Experiments with hydrogen - discovery of the Lamb shift

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Relativistic heavy ion seminar, October 26, 2006
Outline

1 Pre-Lamb experiment
   - The beginning (Bohr’s formula)
   - Fine structure (Dirac’s equation)
   - Zeeman effect and HFS

2 Lamb experiment
   - Phys. Rev. 72, 241 (1947)
   - Phys. Rev. 72, 339 (1947)

3 Post-Lamb experiment
   - New results
   - High-Z experiment
   - Other two body systems
   - Theory

4 Summary
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The beginning

- why hydrogen?
- "simple" object, only tho bodies: proton and electron
- easy to test theories ⇒ established and ruled out many
The beginning

- why hydrogen?
- "simple" object, only two bodies: proton and electron
- easy to test theories $\Rightarrow$ established and ruled out many

- 1885 Balmer’s simple equation for fourteen lines of hydrogen
- 1887 fine structure of the lines, Michelson and Morley
- 1900 Planck’s quantum theory
Bohr’s formula

- 1913 Bohr derived Balmer’s formula
- point-like character and quantization lead to:

\[ E_n = -\frac{Z^2 \hbar c \text{Ry}}{n^2} \]

- \( \text{Ry} \) is Rydberg wave number
Rydberg constant

- two-photon Doppler-free spectroscopy of hydrogen and deuterium
- measurement of two or more transitions

Value of the Rydberg constant: \( R_{\infty} = 10973731.568 \, [m^{-1}] \)

\[ Ry(1998) = 10973731.568 \, 549(83) \, m^{-1} \]
Fine structure

1916 Sommerfeld: fine structure is the result of relativistic effects
"fine structure", relativistic hydrogen, dependence of energy on eccentricity
for \( n = 2 \) circular \((l = 1)\) and elliptic orbit \((l = 0)\) differ by

\[
\Delta E_2 = \frac{1}{16} \alpha^2 Z^4 \hbar c R \text{y}
\]

fine structure constant is \( \alpha^{-1} = 137.035 \ 999 \ 11(46) \)
Dirac’s equation

1924 De Broglie attributed wave properties to particles
quantum mechanics of hydrogen atom emerge
1925 spin and magnetic moment
1928 Dirac’s equation

\[ E = m_0 c^2 \left[ 1 + \left( \frac{\alpha Z}{n - j - 1/2 + \sqrt{(j + 1/2)^2 - \alpha^2 Z^2}} \right)^2 \right]^{-1/2} \]

- Fine structure \( \Delta j \neq 0 \)
- Lamb shift \( \Delta l \neq 0 \)
- Gross structure \( \Delta n \neq 0 \)
- HFS \( \Delta F \neq 0 \)
Zeeman effect and HFS

- hyperfine structure: interaction of nuclear spin and the angular momentum of orbiting particle
Zeeman effect and HFS

- hyperfine structure: interaction of nuclear spin and the angular momentum of orbiting particle
- external magnetic field

Without spin: \( \Delta E = m_l \frac{e \hbar}{2m} B \)

With spin: \( \Delta E = g \frac{e \hbar}{2m} m_j B \)
Deviations form Dirac theory

- 1938 deviations from Dirac theory for $H_\alpha$ observed by Houston and Williams
- Pasternak suggested that these results could be interpreted as 0.03 $cm^{-1}$ higher $2S_{1/2}$ relative to $2P_{1/2}$
- not enough attention, discrepancies attributed to impurity of the source
Deviations from Dirac theory

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Shelter Island

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- stimulated the creation of QED
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- molecular hydrogen is thermally dissociated
- jet of atoms is cross-bombarded by an electron stream
- one part in a hundred million atoms is excited to $2S_{1/2}$
Lamb shift experiment

- detected by electron ejection from metal target
- between bombardier and detector the atoms are exposed to radio waves
RF region

- good reasons for process being carried out in magnetic field
- energy levels are subject to Zeeman splitting
- frequencies of possible transition depend on magnetic field
- $2S_{1/2}$ lifetime $1/7$ seconds
- $2P_{1/2}$ lifetime $10^{-8}$ seconds
- any perturbation leads to mixing with the $2P_{1/2}$ thus reducing the lifetime
Beam decay

