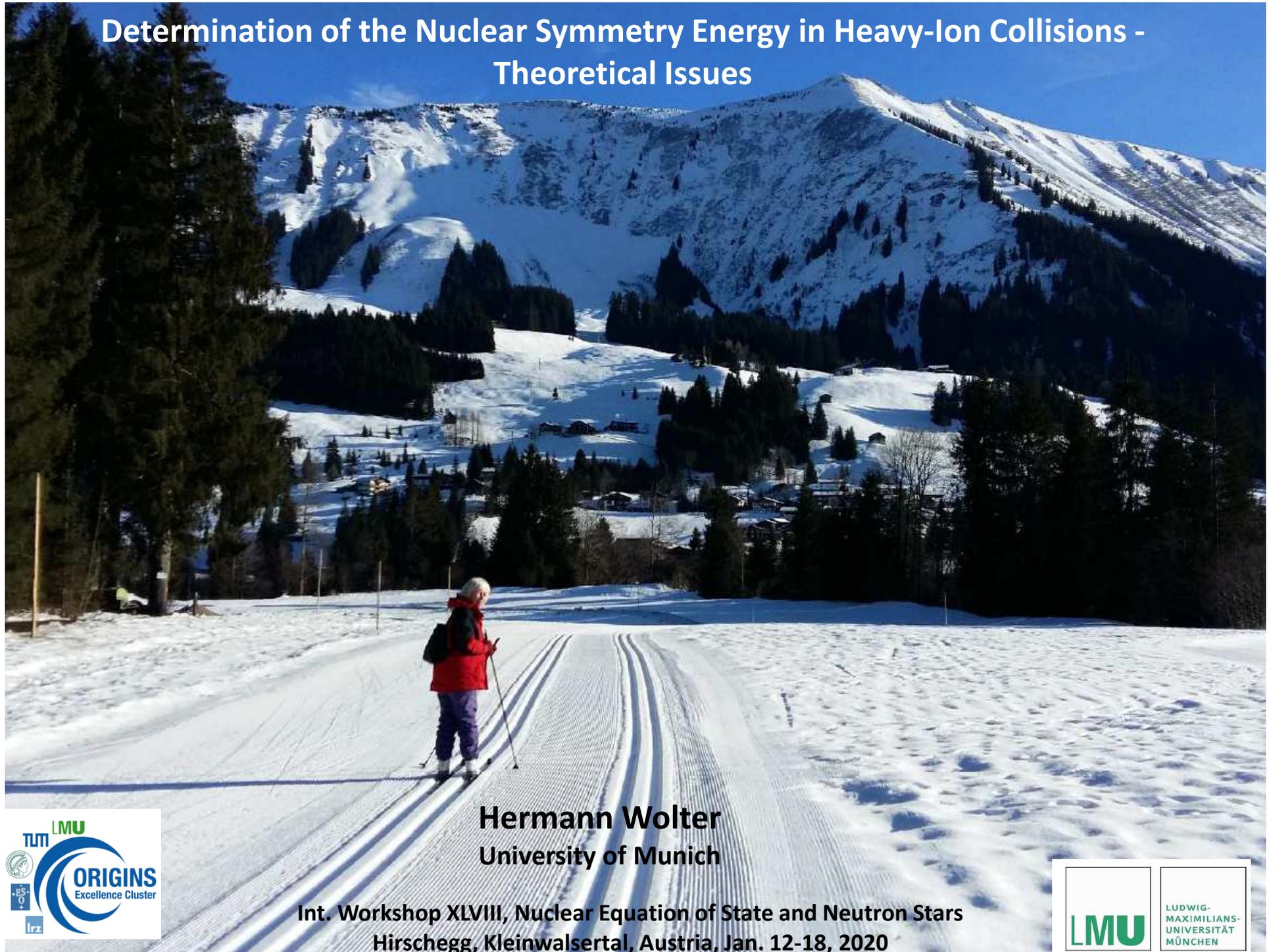


Determination of the Nuclear Symmetry Energy in Heavy-Ion Collisions - Theoretical Issues

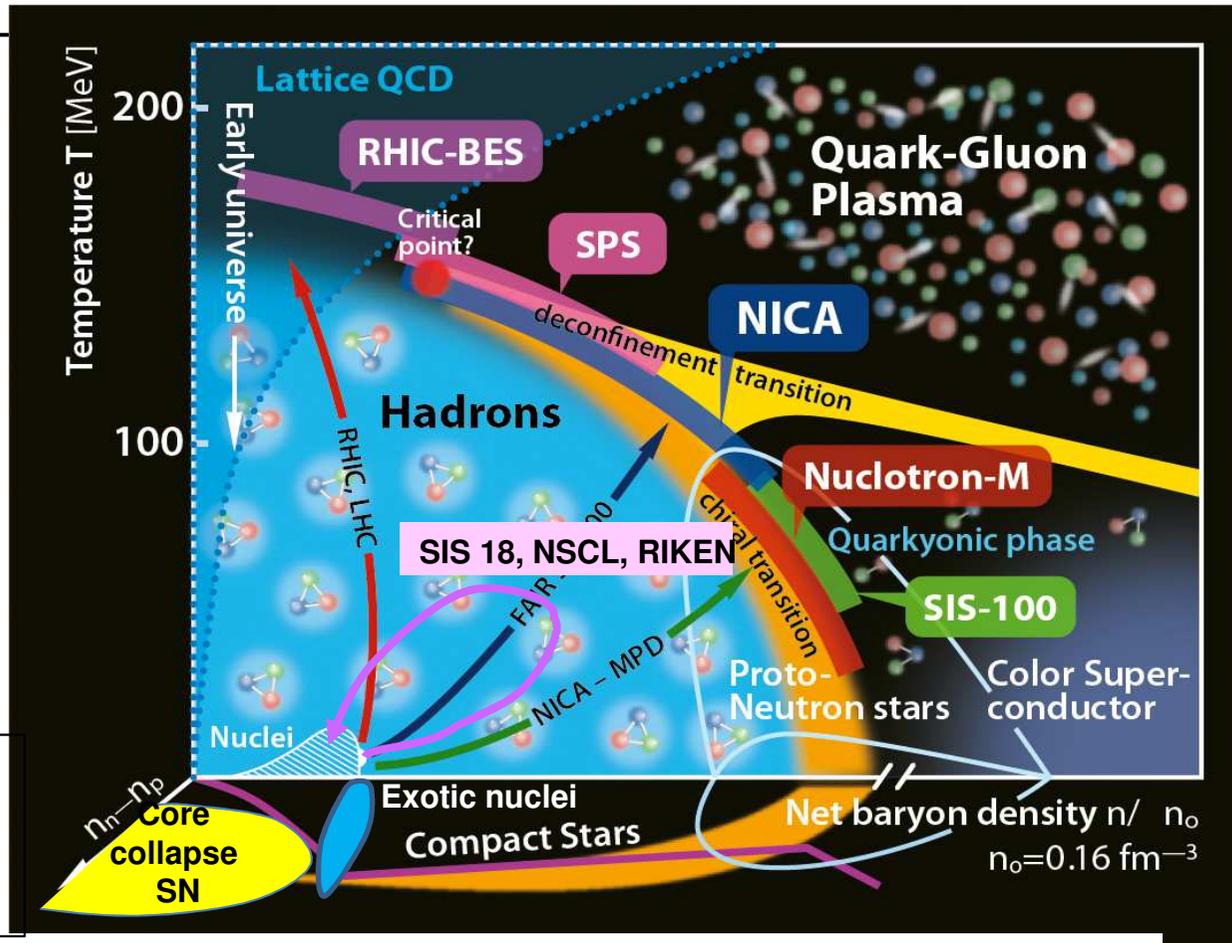


Hermann Wolter
University of Munich

Int. Workshop XLVIII, Nuclear Equation of State and Neutron Stars
Hirschegg, Kleinwalsertal, Austria, Jan. 12-18, 2020



Aim of Heavy-Ion Collisions (HIC): Determine the Phase Diagram of **Strongly Interacting Matter**



Asymmetry axis
--> search for
symmetry
energy

$$E(\rho_B, \delta) / A = E_{nm}(\rho_B) + E_{sym}(\rho_B) \delta^2 + O(\delta^4) + \dots; \quad \delta = \frac{\rho_n - \rho_p}{\rho_n + \rho_p}$$

Extensive efforts by:

- Microscopic theory
- Neutron star observations, NS mergers, NICER
- HI experiments in the hadronic and partonic regime,
**only way to investigate dense, mildly neutron-rich matter
in the lab**

Note: HIC trajectories are non-equilibrium processes
→ **transport theory is necessary**
but should check its robustness

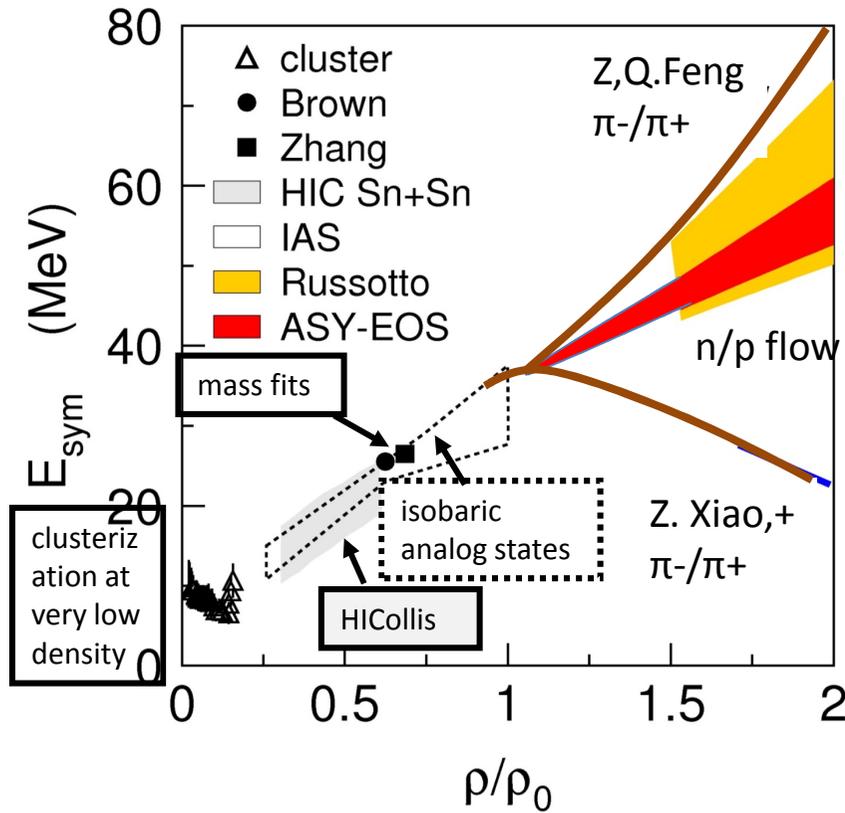
Plan:

Some theoretical issues in the transport description of heavy-ion collisions (HIC).

