Fast Rotating Neutron Stars: Spectra and Stability without Approximation

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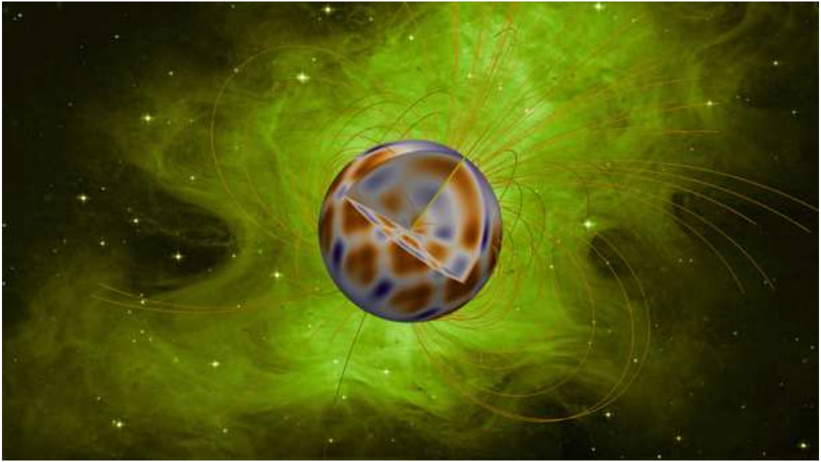
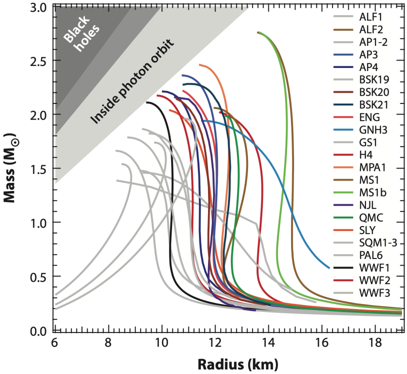
Neutron Stars & their Oscillations



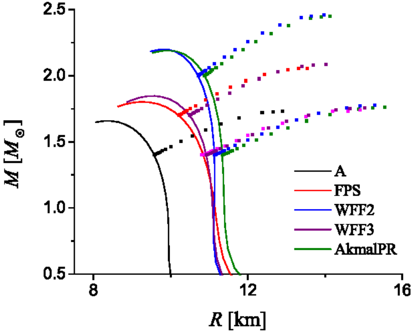
Gravitational Wave Asteroseismology

Static Configurations



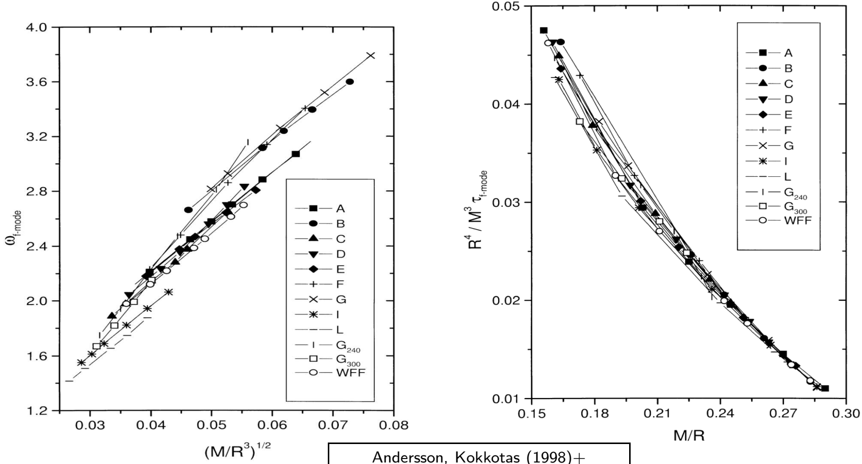
Özel, Freire (2016)

