
- Low-energy constants $\{c_i\}$
- **Renormalization** (of 'nonrenormalizable' interactions)

Goal: systematic expansion for observables

Method 1: perturbative EFT
$\rightarrow$ dilute nuclear matter: $\Lambda \rightarrow \infty$, $c_i^{\text{bare}} = c_i + c.t.$

Few-body systems require resummations!
Method 2: chiral EFT potentials
$\rightarrow$ higher densities: $\Lambda \lesssim \Lambda_\chi$, $c_i^{\text{bare}} = c_i^{\text{bare}}(\Lambda)$



Machleidt; Symm. 8 (2016)

(*Ad hoc*) NN potentials ($\sim$ 1970-1990) from fits to NN data

- long range: pion exchange (I)
- intermediate-range attraction (II)
- strong short-distance repulsion: 'hard core' (III)

$\rightsquigarrow$ nuclear many-body problem <u>nonperturbative!</u>

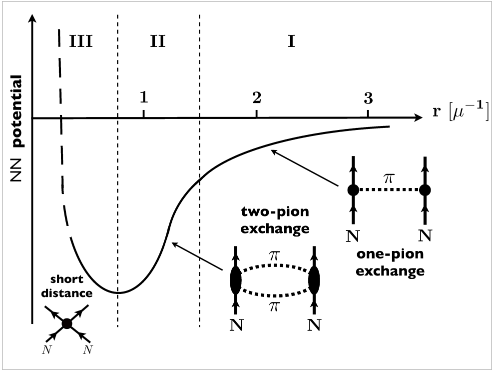
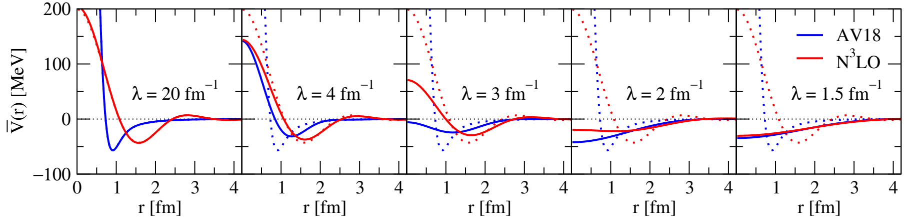


figure adapted from: Holt *et al*; PPNP 73 (2013), Taketani; PTPS 3 (1956)

RG evolution, Cutoff Artifacts, LEC Ambiguities

RG methods to construct **perturbative** NN potentials → MBPT for nuclear matter calculations



Furnstahl; Nucl. Phys. B Proc. Suppl. (2012)

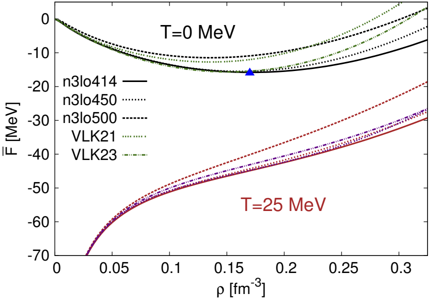
RG evolution induces multi-nucleon forces
→ truncation leads to cutoff dependence!
→ power counting: Λ dependence should decrease with increasing EFT orders

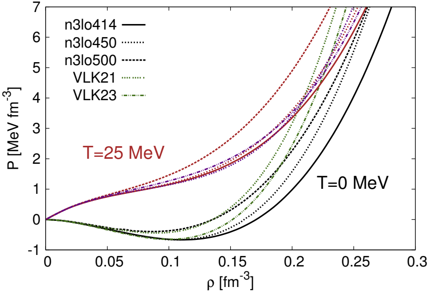
NN+3N potentials used in this work

	Λ (fm^{-1})	c_E	c_D	c_1 (GeV^{-1})	c_3 (GeV^{-1})	c_4 (GeV^{-1})
n3lo414	2.1	-0.072	-0.4	-0.81	-3.0	3.4
n3lo450	2.3	-0.106	-0.24	-0.81	-3.4	3.4
n3lo500	2.5	-0.205	-0.20	-0.81	-3.2	5.4
VLK21	2.1	-0.625	-2.062	-0.76	-4.78	3.96
VLK23	2.3	-0.822	-2.785	-0.76	-4.78	3.96