- **not**: comparison to experimental data (talks of Bill Lynch, Arnauld LeFevre, Abdou Chbihi)
- Transport theory:
approximations, physical ingredients, equation-of-state (EoS)
beyond mean field: fluctuations and correlations
- Robustness of transport model results:
benchmark calculations under controlled conditions:
box calculations with periodic boundary conditions.
- see also: Workshop „Challenges to transport theory for heavy-ion collisions“.
May 20-24, 2019, ECT*, Trento

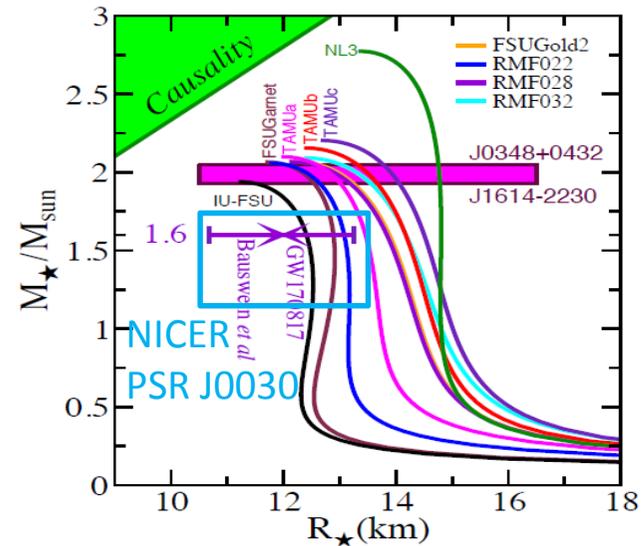
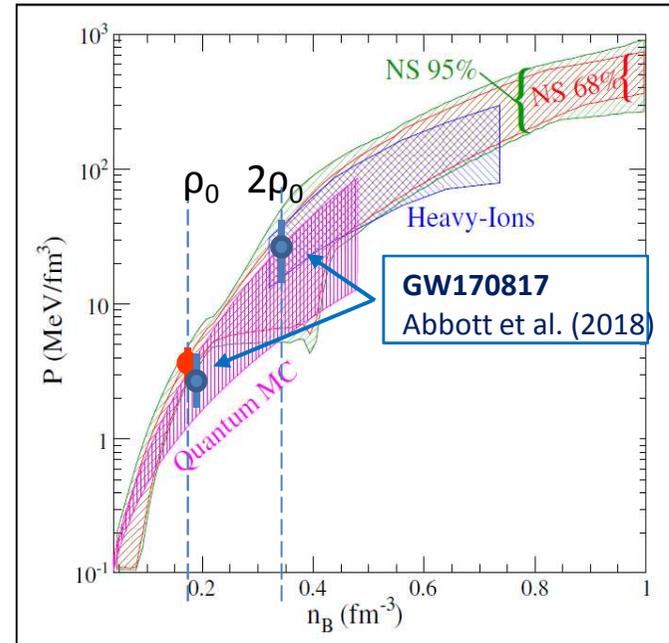
Present state and successes of transport analyses of HIC:

from nuclear physics



constraints from NS masses, radii and mergers

comparison to many-body calc and astrophysics



Remarks on derivation of transport theory for HIC

(e.g. P. Danielewicz, Ann. Phys. 152, 239 (1984), and Transport 2019 workshop, ECT*)

Real-time Green function method: non-equilibrium, many-body

$$G_1(r_1, t_1; r'_1, t'_1) \leftrightarrow G_2(r_1, t_1, r_2, t_2; r'_1, t'_1, r'_2, t'_2) \leftrightarrow G_3(1, 2, 3; 1', 2', 3') \leftrightarrow \dots \text{ BBGKY-Hierarchy}$$

non-equilibrium \rightarrow 2 indep. Greenfcts

Truncation on 1-body level and definition of self energy Σ

$$iG_1(1, 1') = \int d2 d2' d3' G_1(1, 2) \langle 23' | V | 2'3' \rangle G_2(2'3'; 1'3')$$

$$\approx \int d2 d2' G_1(1, 2) \Sigma(2, 2') G_1(2', 1')$$

This neglects higher order correlation effects,

they have to be re-introduced: - in the form of fluctuations (for fragment production)
- explicitly (for light clusters)

Quasi-particle approx.: under slow spatial and temporal changes of the system the Wigner transform of $G^<$ becomes a 1-body phase space density

$$f(r, p; t) = \int dr' e^{ipr'} G_1^<(r + \frac{r'}{2}, r - \frac{r'}{2}; t), \quad r = \frac{1}{2}(r_1 + r_2), \quad r' = (r_1 - r_2)$$

This obeys an evolution equation of the Boltzmann-Vlasov type:

Mean field evolution plus collision term

$$\boxed{\frac{df}{dt} = \frac{\partial f}{\partial t} + \frac{\vec{p}}{m} \vec{\nabla}^{(r)} f - (\vec{\nabla}^{(r)} U(r, p) \vec{\nabla}^{(p)} + \vec{\nabla}^{(p)} U(r, p) \vec{\nabla}^{(r)}) f(\vec{r}, \vec{p}; t) = I_{coll}}$$

Physics ingredients in the transport equation

$$\frac{df}{dt} = \frac{\partial f}{\partial t} + \frac{\vec{p}}{m} \vec{\nabla}^{(r)} f - (\vec{\nabla}^{(r)} U(r, p) \vec{\nabla}^{(p)} + \vec{\nabla}^{(p)} U(r, p) \vec{\nabla}^{(r)}) f(\vec{r}, \vec{p}; t) = I_{coll}$$

$$I_{coll} = \int d\vec{p}_2 d\vec{p}_1 d\vec{p}_2' v_{21} \sigma_{12}^{in-med}(\Omega) (2\pi)^3 \delta(p_1 + p_2 - p_1' - p_2') [f_1' f_2' \bar{f}_1 \bar{f}_2 - f_1 f_2 \bar{f}_1' \bar{f}_2']$$

$\bar{f}_i = (1 - f_i)$ Pauli blocking factors,

a) mean field, EOS,
energy density functional

$$\varepsilon(\rho, \delta) = E_0(\rho) + E_{sym}(\rho) \delta^2 + \dots$$

→ potential,
momentum-dependent

$$U(r, p; \rho, \delta) = \frac{\partial \varepsilon(\rho, \delta)}{\partial f(r, p)} = U_0(r, p; \rho, \delta) + U_{sym}(r, p; \rho, \delta) \delta + \dots$$

↳ effective mass m_n^* , p/n eff. mass splitting $m_n^* - m_p^*$

How to choose EDF:

usual: model with parameters, e.g. Skyrme functional, Relativistic mean field (RMF)

determine EOS by comparison with experiment

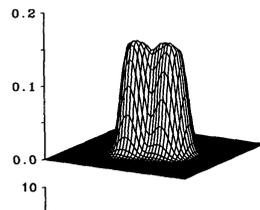
„ab-initio“: Brückner theory, Brückner G-matrix:

$$U = \int G \cdot f; \sigma^{in-med} = |G|^2$$

Temperature

$$U(r, p; \rho, \delta, T)??$$

but non-equilibrium!



in the Brückner approach is taken (partly) into account
by the folding with the non-equilibrium f .