Rotating Configurations



Gravitational Wave Asteroseismology

Universal relations for f-mode frequency and damping time



Andersson, Kokkotas (1998)+

The Rich Spectrum of Neutron Stars

- f/p-modes: main restoring force is pressure
- w-modes: spacetime modes with no Newtonian counterpart
- g-modes: sensitive to thermal/composition gradients
- s-modes: shear waves in the solid crust
- Alfvèn-modes: due to magnetic field
- i-modes: inertial modes present in rotating stars
- Some modes may become unstable due to rotation (r-mode, f-mode, w-mode)

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CFS-Instability

- Non-axisymmetric modes have forward and backward moving pattern as seen by a comoving observer.
- The retrograde mode has negative angular momentum ($J_{comov} < 0$) in the comoving frame.
- For sufficiently fast rotating stars, an inertial observer will also see the retrograde pattern propagate along with the star's rotation ($\omega_{inert} = -\omega_{comov} + m\Omega$), i.e. with a positive angular momentum ($J_{inert} > 0$).
- Emission of GW will radiate angular momentum away from this mode, increasing its amplitude $\rightarrow$ mode becomes unstable (Chandrasekhar 1969, Friedman & Schutz 1978).

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CFS-Instability of the f-mode

- fundamental acoustic mode, present even in uniform density stars.
- Frequency of $1.2 - 3\,\text{kHz}$ for $l = 2$.
- Becomes CFS-unstable for $\beta = T/W \gtrsim 0.14$ in Newtonian theory.
- In GR, the $l = m = 2$ mode becomes unstable already at $\beta \gtrsim 0.06 - 0.08$.

$\rightarrow$ GR enhances CFS-instability!

CFS-Instability of the r-mode

- Has non-zero frequency only in rotating stars (Coriolis force) where inertial modes exist.
- The $l = m = 2$ inertial mode is called r-mode.
- In the comoving frame, the frequency is $\omega = \frac{2m}{l(l+1)}\Omega$.
- $\rightarrow$ r-mode is generically unstable at any rotation rate (Andersson 1998).
- r-mode instability is largely suppressed due to mode couplings.

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Previous Studies

- Cowling (1942): Mode classification
- The CFS-Instability: Chandrasekhar-Friedman-Schutz 1970+
- Spacetime (w-)modes: Kokkotas-Schutz 1986-90+
- Gravitational Wave Asteroseismology: Andersson-Kokkotas 1996+
- *r*-mode instability: Andersson, Friedman-Morsink 1998+
- Determination of QNMs: Detweiler-Lindblom 1983+, Andersson et al. 1995+

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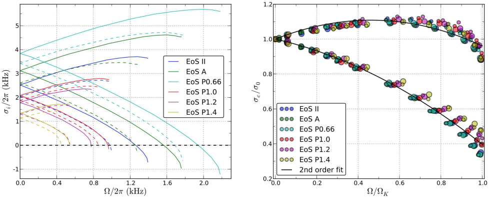
These studies were done for
Non-rotating and
Slowly rotating stars

Previous Studies - Fast Rotation

- Equilibrium Configurations of fast rotating NSs: Bonazzola (1974), Komatsu-Eriguchi-Hachisu (1988), Cook-Shapiro-Stergioulas-Friedman (1995)
- Onset of CFS-instability: Stergioulas-Friedman (1995)
- Oscillations of fast rotating NS (Cowling): Gaertig-Kokkotas (2008)

f-mode in the Cowling Approximation

Frequency



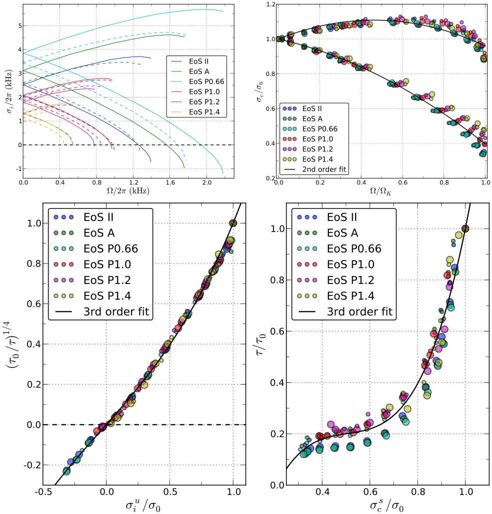
Gaertig, Kokkotas (2008, 2010, 2011)
Doneva, Gaertig, Kokkotas, Krüger (2013)

f-mode in the Cowling Approximation

Frequency

Gaertig, Kokkotas (2008, 2010, 2011)
Doneva, Gaertig, Kokkotas, Krüger (2013)

Damping/Growth Time



f-mode in the Cowling Approximation

Frequency

Gaertig, Kokkotas (2008, ...