Entem, Machleidt; PRC 68 (2003), Gazit; Phys.Lett.B 666 (2008), Coraggio, Holt *et al.*; PRC 87 (2013) + PRC 89 (2014)
Bogner, Furnstahl, Schwenk, Nogga; NPA 763 (2005), Nogga, Bogner, Schwenk; PRC 70 (2004)

Many-Body Perturbation Theory: $F(\rho) = F_0(\rho) + F_1(\rho) + F_2(\rho) + \ldots \xrightarrow{T\to 0} E(k_F)$



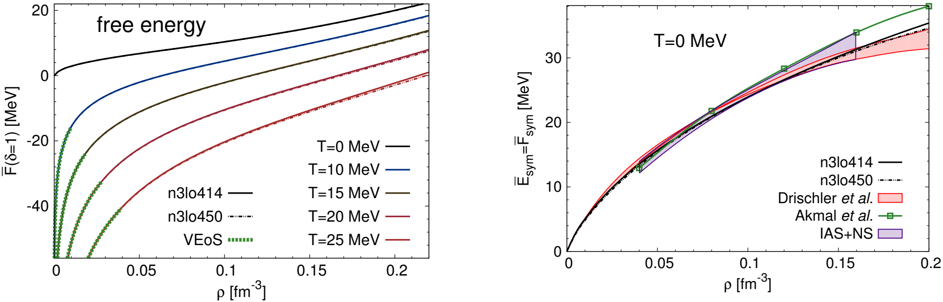


- empirical saturation point $(E_0, \rho_0) \approx (16\,\mathrm{MeV}, 0.16\,\mathrm{fm}^{-3})$
- negative thermal expansion at high ρ (?) ($\sim$ water below $4^\circ C$)

  → **astrophysics:** thermal index $\Gamma = 1 + \frac{P(T)-P(0)}{\epsilon(T)-\epsilon(0)} < 1$
- $\rho \gtrsim \rho_{sat}$: model dependence sizeable, dominated by 3N contributions

Wellenhofer, Holt, Kaiser, Weise; PRC 89 (2014)
see also: Carbone et al.; PRC 98 (2018)

Pure Neutron Matter, Symmetry Energy



Good agreement with

- virial expansion for dilute neutron matter Horowitz & Schwenk; Phys.Lett.B 638 (2006)
- empirical constraints on symmetry energy $\bar{E}_{sym}$ from measurements of isobaric analog states (IAS) and neutron skins (NS)
  Danielewicz & Lee; Nucl. Phys. A 922 (2013)

Wellenhofer, Holt, Kaiser; PRC 92 (2015)
see also: Tews et al.; PRL 110 (2013)
see also: Drischler et al.; PRC 94 (2016)

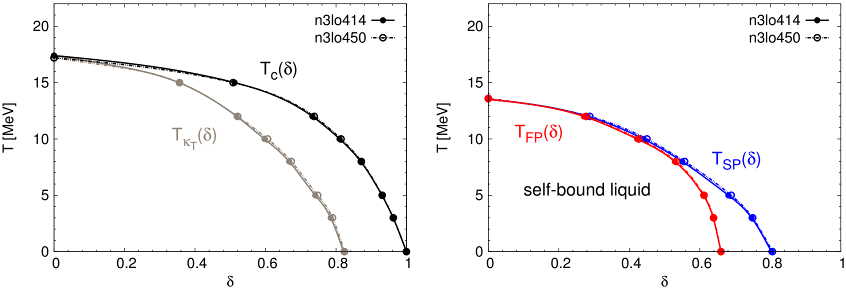
Nuclear Liquid-Gas Phase Transition

Neutron star crust-core transition – Coulomb = nuclear liquid-gas transition

- spinodal instability $\rightsquigarrow$ multifragmentation experiments
  $\rightsquigarrow T_c(\delta = 0) \approx 15 - 20$ MeV Karnaukhov *et al.*, Phys.Atom.Nucl. 71 (2008)

Dependence of T_c on isospin-asymmetry $\delta = (\rho_n - \rho_p)/\rho$

- **isospin distillation**: $F(\delta) \sim F(0) + \delta^2 F_{\text{sym}}(0) + \ldots$ does not work!
- (metastable) self-bound states at low T and δ