- metastable atoms in the beam are quenched
- atoms in the ground state are unable to eject electrons from the detector
- transition $2S \rightarrow 2P$ induced by RF radiation
- $2P$ decays while moving through a very small distance
Result

- experimental results are shown by circles
- resonant magnetic fields
- solid curves are theoretical predictions
- dashed are solid – 1000 Mc/sec
- not a "best fit"
Hydrogen levels

\[ \begin{align*}
2S_{1/2} & \quad F=0, 1 \\
& \quad 243 \text{ nm}
\end{align*} \]

\[ \begin{align*}
1S_{1/2} & \quad F=0, 1 \\
& \quad 8173 \text{ MHz}
\end{align*} \]

\[ \begin{align*}
2P_{1/2} & \quad F=0, 1 \\
& \quad 2466 \text{ THz}
\end{align*} \]

\[ \begin{align*}
2P_{3/2} & \quad F=0, 1, 2 \\
& \quad 9910 \text{ MHz}
\end{align*} \]

\[ \begin{align*}
& \quad 1058 \text{ MHz}
\end{align*} \]

\[ \begin{align*}
& \quad 59 \text{ MHz}
\end{align*} \]
Bethe’s calculation

- Bethe H.A., Phys. Rev. 72, 339 (1947)
- interaction of electron with radiation field
- "However, it is possible to identify the most strongly (linearly) divergent term in the level shift with an electromagnetic mass effect which must exist for a bound as well as for a free electron."
- subtract it from theoretical expression
- only logarithmic divergence remains in non-relativistic theory
- a relativistic theory should converge
Self-energy

Self-energy of electron in quantum state m

\[ W' = W - W_0 = \frac{2e^2}{3\pi\hbar c^3} \sum_n |v_{mn}|^2 (E_n - E_m) \ln \frac{K}{|E_n - E_m|} \]

- logarithm is very large, independent of n

For S state

\[ W'_ns = \frac{8}{3\pi} \left( \frac{e^2}{\hbar c} \right)^3 Ry \frac{Z^4}{n^3} \ln \frac{K}{<E_n - E_m>_{Av}} = 1040 \text{ megacycles} \]

- \( <E_n - E_m>_{Av} \) is average excitation energy
- \( K \approx mc^2 \) is the natural cut-off from relativity theory
Bethe’s calculation

Conclusions

- Level shift due to interaction with radiation is a real effect.
- Effect of infinite electromagnetic mass can be eliminated.
- Accurate investigation may establish relativistic effects.
- A first major success of the "renormalization idea".
Bethe’s calculation

Conclusions
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- effect of infinite electromagnetic mass can be eliminated
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- a first mayor success of the "renormalization idea"

Consequences
Nobel pize acceptance speech (1955): "It is very important that this problem should receive further experimental and theoretical attention. When an accuracy of comparison of 0.1 \( Mc/sec \) has been reached, it will mean that the energy separation of the 2S and 2P states of hydrogen agree with theory to a precision of few parts in \( 10^9 \) of their binding energy or that the exponent in Coulomb law of force is two with comparable accuracy."
Last results for $H$ and $D$

- hydrogen, hydrogen-like atoms, positronium, muonic atoms
- microwave spectroscopy has been surpassed by optical measurements in the last decade
- accuracy of the radiofrequency measurements is limited by width of the $2P$ state (about 100 MHz)
- direct measurements and fine structure
- optical, two photon Doppler free spectroscopy
Last results for $H$ and $D$

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- last experimental result $H$ (1999): 1057.845(3) MHz
- last theoretical result $H$ (1999): 1057.841(5) MHz
- last experimental result deuterium: 1059.2337(29) MHz
- $He$ exp: 14 041.13(17) MHz
- $He$ th: 14 041.18(13) MHz
U Lamb shift

- 1S (ground state)
  Lamb shift is difference between what Dirac equation predicts and real value (experiment or QED)
- most highly charged ion available in laboratory
- highly relativistic, approaches $Z = 137$

![Graph showing 1s Lamb Shift in U$^{91+}$ with data points and theoretical value.](image)
Lamb shift overview