Doneva, Gaertig, Kokkot...

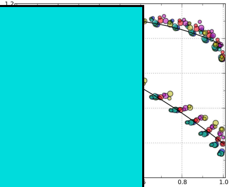
Damping/Growth Time

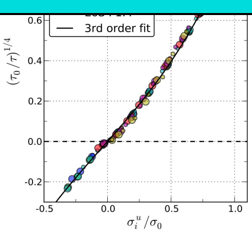
Fitting formulae:

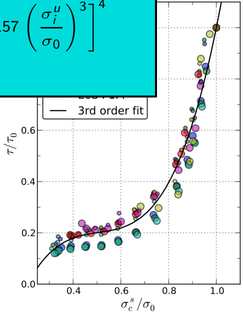
$$\frac{\sigma_c^s}{\sigma_0} = 1 - 0.235\left(\frac{\Omega}{\Omega_K}\right) - 0.358\left(\frac{\Omega}{\Omega_K}\right)^2$$

$$\frac{\sigma_c^u}{\sigma_0} = 1 + 0.402\left(\frac{\Omega}{\Omega_K}\right) - 0.406\left(\frac{\Omega}{\Omega_K}\right)^2$$

$$\frac{\tau_0}{\tau} = \mathrm{sgn}(\sigma_i^u)\left[0.900\left(\frac{\sigma_i^u}{\sigma_0}\right) - 0.057\left(\frac{\sigma_i^u}{\sigma_0}\right)^2 + 0.157\left(\frac{\sigma_i^u}{\sigma_0}\right)^3\right]^4$$







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Gaertig, Kokkotas (2008,
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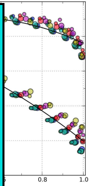

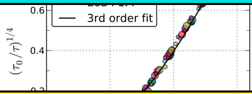

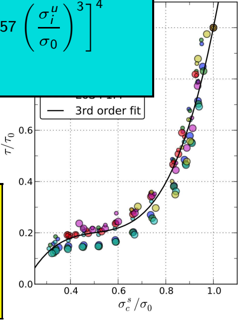

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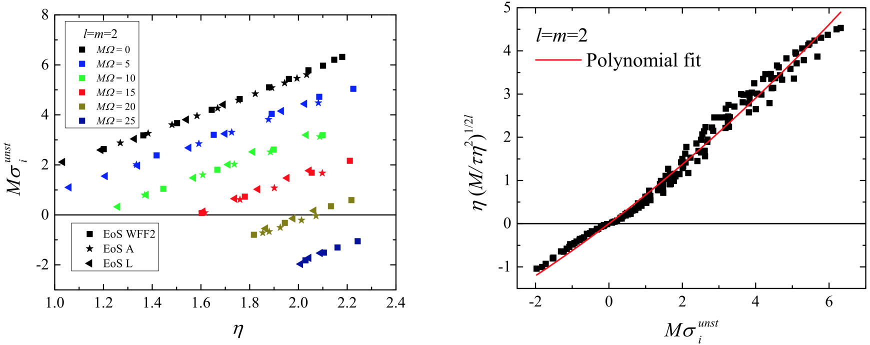
Issue:
20-40% error in frequencies
300% error in damping times

f-mode in the Cowling Approximation



$$M\sigma_i^u = \left(-1.76 - 0.143\hat{\Omega} - 0.0067\hat{\Omega}^2\right) + \left(3.64 - 0.0436\hat{\Omega} + 0.0020\hat{\Omega}^2\right)\eta$$

Independent of particular sequences of stars!