C. Fuchs, et al., NPA 601 (1996) 473 and 505

Physics ingredients in the transport equation

$$\frac{df}{dt} = \frac{\partial f}{\partial t} + \frac{\vec{p}}{m} \vec{\nabla}^{(r)} f - (\vec{\nabla}^{(r)} U(r, p) \vec{\nabla}^{(p)} + \vec{\nabla}^{(p)} U(r, p) \vec{\nabla}^{(r)}) f(\vec{r}, \vec{p}; t) = I_{coll}$$

$$I_{coll} = \int d\vec{p}_2 d\vec{p}_1 d\vec{p}_2' v_{21} \sigma_{12}^{in-med}(\Omega) (2\pi)^3 \delta(p_1 + p_2 - p_1' - p_2') [f_1' f_2' \bar{f}_1 \bar{f}_2 - f_1 f_2 \bar{f}_1' \bar{f}_2']$$

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↳ effective mass m_n^* , p/n eff. mass splitting $m_n^* - m_p^*$

b) collisions

$$\sigma_{NN}^{in-med}$$

medium-modified elast, cross sect., e.g. Brueckner G-Matrix,

inelastic collisions: e.g.

$$NN \leftrightarrow N\Delta \leftrightarrow N\Lambda K$$

$$\downarrow \Delta \leftrightarrow N\pi$$

transfer of
asymmetry

$$\frac{n}{p} \rightarrow \frac{\Delta^{-,0}}{\Delta^{+,++}} \rightarrow \frac{\pi^-}{\pi^+}$$

Coupled transport eqs.

$$Df^{(N)} = I_{coll}(NN \leftrightarrow NN) + I_{coll}(NN \leftrightarrow N\Delta)$$

$$Df^{(\Delta)} = I_{coll}(NN \leftrightarrow N\Delta) + I_{decay}(\Delta \rightarrow N\pi)$$

$$Df^{(\pi)} = I_{coll}(\pi N \leftrightarrow \pi N) + I_{coll}(N\pi \leftrightarrow \Delta)$$

new physics input:
 π, Δ potentials and
inelastic cross sections

Transport eq.: dissipative mean field dynamics, deterministic in principle

In practice two main transport approaches

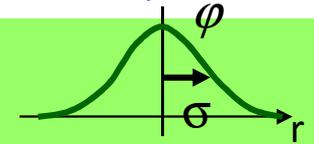
Boltzmann-Vlasov-like (BUU)

$$\left(\frac{\partial}{\partial t} + \frac{\vec{p}}{m} \vec{\nabla}^{(r)} - \vec{\nabla} U(r) \vec{\nabla}^{(p)} \right) f(\vec{r}, \vec{p}; t)$$

$$= I_{coll} [\sigma^{in-med}]$$

Dynamics of the 1-body phase space distribution function f with 2-body dissipation,
 solution by test particle method
 in principle deterministic,
 in practice stochastic simulation

Molecular-Dynamics-like (QMD/AMD)

$$|\Phi\rangle = \mathbf{A} \prod_{i=1}^A \phi(r; r_i, p_i) |0\rangle$$


$$\dot{r}_i = \{r_i, H\}; \quad \dot{p}_i = \{p_i, H\}; \quad H = \sum_i t_i + \sum_{i,j} V(r_i - r_j)$$

TD-Hartree(-Fock)

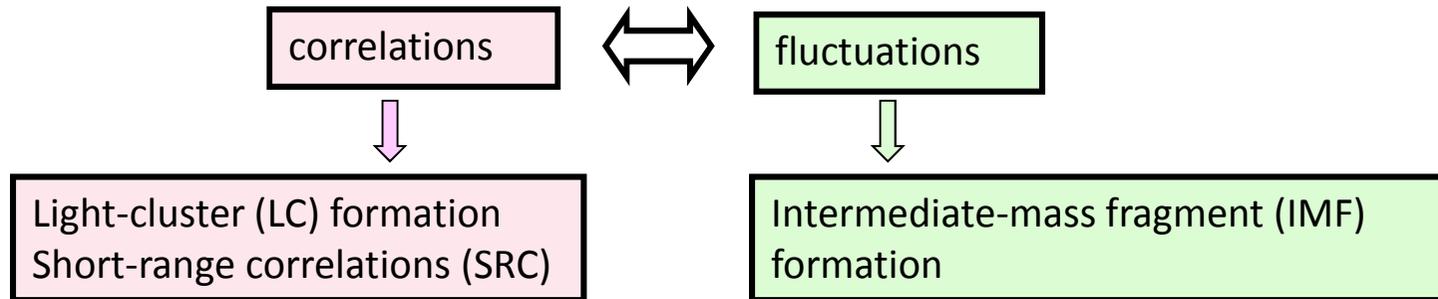
plus stochastic NN collisions

No quantum correlations,
 but classical N-body correlations, damped by
 the smoothing. by wp width

Both BUU and QMD do not naturally have the correct fluctuations and no quantum correlations (except Pauli correlations in AMD))

fluctuations and correlations are effects **beyond a 1-body theory**,
 - but become important in description of HIC, and have to be reintroduced

The issue of fragment and cluster formation in HIC collisions:



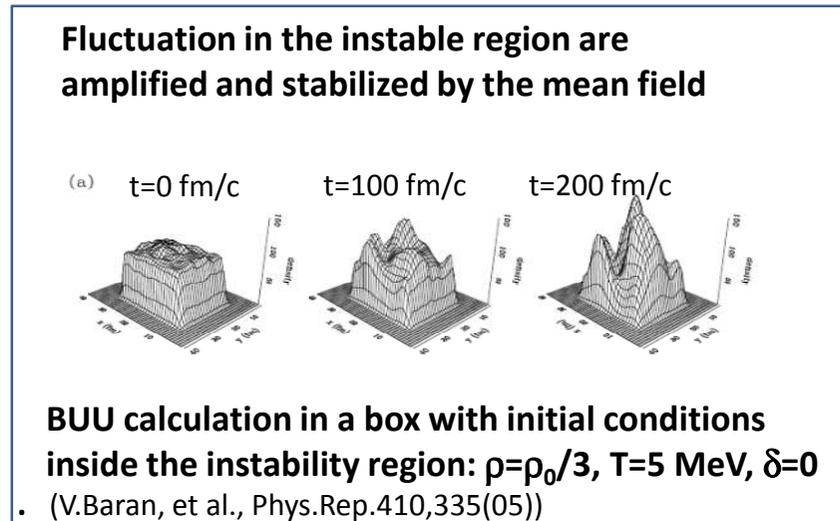
a large fractions of particles in clusters, e.g.

Partitioning of protons

	Xe + Sn 50 MeV/u	Au + Au 250 MeV/u
p	≈10%	21%
α	≈20%	20%
d, t, ^3He	≈10%	40%
$A > 4$	≈60%	18%

INDRA data, Hudan et al., PRC67 (2003) 064613.
FOPI data, Reisdorf et al., NPA 848 (2010) 366.

LC's are not stabilized by the mean field but by few-body correlations.
Introduce **as explicit degrees of freedom**,



Inject fluctuations with amplitudes motivated by physical (not numerical) principles.

We will see that fluctuations are also important in stable situations

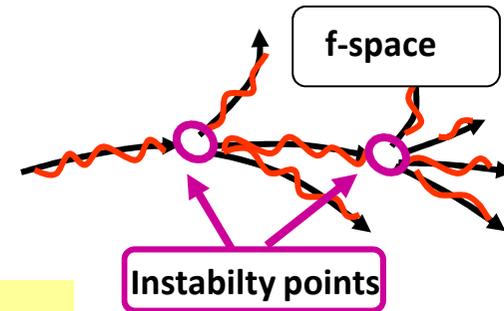
Methods to introduce fluctuations

BUU: phase space distribution f is a statistical quantity with fluctuations, where the av. value $\langle f \rangle$ is determined by the BUU eq.

$$f = \bar{f} + \delta f \quad ; \quad \langle \delta f \rangle = 0, \quad \langle \delta f \delta f \rangle = \sigma^2$$

f now obeys the Boltzmann-Langevin eq. (BL)

$$\frac{df}{dt} = I_{coll} + I_{fluct}$$



The amplitude and spectrum of the fluctuations is the critical question. How to specify?

- fluctuation-dissipation theorem, is given by the 2-body collisions
- general thermal statistical fluctuations of a Fermi system. since fluctuations also arise from the neglected higher order correlations $\sigma^2(r, p) = \bar{f}(r, p)(1 - \bar{f}(r, p))$

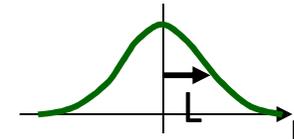
Implementations in BUU: minimize numerical noise (many TPs)

- **SMF (stochastic mean field):** project on density fluctuations (Colonna)
- **BLOB (Boltzmann-Langevin One-Body dynamics)**

Move N_{TP} test particles simultaneously (in p-space) to simulate fluctuation connected to NN collisions (Napolitani)

- **Fokker-Planck-eq.** , apply locally , invoking the dissipation-fluctuation theorem (Hao Li, PD)
- ⇒ ensure global conservation laws

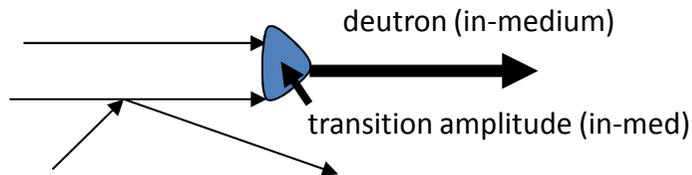
QMD: fluctuations are given by classical molecular dynamics but controlled by wave packet width L :
 „empirical“: $L^2 \sim 2fm^2$



Methods to introduce light cluster (LC) correlations:

pBUU (Danielewicz)

LC as explicit degrees of freedom



→ coupled transport equations for LC

Medium modification of properties and transition amplitudes of light clusters in heavy ion reactions

C. Kuhrt, et al., PRC63 (2001), Typel, Röpke, et al., PRC81 (2010)

Continue for heavier clusters, like t, ^3He , α
 --> coupled transport eqs. but increasingly complicated collision terms and unknown amplitudes

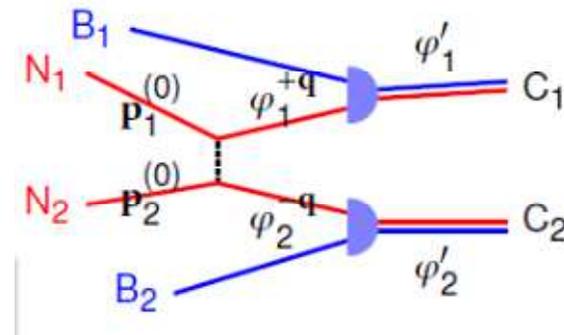
not (yet) included α particle

AMD (Ono)

1. formation of clusters in terms of overlap with cluster wave function

2. put wave packets into the configuration of the cluster, satisfying Pauli principle, but propagate as nucleons

3. include also cluster-cluster collisions to form bigger clusters



Plan:

Some theoretical issues in the transport description of heavy-ion collisions (HIC).