Wellenhofer, Holt, Kaiser, PRC 92 (2015) & PRC 93 (2016), see also: Carbone et al., PRC 98 (2018)

MBPT Binary System: Asymmetry Expansion

Explicit parametrization via expansion about $\delta = 0$, where $\delta = (\rho_n - \rho_p)/\rho$

$$F(\delta) \sim \overbrace{F(\delta = 0) + A_2\,\delta^2}^{\gtrsim 99\%\text{ of literature}} + A_4\,\delta^4 + A_6\,\delta^6 + \ldots$$

MBPT Binary System: Asymmetry Expansion

Explicit parametrization via expansion about $\delta = 0$, where $\delta = (\rho_n - \rho_p)/\rho$

$$F(\delta) \sim \overbrace{F(\delta = 0) + A_2\,\delta^2}^{\gtrsim 99\% \text{ of literature}} + A_4\,\delta^4 + A_6\,\delta^6 + \ldots$$

Higher-order coefficients A_4, A_6 are singular at zero temperature!

$$F_2(T = 0, \rho, \delta) = A_0(0, \rho) + A_2(0, \rho)\,\delta^2 + \sum_{n=2}^{\infty} A_{2n,\text{reg}}(\rho)\,\delta^{2n} + \sum_{n=2}^{\infty} A_{2n,\log}(\rho)\,\delta^{2n} \ln|\delta|$$

Kaiser; PRC 92 (2015) Wellenhofer, Kaiser, Weise; PRC 95 (2016)

MBPT Binary System: Asymmetry Expansion

Explicit parametrization via expansion about $\delta = 0$, where $\delta = (\rho_n - \rho_p)/\rho$

$$F(\delta) \sim \overbrace{F(\delta=0) + A_2\,\delta^2}^{\gtrsim 99\%\text{ of literature}} + A_4\,\delta^4 + A_6\,\delta^6 + \dots$$

Higher-order coefficients A_4, A_6 are singular at zero temperature!

$$F_2(T=0,\rho,\delta) = A_0(0,\rho) + A_2(0,\rho)\,\delta^2 + \sum_{n=2}^{\infty} A_{2n,\mathrm{reg}}(\rho)\,\delta^{2n} + \sum_{n=2}^{\infty} A_{2n,\log}(\rho)\,\delta^{2n}\ln|\delta|$$

Kaiser; PRC 92 (2015) Wellenhofer, Kaiser, Weise; PRC 95 (2016)

What is the origin of the logarithmic terms at $T=0$? What happens at finite T?

→ energy denominators in contributions beyond first order, e.g.,

$$E_{0;2} = -\frac{1}{4}\sum_{ijab} \bar{V}_{\mathrm{NN}}^{ij,ab}\,\bar{V}_{\mathrm{NN}}^{ab,ij}\,\frac{\Theta_i^-\Theta_j^-\Theta_a^+\Theta_b^+}{\varepsilon_a+\varepsilon_b-\varepsilon_i-\varepsilon_j}$$

$$F_2 = -\frac{1}{8}\sum_{ijab} \bar{V}_{\mathrm{NN}}^{ij,ab}\,\bar{V}_{\mathrm{NN}}^{ab,ij}\,\frac{\tilde{f}_i^-\tilde{f}_j^-\tilde{f}_a^+\tilde{f}_b^+ - \tilde{f}_i^+\tilde{f}_j^+\tilde{f}_a^-\tilde{f}_b^-}{\varepsilon_a+\varepsilon_b-\varepsilon_i-\varepsilon_j}$$

integrand diverges at integral boundary
$\rightsquigarrow E_{0;2} \in C^3$

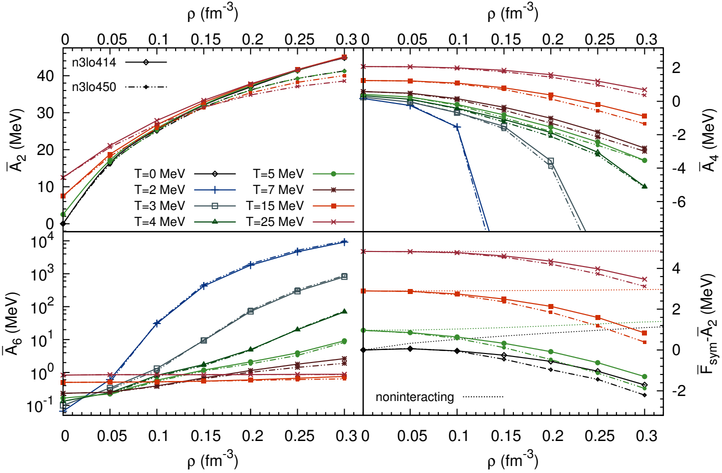
smooth integrand
$\rightsquigarrow F_2 \in C^\infty$

Logarithmic terms also when ladders are resummed to all orders!