$$\eta \sim \sqrt{M^3/I}$$

Doneva, Kokkotas (2015)

Equations for Equilibrium Configurations

- Einstein equations

$$G_{\mu\nu} = 8\pi T_{\mu\nu} \, , \qquad \nabla_\mu T^{\mu\nu} = 0$$

- Neutron star modelled as perfect fluid

$$T^{\mu\nu} = (\epsilon + p) u^\mu u^\nu + p g^{\mu\nu}$$

- Metric describing a rotating star can be written as

$$ds^2 = -e^{2\nu} dt^2 + e^{2\psi} r^2 \sin^2\theta (d\phi - \omega dt)^2 + e^{2\mu}(dr^2 + r^2 d\theta^2)$$

- Use the rns-code to solve these equations and construct equilibrium models.

Perturbation Equations

- Perturbed Einstein Equations & Conservation of Energy-Momentum

$$\delta G_{\mu\nu} = 8\pi\delta T_{\mu\nu},$$
$$\delta\left(\nabla_\nu T^{\mu\nu}\right) = 0.$$

- Choose Hilbert Gauge:

$$\nabla^\mu h_{\mu\nu} = 0.$$

with $h_{\mu\nu}$ the trace-reversed metric perturbations.

- Wave equations for the metric components:

$$-2\delta G_{\mu\nu} = \Box h_{\mu\nu} + 2R^{\alpha}{}_{\mu}{}^{\beta}{}_{\nu} h_{\alpha\beta} - R^{\alpha}_{[\mu} h_{\nu]\alpha} + R h_{\mu\nu} - g_{\mu\nu} R^{\alpha\beta} h_{\alpha\beta}$$

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Perturbation Equations

$$e^{2\psi-2\nu}\frac{\partial^2}{\partial t^2}\mathcal{P} = \frac{\partial^2}{\partial r^2}\mathcal{P} + \frac{1}{r^2}\frac{\partial^2}{\partial\theta^2}\mathcal{P} + 8e^{4\psi}\pi Q_6 - \left[2\psi_r^2 + \kappa + \frac{4}{r}\psi_r - 2\frac{\cot^2\theta}{r^2}\right]\mathcal{W} + \frac{2}{r}\left[\psi_r + \frac{1}{r}\right]\frac{\partial}{\partial\theta}\mathcal{Q}$$
$$- \left[2\psi_r\nu_r - \kappa + \frac{2\nu_r}{r}\right]\mathcal{H} - \left[2\nu_r\psi_r - 2\psi_r^2 - \kappa + \frac{2}{r}\nu_r - \frac{4}{r}\psi_r - \frac{2}{r^2}\right]\mathcal{K}$$
$$- \left[2\psi_r^2 + \kappa + \frac{4}{r}\psi_r + \frac{2\cot^2\theta}{r^2} + \frac{2}{r^2}\right]\mathcal{P} + \left[\nu_r + \psi_r + \frac{2}{r}\right]\frac{\partial}{\partial r}\mathcal{P} + \frac{\cot\theta}{r^2}\frac{\partial}{\partial\theta}\mathcal{P}$$

$$e^{2\psi-2\nu}\frac{\partial^2}{\partial t^2}\mathcal{Q} = \frac{\partial^2}{\partial r^2}\mathcal{Q} + \frac{\partial^2}{\partial\theta^2}\mathcal{Q} + \frac{4}{r^2}(\psi_r r + 1)\frac{\partial}{\partial\theta}\mathcal{K} - \frac{4}{r^2}(\psi_r r + 1)\frac{\partial}{\partial\theta}\mathcal{P} + \frac{4\cot\theta}{r^2}(\psi_r r + 1)\mathcal{W}$$
$$- \frac{4\cot\theta}{r^2}(\psi_r r + 1)\mathcal{P} - \left[\nu_r^2 + 5\psi_r^2 - 2\nu_r\psi_r + 2\kappa + \frac{10\psi_r - 2\nu_r}{r} + \frac{5 + cot\theta}{r^2}\right]\mathcal{Q} + \frac{2\nu_r}{e^{2\nu}}\frac{\partial}{\partial t}\mathcal{M}$$
$$+ \left[\nu_r + \psi_r + \frac{2}{r}\right]\frac{\partial}{\partial r}\mathcal{Q} + \frac{\cot\theta}{r^2}\frac{\partial}{\partial\theta}\mathcal{Q}$$