- **not**: comparison to experimental data (talks of Bill Lynch, Arnaud LeFevre, Abdou Chbihi)

- Transport theory:
approximations, physical ingredients, equation-of-state (EoS)
beyond mean field: fluctuations and correlations

- robustness of transport model results:
benchmark calculations under controlled conditions:
box calculations with periodic boundary conditions.

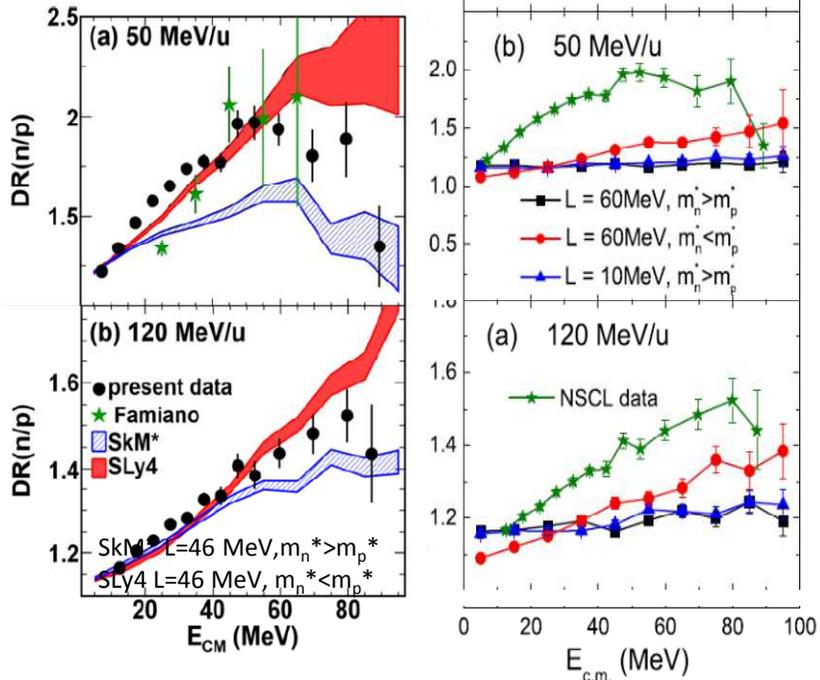
- see also: Workshop „Challenges to transport theory for heavy-ion collisions“.
May 20-24, 2019, ECT*, Trento

Solving the Transport theory by simulations: A need for more consistency. Examples

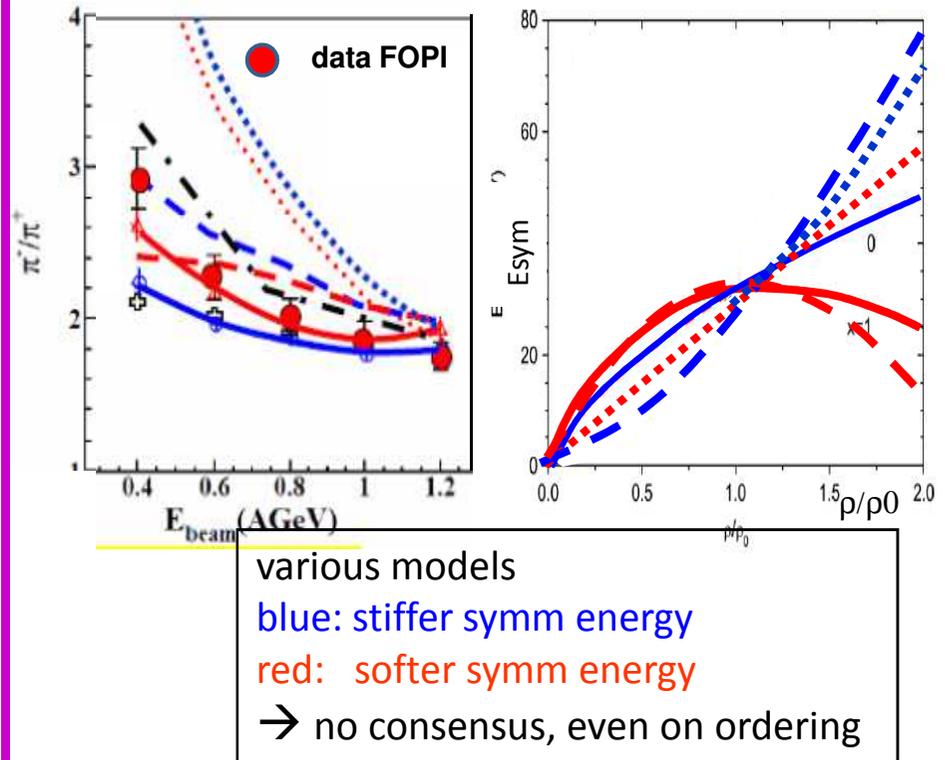
double ratio of n/p pre-equilibrium emiss.

D.D.S.Coupland, et al., PRC94, 011601(R) (2016)

H.J.Kong, et al., PRC91,047601 (2015)



ratio of pion yields, Au+Au, 0.4-1.2 GeV/A



Reasons for differences often not clear, since calculations slightly different in the physical parameters.

→ therefore comparison of calculations with same physical input, i.e. under controlled conditions

Transport Model Evaluation Project

Comparison of all major transport codes under controlled and as far as possible identical conditions.
Steps:

1. Full heavy ion collisions (Au+Au)

a) high energy ~ 1 AGeV: attention to π, K production (Trento 2004)

E. Kolomeitsev, et al., J. Phys. G 31 (2005) S741

b) intermediate energy, 100, 400 AMeV, attention to flow and NN collision rates (Trento 2009 and Shanghai 2014)

J. Xu et al., Phys. Rev. C 93, 064609 (2016)

-> considerable discrepancies, but difficult to disentangle reasons

2. Calculations of nuclear matter (box with periodic boundary conditions)

test separately ingredients in a transport approach:

a) collision term without and with blocking (Cascade) (MSU 2017)

Y.X. Zhang, et al., Phys. Rev. C 97, 034625 (2018)

b) π, Δ production in Cascade (Busan 2018)

A. Ono, et al., Phys. Rev. C 100, 044617 (2019)

c) π, Δ production in a full HIC : Sn+Sn, 270 AMeV, **close to submission**

d) mean field propagation (Vlasov), **in preparation**

e) instabilities, fragmentation **planned**

→ up to 19 codes of BUU- and QMD-type, 1 AMD code

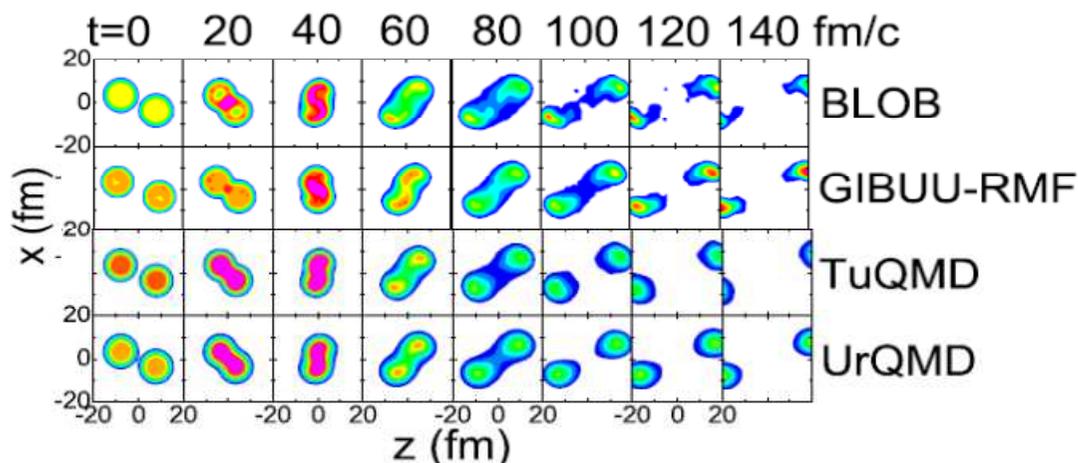
→ non-rel. and relativistic codes

→ BUU codes with explicit fluctuations: SMF, BLOB

→ many new Chinese codes: QMD-XXX: much activity in China, originally closely related

homework 1: full heavy ion collision with identical physics input

Au+Au at $b=7\text{fm}$ (midcentral)
 100 and 400 AMeV,
 selected contour plots;
 different evolution apparent