Kaiser; EPJA 48 (2014), Wellenhofer; arXiv:1707.09222

MBPT Binary System: Asymmetry Expansion

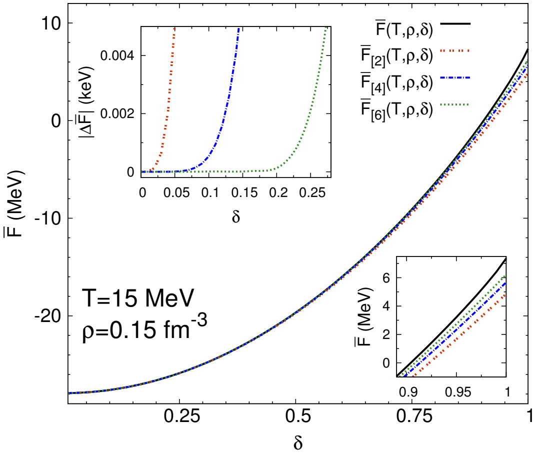
- $A_2 > A_4 > A_6 > \ldots$ at high $T\mu$, $A_2 \ll A_4 \ll A_6 \ll \ldots$ at low $T\mu$
  $(A_{2n\geq 4} \xrightarrow{T\to 0} \pm\infty)$



Wellenhofer, Holt, Kaiser; PRC 93 (2016)

- **bottom-right:** accuracy of quadratic approximation governed by $F_{sym} - A_2$

Asymmetry Expansion (**High** Temperature)

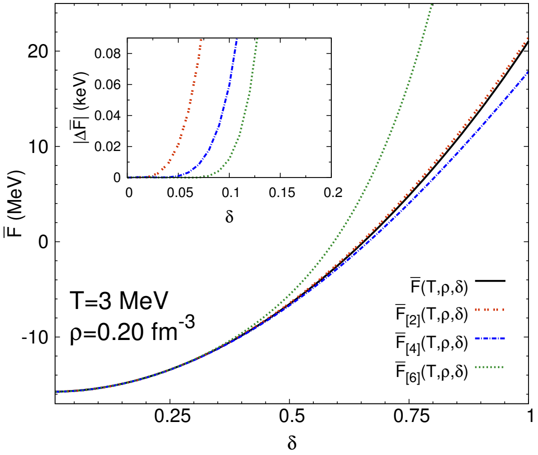


Wellenhofer, Holt, Kaiser; PRC 93 (2016)

Main Plot: Exact $F(T,\rho,\delta)$ vs different orders in the expansion $F_{2,4,6}(T,\rho,\delta)$

Insets: Deviation $\Delta F = F - F_{2,4,6}$

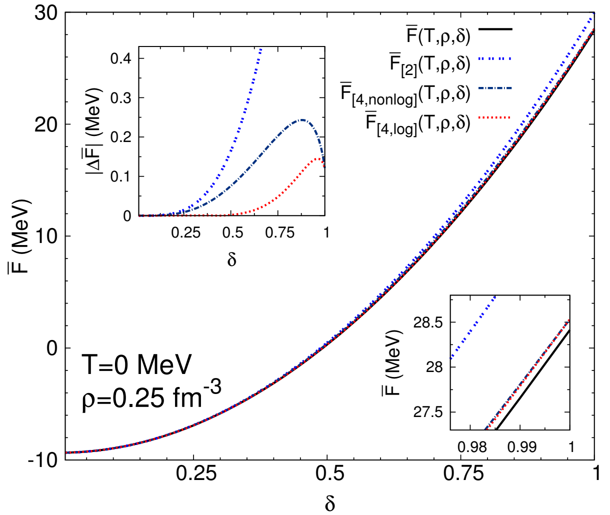
Asymmetry Expansion (**Low** Temperature)