$$e^{2\psi-2\nu}\frac{\partial^2}{\partial t^2}\mathcal{W} = [\ldots]$$

$$e^{2\psi-2\nu}\frac{\partial^2}{\partial t^2}\mathcal{H} = [\ldots]$$

$$\vdots = \vdots$$

Krüger, Kokkotas (2019)

Perturbation Equations

$$e^{2\psi-2\nu}\frac{\partial^2}{\partial t^2}\mathcal{P} = \frac{\partial^2}{\partial r^2}\mathcal{P} + \frac{1}{r^2}\frac{\partial^2}{\partial \theta^2}\mathcal{P} + 8e^{4\psi}\pi Q_6 - \left[2\psi_r^2 + \kappa + \frac{4}{r}\psi_r - 2\frac{\cot^2\theta}{r^2}\right]\mathcal{W} + \frac{2}{r}\left[\psi_r + \frac{1}{r}\right]\frac{\partial}{\partial \theta}\mathcal{Q}$$

$$\dots \mathcal{P}$$

These are for non-rotating configurations only!

$$e^{2\psi-2\nu}\frac{\partial^2}{\partial t^2}\mathcal{Q} = \dots + \frac{4\cot\theta}{r^2}(\psi_r r + 1)\mathcal{W}$$

$$- \dots + \frac{5+\cot\theta}{r^2}\Big]\mathcal{Q} + \frac{2\nu_r}{e^{2\nu}}\frac{\partial}{\partial t}\mathcal{M}$$

$$+ \Big[\dots$$

- \+ rotational corrections
- \+ azimuthal dependence
- \+ 8 more equations for spacetime
- \+ 4 fluid equations

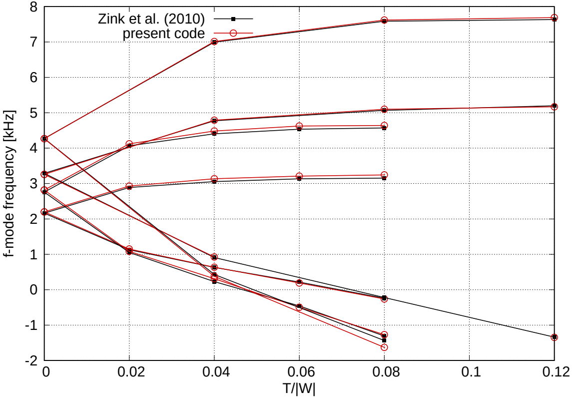
$$e^{2\psi-2\nu}\frac{\partial^2}{\partial t^2}\mathcal{W} = [\dots]$$

$$e^{2\psi-2\nu}\frac{\partial^2}{\partial t^2}\mathcal{H} = [\dots]$$

$$\vdots = \vdots$$

Krüger, Kokkotas (2019)

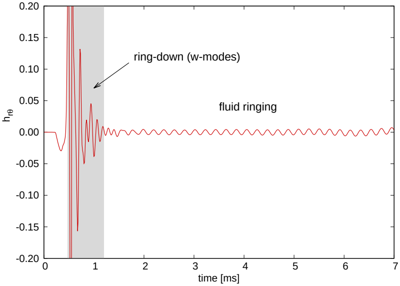
Comparison with Published Values

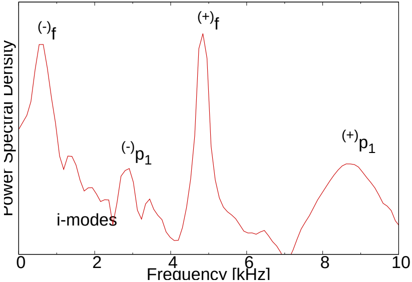


Excellent agreement with values from non-linear codes.