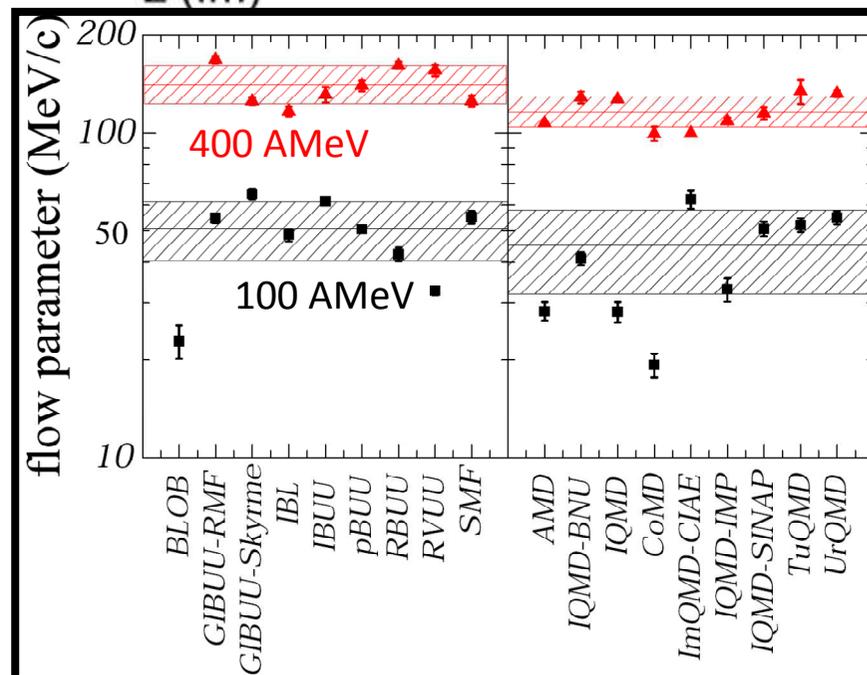


- Differences between codes seen in
 - initialization,
 - density evolution
 - collision rates and blocking of collisions
 --> differences in observables

quantify spread of simulations by value of flow parameter = slope of transverse flow at midrapidity
 BUU and QMD approx. consistent

uncertainty

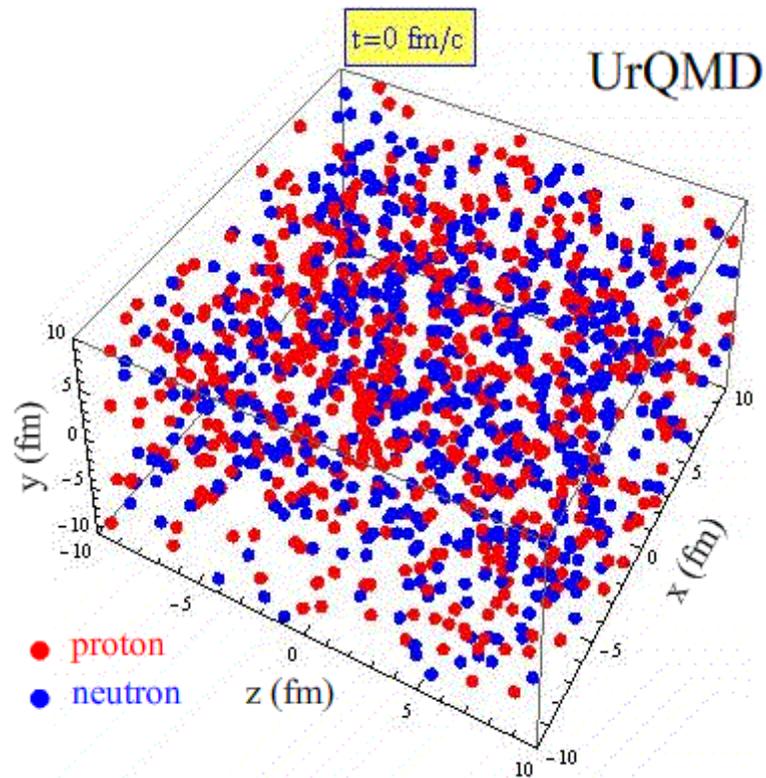
100 AMeV: ~30%
 400 AMeV: ~13%



Difficult to disentangle origin of discrepancies, since effects interact

Box calculation comparison

simulation of the static system of infinite nuclear matter,
→ solve transport equation in a periodic box



Useful for many reasons:

- check thermodynamical consistency of calculation
 - check consistency of simulation:
(often exact limits from kinetic theory)
 - check different aspects of simulation separately
 - Cascade: only collisions
without/with blocking
 - Vlasov: only mean field propagation
- Strategies for particle production, e.g. pions
etc

Box calculations are an important tool to understand transport simulations
They should lead to an improvement and development of transport approaches:

homework 2: Collision term in box calculations

collision probability

$$I_{coll} = \int d\vec{p}_2 d\vec{p}_1 d\vec{p}_2' v_{21} \sigma_{12}^{in-med}(\Omega) (2\pi)^3 \delta(p_1 + p_2 - p_1' - p_2') \left[f_1' f_2' (1-f_1)(1-f_2) - f_1 f_2 (1-f_1')(1-f_2') \right]$$

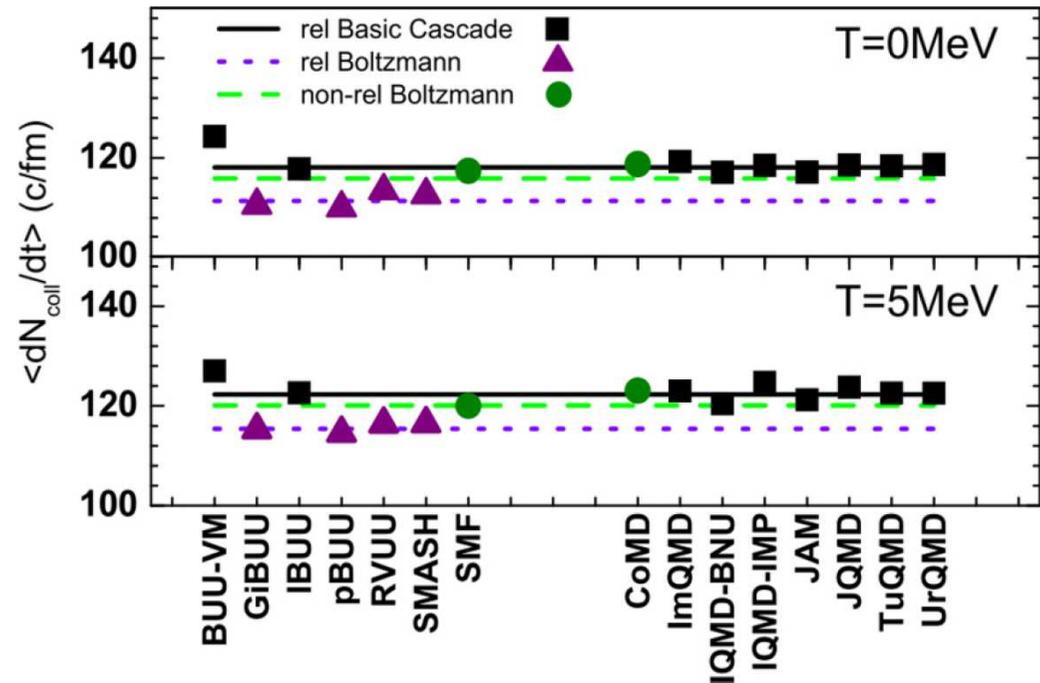
no blocking

Collision rates in a cascade box calculation (w/o mean field, T=0 and 5 MeV)

without blocking
Comparison to exact limit

$$\begin{aligned} \frac{dN_{coll}}{dt} &= \frac{A}{2\rho} g^2 \int \frac{d^3 p d^3 p_1}{(2\pi \hbar)^6} v_{rel} \sigma^{med} f(p) f(p_1) \\ &= \frac{1}{2} A \rho \langle v_{rel} \sigma^{med} \rangle. \end{aligned}$$

(v_{rel} and average depend on treatment of relativity)



good agreement with corresponding exact result
collision probability generally ok
(small deviations can be understood, higher order correlations between collisions)

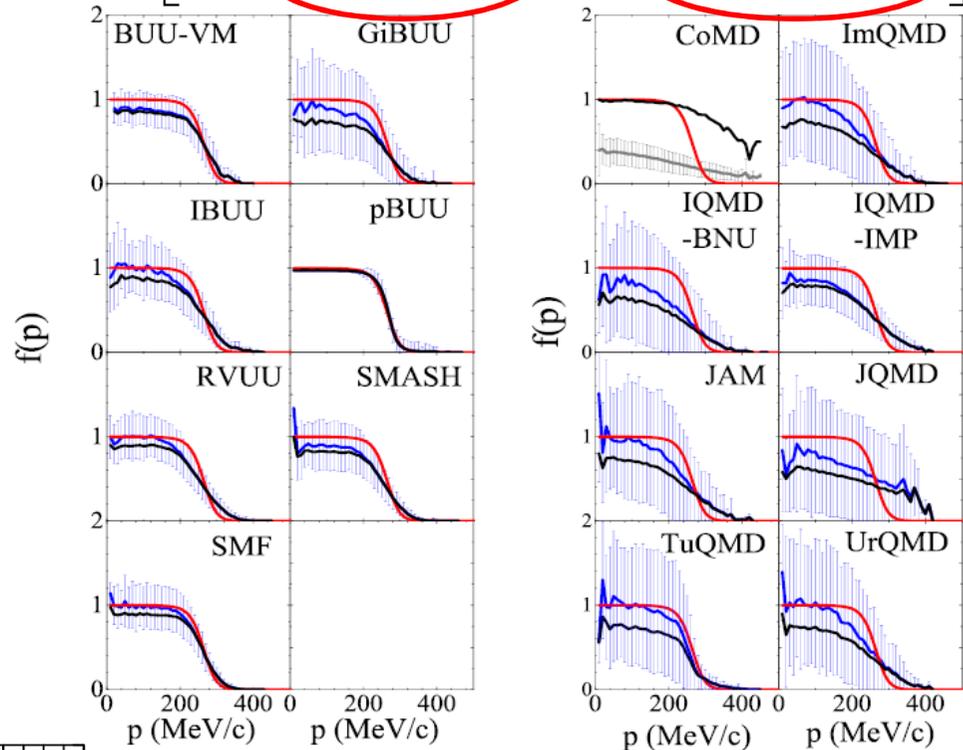
Test of collision integral: Cascade calculation in a box with Pauli blocking

$$I_{coll} = \int d\vec{p}_2 d\vec{p}_1 d\vec{p}_2' v_{21} \sigma_{12}^{in-med}(\Omega) (2\pi)^3 \delta(p_1 + p_2 - p_1' - p_2') \left[f_1' f_2' (1-f_1)(1-f_2) - f_1 f_2 (1-f_1')(1-f_2') \right]$$