Wellenhofer, Holt, Kaiser; PRC 93 (2016)

Main Plot: Exact $F(T,\rho,\delta)$ vs different orders in the expansion $F_{2,4,6}(T,\rho,\delta)$

Inset: Deviation $\Delta F = F - F_{2,4,6}$



Wellenhofer, Holt, Kaiser; PRC 93 (2016)

$T = 0$: Exact $F(T, \rho, \delta)$ vs 'logarithmic' expansion

Statistical Quasiparticles

$$\mathcal{H} = \mathcal{T}_{\text{kin}} + \mathcal{V} = \underbrace{(\mathcal{T}_{\text{kin}} + \mathcal{U})}_{\substack{\text{reference system} \\ \text{“mean-field theory”}}} + \underbrace{(\mathcal{V} - \mathcal{U})}_{\substack{\text{perturbation} \\ \text{“correlations”}}}, \quad \text{with SP potential } \mathcal{U} = \sum_{\boldsymbol{k}} U_{\boldsymbol{k}}\, a^\dagger_{\boldsymbol{k}} a_{\boldsymbol{k}}$$

→ **usually:** Hartree-Fock potential: $U_{\boldsymbol{k}} = \sum_{\boldsymbol{p}} \langle \boldsymbol{k}\boldsymbol{p}|V|\boldsymbol{k}\boldsymbol{p}\rangle = \dfrac{\delta\Omega_1}{\delta n_{\boldsymbol{k}}}$

Order-by-order renormalization of SP potential: $U_{\boldsymbol{k}} = \sum_{n=1}^{N} \dfrac{\delta\Omega^{*,**,***}_{n,\text{normal}}}{\delta n_{\boldsymbol{k}}}$

Balian & de Dominicis; Comp. Rend. (1960)
Wellenhofer; PRC 99 (2019)

- thermodynamic relations of Fermi-liquid theory (∼ Landau), valid $\forall T$

$$\varrho = \sum_{\boldsymbol{k}} n_{\boldsymbol{k}}, \quad S = -\sum_{\boldsymbol{k}} \left(n_{\boldsymbol{k}} \ln n_{\boldsymbol{k}} + \bar{n}_{\boldsymbol{k}} \ln \bar{n}_{\boldsymbol{k}}\right), \quad \frac{\delta E}{\delta n_{\boldsymbol{k}}} = \epsilon_{\boldsymbol{k}}, \quad F = F_0 + F_{\text{int}}$$

- grand-canonical and $T = 0$ MBPT consistent: $F(T,\mu) \xrightarrow{T\to 0} E(k_{\mathrm{F}})$
- “*at each new order, not only is new information about interaction effects included, but this information automatically improves the reference point*”
- Fermi-liquid relations: ⇝ phenomenological parametrizations, Sommerfeld
- Second-order contribution $U_{2,k}$ has significant effects! Holt, Kaiser; PRC 95 (2017)
- **But**: convergence rate with higher-order $\mathcal{U}$? → Current Work!

Summary

Dilute Fermi Systems

- perturbative EFT: systematic uncertainties via EFT orders
- k_{F} expansion for $E(k_{\mathrm{F}})$ evaluated up to fourth order

  $\rightsquigarrow$ converged results for $k_{\mathrm{F}} a_s \lesssim 0.5$, Padé extrapolations for larger $k_{\mathrm{F}} a_s$

$\rightarrow$ improved large $k_{\mathrm{F}} a_s$ extrapolations? (current work with Daniel Phillips & Achim Schwenk)

Nuclear Matter Thermodynamics

- **Realistic nuclear thermodynamics from chiral EFT potentials**

  $\rightarrow$ astrophysical EOS tables (current work with Sabrina Schäfer & Achim Schwenk)

  $\rightarrow$ improved calculations (current work with Christian Drischler, Jonas Keller, Kai Hebeler, Achim Schwenk)
- **3N contributions** enhanced at high densities, dominate uncertainties

Many thanks to my collaborators: Christian Drischler, Kai Hebeler, Jeremy Holt, Norbert Kaiser, **Jonas Keller**, Daniel Phillips, **Sabrina Schäfer**, Achim Schwenk, Wolfram Weise