Compared to: Zink et al. (2010), Stavropoulos (2017)

Characteristic example

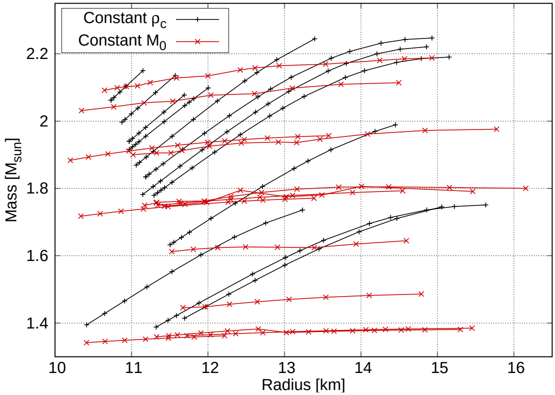




EoS SLy	$M = 2.02\, M_\odot$
$\epsilon_c = 1.2e15$ g/cm^3	$\Omega = 1.3$ kHz
$r_e/r_p = 0.56$	$\Omega/\Omega_K = 0.98$

Krüger, Kokkotas (2019)

Overview of Equilibrium Models



APR
3 - 39 fixed M_0
6 - 58 fixed ϵ_c

SLy
7 - 65 fixed M_0
5 - 63 fixed ϵ_c

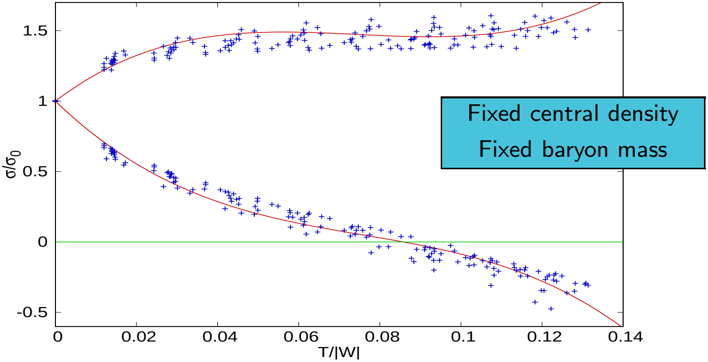
WFF1
3 - 28 fixed M_0
1 - 9 fixed ϵ_c

Polytropes
5 - 52 fixed ϵ_c

- Considered more than 300 equilibrium configurations.

Krüger, Kokkotas (2019)

Fitting formulae – σ/σ_0 vs. T/W



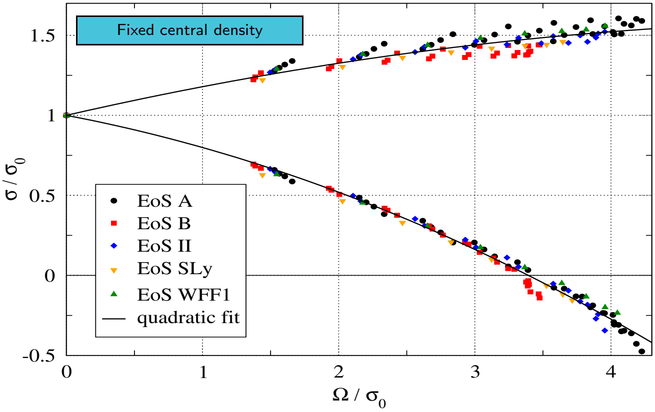
$$\frac{\sigma^u}{\sigma_0} = 1 - 27.8\left(\frac{T}{|W|}\right) + 302\left(\frac{T}{|W|}\right)^2 - 1320\left(\frac{T}{|W|}\right)^3$$

$$\frac{\sigma^s}{\sigma_0} = 1 + 21.9\left(\frac{T}{|W|}\right) - 314\left(\frac{T}{|W|}\right)^2 + 1410\left(\frac{T}{|W|}\right)^3$$

f-mode becomes CFS-unstable when $T/|W| \gtrapprox 0.08$.