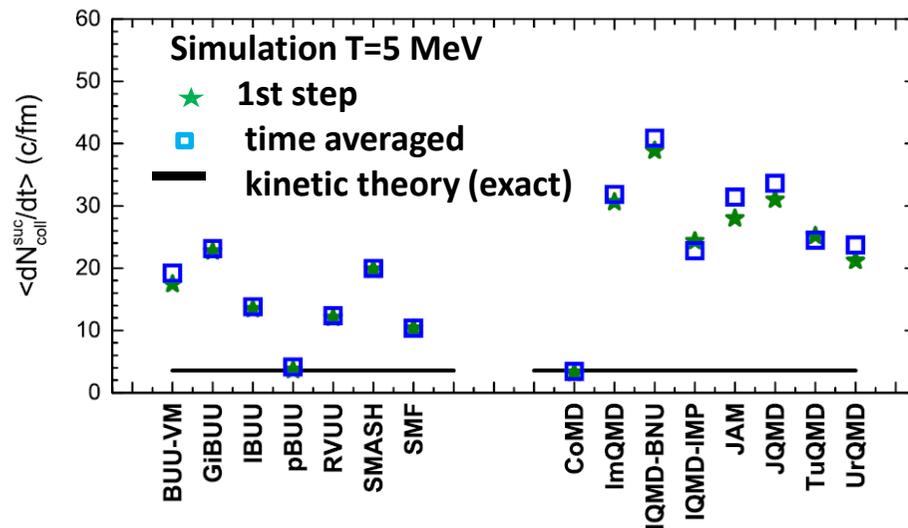
Sampling of occupation prob.
in comp. to prescribed FD distribution (red):
-> large fluctuation, depending on code

width and averages of calculated occupation numbers in different codes

- prescribed occupation
- average calculated occupation
- average of $f < 1$ occupation (used for the blocking)



Collision rates

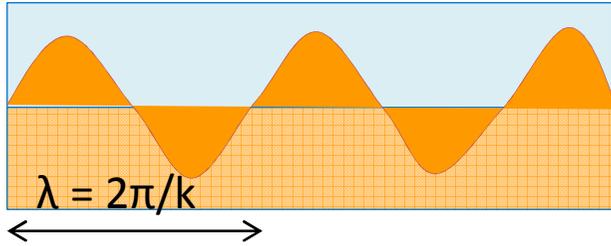


- almost all codes have too little blocking, i.e. allow too many collisions,
- QMD codes more, because of larger fluctuations

- Fluctuations influence dynamics of transport calculations.
- Codes differ in treatment of fluctuations
- Proper treatment of fluctuations in transport under debate (as discussed above).

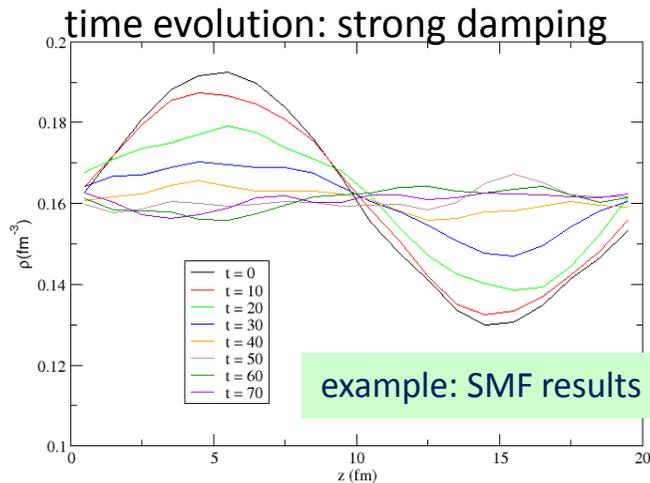
homework 2: Box simulations: test of m.f. dynamics, Vlasov (in preparation)

1. Density oscillations in the **stable** regime, time evolution of $\rho(z)$;



$$\rho(z, t=t_0) = \rho_0 + a_\rho \sin(k_i z)$$

$$k_i = n_i 2\pi/L, \quad a_\rho = 0.2 \rho$$



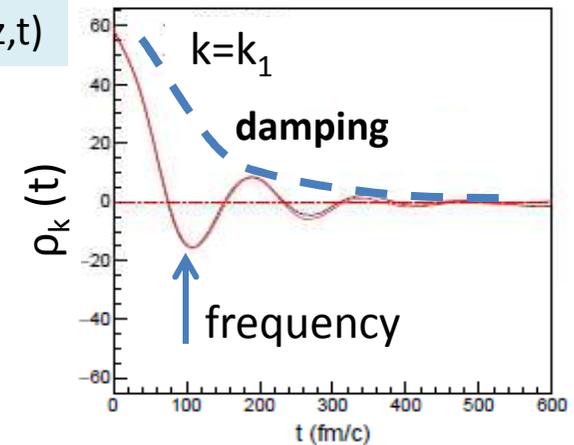
Maria Colonna

-- Symmetric matter --

- Only mean-field potential
- No surface terms
- Compressibility $K=240$ and 500 MeV

2. Fourier transform in space, modes

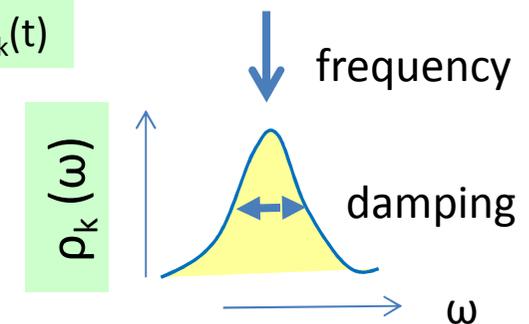
$$\rho_k(t) = \int dz \sin(kz) \rho(z, t)$$



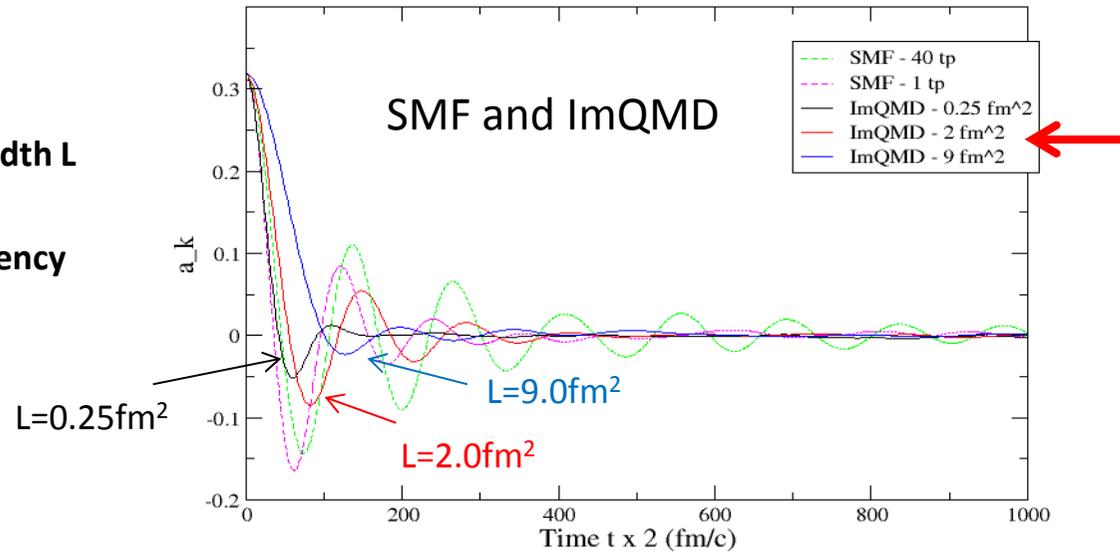
3. Fourier transform in time:

extract response funct., compare to exact result

$$\rho_k(\omega) = \int dt \cos(\omega t) \rho_k(t)$$

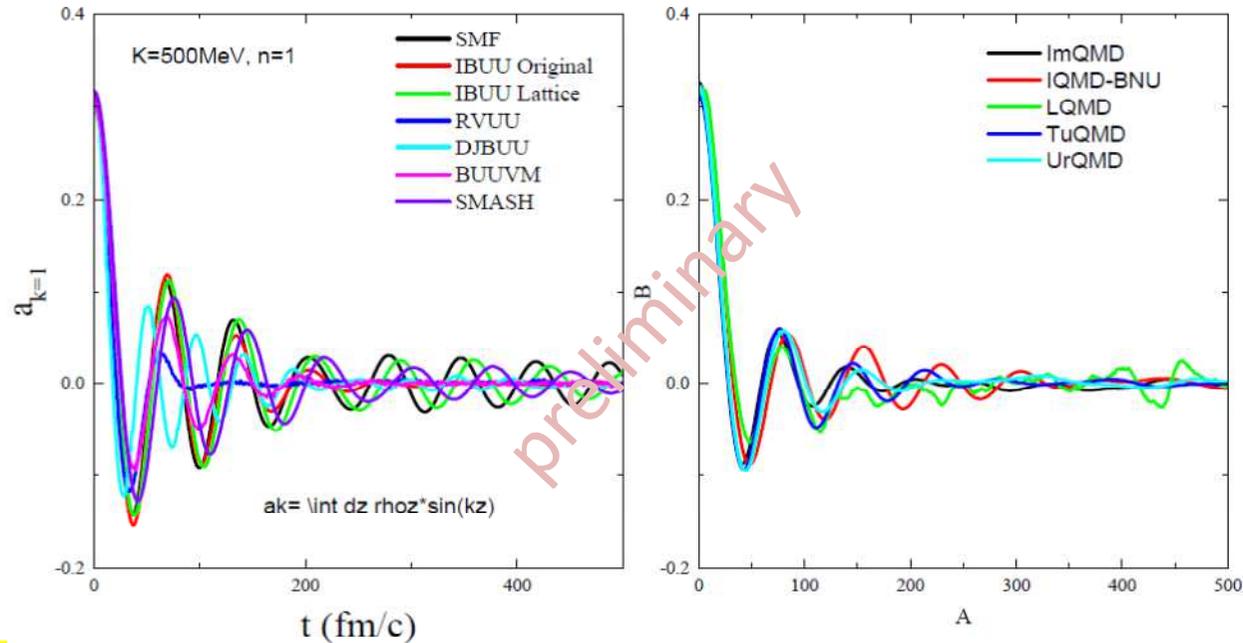


Increasing fluctuation with:
 SMF(BUU) decreasing TP no.
 QMD (ImQMD) decreasing width L
 --> increase damping
 increase somewhat frequency



all codes in the comparison

differences in frequency connected with effect of fluctuations on the calculation of the forces (gradients of potentials)



Fluctuations influence the mt dynamics. The effect will be much stronger in a regime of instability (spinodal region)

homework 3: Pion production in a box

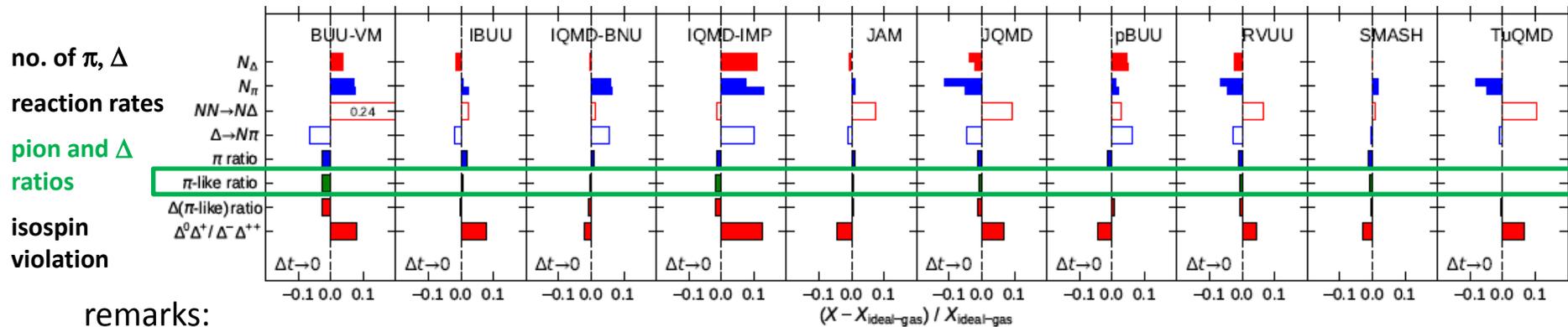
A. Ono, et al., Phys. Rev. C 100, 044617 (2019)

Box calculation, $\rho=\rho_0$, $T=60$ MeV, asymmetry $\delta=0., 0.2$

Cascade calculation: no m.f., no Pauli-blocking, standard x-sections and widths



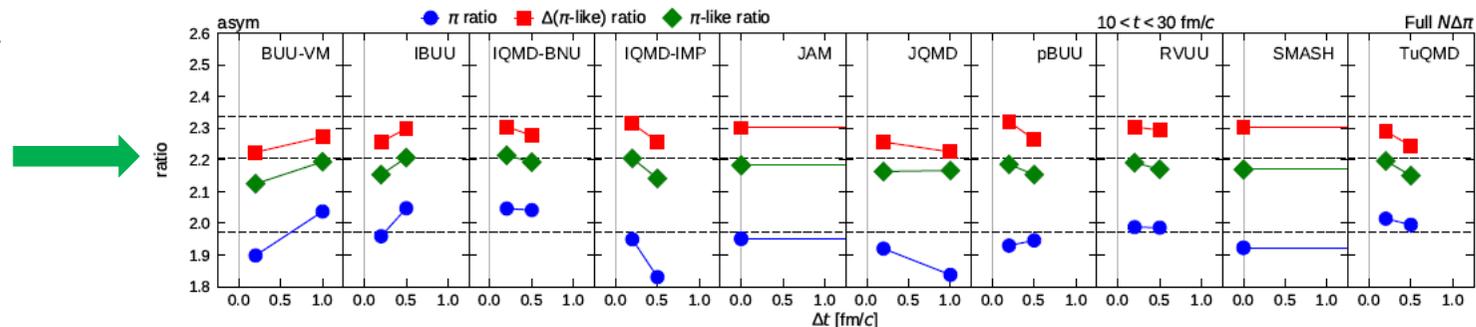
Comparison of results of different codes to exact limit (Boltzmann equilibrium distributions)



remarks:

1. time step in the solution is important. Results extrapolated to $\Delta t \rightarrow 0$
2. Differences in no. of π, Δ ; mostly understood from strategies in handling the sequence of collisions, this may result in unphysical isospin violations
3. simulation and Boltzmann statistics may be different, because of higher order correlations between collisions
4. The pion-like ratio (green) corresponds to the π^-/π^+ ratio in a HIC. A good correspondence between the codes

Still valid in non-equilibrium?



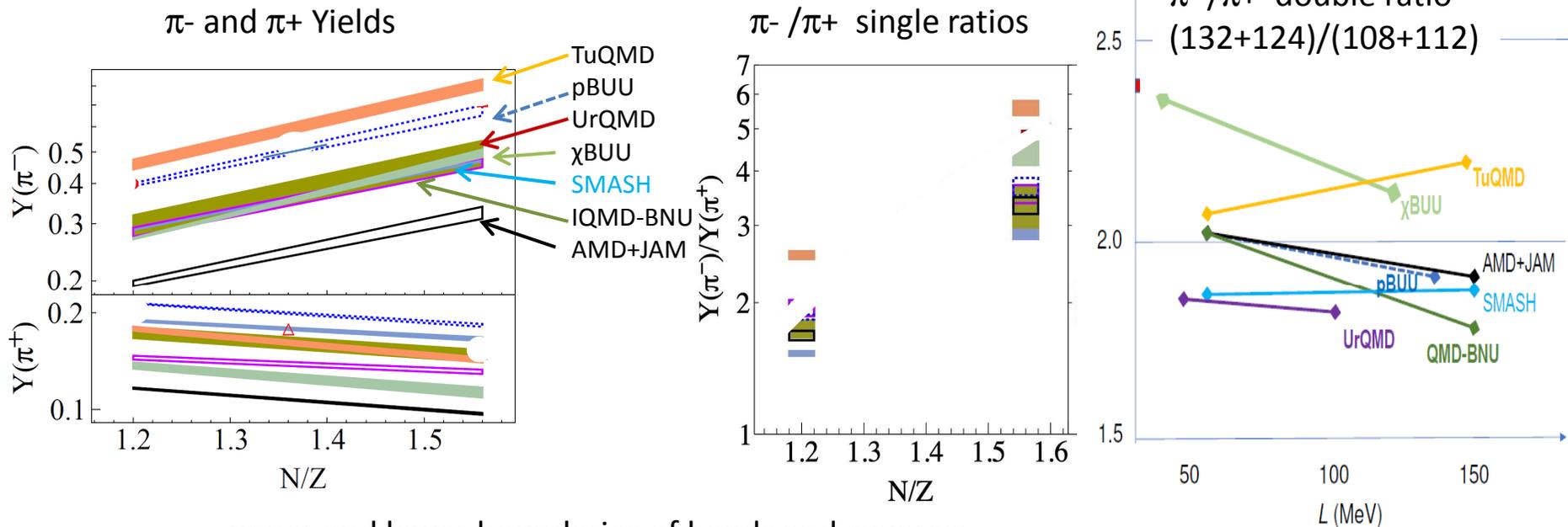
the same comparison (in a different representation) for earlier time (box not equilibrated) shows larger differences (green diamonds).

Pion production in HIC:

$S\pi$ RIT experiment, B. Lynch talk, to be submitted soon
 Sn+Sn @270 AMeV, $b=3$ fm

$^{132}\text{Sn}+^{124}\text{Sn}$ ($N/Z=1.56$)
 $^{112}\text{Sn}+^{124}\text{Sn}$ ($N/Z=1.36$)
 $^{108}\text{Sn}+^{112}\text{Sn}$ ($N/Z=1.2$)

predictions of codes prior to the data



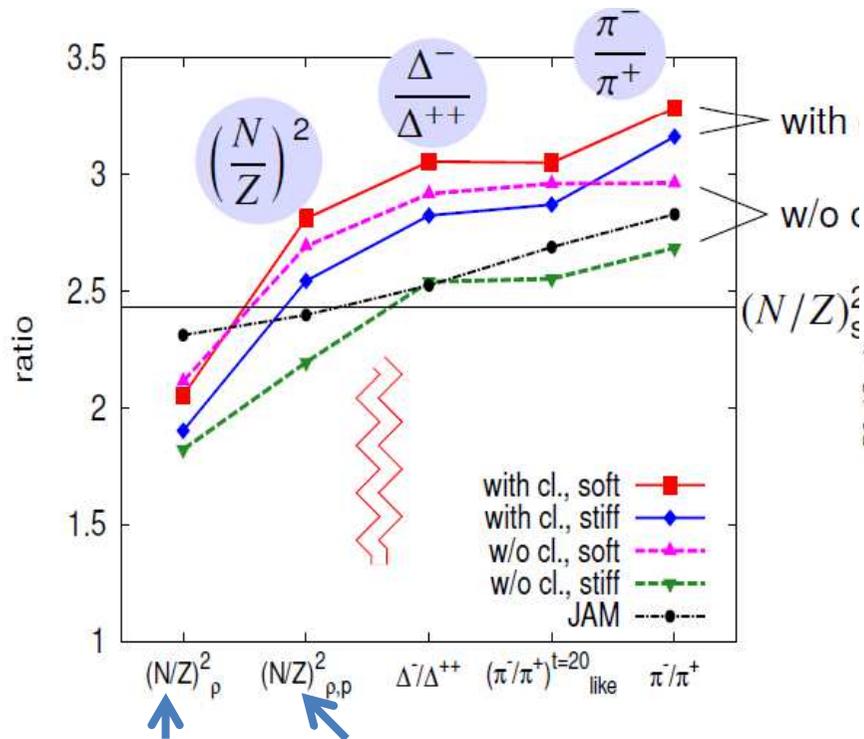
upper and lower boundaries of bands and squares:
 --> stiff and soft symmetry energies for each code

Differences between codes is larger than difference between stiff and soft SE for each code!
 Need to understand better.

Predictions for SpiRIT exp:

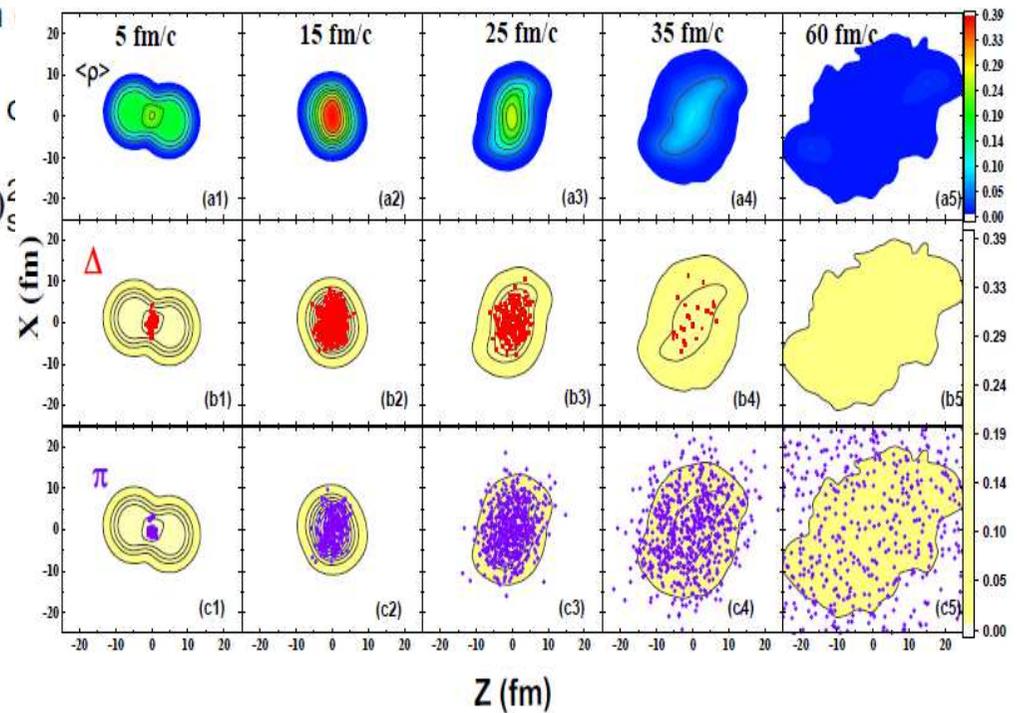
also spectra are not very sensitive

further selection of pion production improves sensitivity (Ono, priv. commun.)

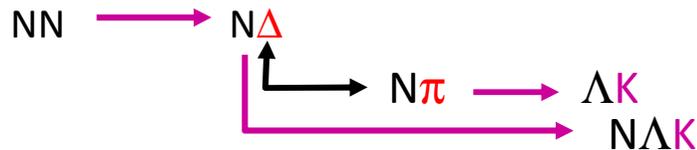


high density and momentum regions

Au+Au $b_0 = 0.00 \sim 0.25$ $E_{\text{beam}} = 0.4$ GeV/nucleon

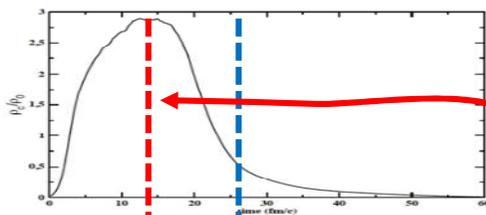


Comment: consider Kaon production again:

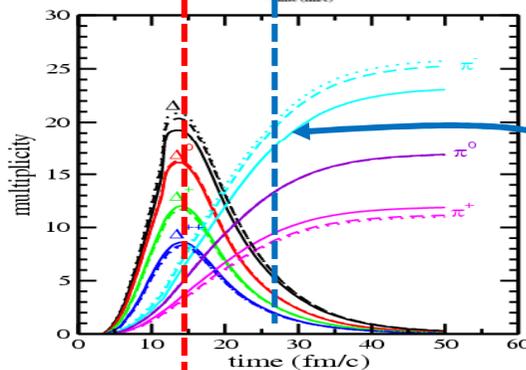


Dynamics

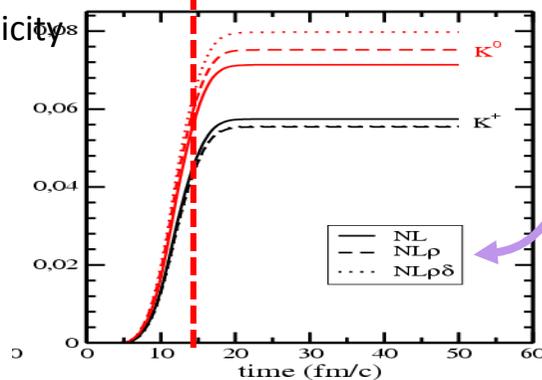
Central density



π and Δ multiplicity



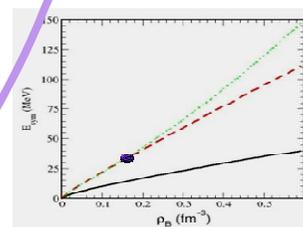
$K^{0,+}$ multiplicity



Δ and K: production in high density phase

Pions: low and high density phase

Sensitivity to asy-stiffness



stiffer



softer

Conclusions:

- Transport theory necessary to interpret HICs
- Allow to investigate the EoS, particularly the symmetry energy, away from saturation
- Important successes, but also
 - questions about effects beyond mean field dynamics
 - consistency of predictions of different codes (implementations)

Review of foundation, ingredients and extensions of transport approach

- ❑ Momentum (temperature?) dependence of potential,
- ❑ In-medium cross sections, elastic, inelastic, treatment of collision term
- ❑ Fluctuations (fragment production, but also blocking, mean field damping)
- ❑ Few-body correlations (Light cluster production, important probes for the symmetry energy and the state of the system), short-range correlations

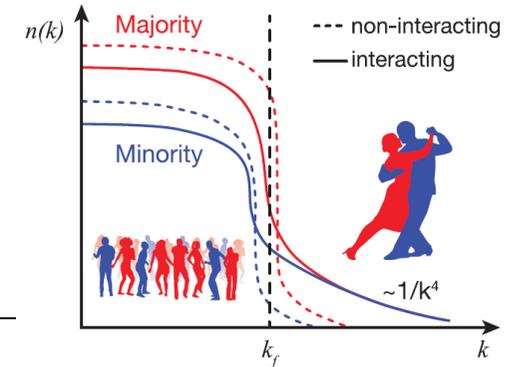
Transport model evaluation project:

- Estimate systematical theoretical uncertainty of transport analyses
- Study effects on results (e.g. fluctuations, higher-order correlations, etc.)
- pion production is interesting but needs better understanding

Thank you for the attention !

Role of Short-Range-Correlations

High momentum tail due to short range correlations.
 In asymmetric nuclear matter, this is different for neutrons and protons, for $k < k_F$, but similar for $k > k_F$. -> **Symmetry energy effect**.
 Could be important in HIC in particle production.



Current debate, how to take into account in transport:

1. Initialize momentum distribution with high momentum tail, e.g. GC Yong, PLB 765 (2017) 104
 Should be quickly lost due to collisions and is not regenerated.
2. Subtract correlation energy from mean field potential, e.g. B.A.Li+, PRC 91, 044601 (2015).

$$E_{\text{sym}}(\rho) = \eta \cdot E_{\text{sym}}^{\text{kin}}(\text{FG})(\rho) + [S_0 - \eta \cdot E_{\text{sym}}^{\text{kin}}(\text{FG})(\rho_0)] \left(\frac{\rho}{\rho_0} \right)^\gamma$$

argument: determination of U_{sym} more realistic, since correlation energy not assumed as symmetry energy.

but: but does not affect kinetic energy and produces no high momentum tails

- 3.? Treat explicitly in 3-body collision, in a sense similar to problem of LC production.