Krüger, Kokkotas (2019)

Fitting formulae – σ/σ_0 vs. Ω/σ_0



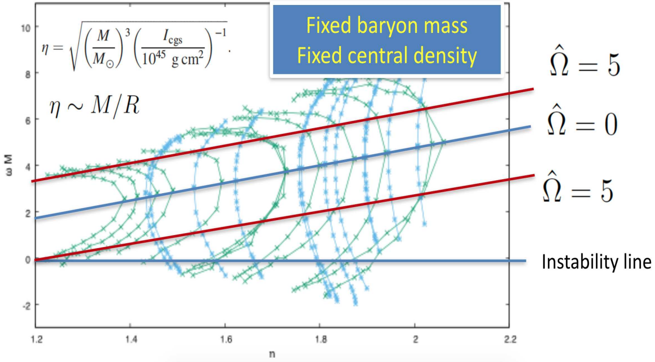
$$\frac{\sigma^u}{\sigma_0} = 1 - 0.162\left(\frac{\Omega}{\sigma_0}\right) - 0.039\left(\frac{\Omega}{\sigma_0}\right)^2$$

$$\frac{\sigma^s}{\sigma_0} = 1 + 0.195\left(\frac{\Omega}{\sigma_0}\right) - 0.016\left(\frac{\Omega}{\sigma_0}\right)^2$$

f-mode becomes CFS-unstable when $\Omega \gtrsim 3.4\sigma_0$.

Krüger, Kokkotas (2019)

Fitting formulae – $M\sigma$ vs. $\hat{\Omega}$ vs η



$$M\sigma_i^u = \left(-2.10 - 0.199\hat{\Omega} - 8.51 \cdot 10^{-3}\hat{\Omega}^2\right) + \left(3.40 + 2.2 \cdot 10^{-3}\hat{\Omega}^2\right)\eta \qquad \hat{\Omega} = M\Omega$$

$$M\sigma_i^u = \left(-2.10 + 0.235\hat{\Omega} - 2.26 \cdot 10^{-3}\hat{\Omega}^2\right) + \left(3.40 + 10.9 \cdot 10^{-3}\hat{\Omega}^2\right)\eta$$

Krüger, Kokkotas (2019)

Non-rotating case: Tsui, Leung (2005) **Cowling case:** Doneva, Kokkotas (2015)

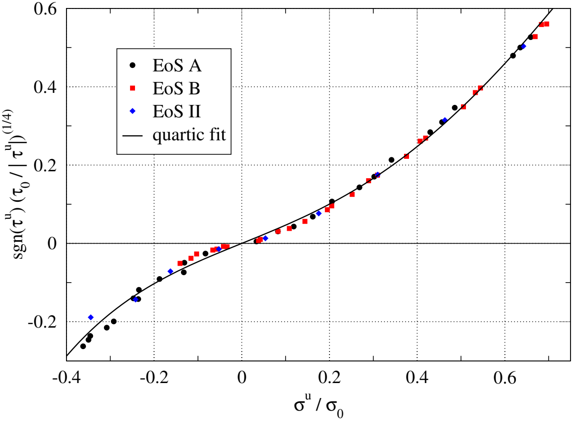
Damping time of the f-mode

- Difficult to determine from the simulation
- Approximate using quadrupole formula:

$$\tau_{\mathrm{GW}} = -\frac{2E_{kin}}{< dE/dt >_{GW}}$$

- Competes with bulk and shear viscosity

Damping time of the f-mode



Very similar relations in Cowling approximation
Error now reduced from 300% in Cowling to 5-15%

Krüger, Kokkotas (2019)

Relevant Astrophysical Scenarios

- Collapse scenarios: excitation of f- and g-modes.
- Tidal effects during inspiral phase of binary mergers:
  - Love numbers (and f-Love-relations)
  - Impact on phase from f-mode resonance
- Early Post-Merger Phase
  - Useful for asteroseismology and constraining EoS
  - Need to implement differential rotation
  - Detection expected towards end of next decade
- Late Post-Merger Phase
  - GW emission depends on the dipole component of the B-field
  - f-mode instability

Thank you

Violation of the Hilbert Gauge