- perturbative treatment no longer applicable, all orders must be considered
ESR at GSI

- ions injected at 358 MeV/u
- electron cooler is used to cool ions
- up to $10^8$ ions
- ions decelerated to final beam energy of 68 and 49 MeV/u
ESR at GSI

- gas jet target
- electron capture on bare ions
- four Ge detector
- forward left/right symmetry and forward backward symmetry
- fast plastic scintillator
- $469 \pm 13$ eV
Doppler shift

- main limitation: uncertainty in correction for the Doppler shift
- ions striped at high energy (360 MeV/u)
- decelerating ions after striping is improvement

\[
E_{proj} = \gamma (1 - \beta \cos \theta) E_{lab}
\]
\[
\Delta \Omega_{proj} = [\gamma (1 - \beta \cos \theta)]^2 \Delta \Omega_{lab}
\]
The 1s Lamb Shift in Hydrogen-Like Ions

(Experiment - Theory) / Theory

Relative Deviation

Z

Double Laser
Recoil Ions
Beam (decel) Gas
Beam Foil
Electron Cooler
Beam Foil (FRS)
Electron Cooler
Gas Target
Bevalac

5 ppm
1.5%
2%
4%
3.4%
2.8%
Positronium and muonium

- Muonium is a bound system of muon and electron.
  - Lifetime: $2.2 \cdot 10^{-6}$ sec
  - Experimental: $2\,455\,528\,941.0(9.8)$ MHz, Theoretical: $2\,455\,528\,934.0(0.3)$ MHz
- Positronium is a bound system of positron and electron.
  - Lifetime: para $1.25 \cdot 10^{-10}$ sec, ortho $1.4 \cdot 10^{-7}$ sec
  - Experimental: $1\,233\,607\,216.4(3.2)$ MHz, Theoretical: $1\,233\,607\,222.2(6)$ MHz

Hydrogen atom

- Fine structure:
  - $2p_{3/2}$
  - $2s_{1/2}$
  - $2p_{1/2}$

- Lamb splitting

Positronium

- Fine structure:
  - $2^3S_1$
  - $2^3P_2$
- Lamb splitting:
  - $2^3P_1$
  - $2^3P_0$
  - $2^1S_0$

Muonic hydrogen

- Fine structure:
  - $2p_{3/2}$
  - $2p_{1/2}$
- Lamb splitting:
  - $2^3S_1$
  - $2^3P_2$
  - $2^1P_1$
  - $2^3P_1$
  - $2^3P_0$
  - $2^1S_0$
Theory

- gross structure: \( E_n = -\frac{(Z\alpha)^2 mc^2}{2n^2} \)
- reduced mass correction: \( m \rightarrow \frac{Mm}{M+m} \)
- relativistic corrections, hyperfine structure, recoil correction, nuclear-structure correction
- QED corrections: self energy, radiative with, vacuum polarization, etc.
## Various contributions

<table>
<thead>
<tr>
<th>Contribution</th>
<th>Hydrogen-like electronic atom</th>
<th>Positronium</th>
<th>Hydrogen-like muonic atom</th>
</tr>
</thead>
<tbody>
<tr>
<td>Schrödinger eq.</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>- with $M = \infty$</td>
<td>1</td>
<td>1</td>
<td>1</td>
</tr>
<tr>
<td>- with $m_R$ (corr.)</td>
<td>$m/M$</td>
<td>1</td>
<td>$m/M$</td>
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<tr>
<td>Relativistic corr.</td>
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<td></td>
<td></td>
</tr>
<tr>
<td>- Dirac equation</td>
<td>$(Z\alpha)^2$</td>
<td>$\alpha^2$</td>
<td>$(Z\alpha)^2$</td>
</tr>
<tr>
<td>- Two-body effects</td>
<td>$(Z\alpha)^2 m/M$</td>
<td>$\alpha^2$</td>
<td>$(Z\alpha)^2 m/M$</td>
</tr>
<tr>
<td>QED</td>
<td></td>
<td></td>
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<tr>
<td>- Self energy</td>
<td>$\alpha(Z\alpha)^2 \ln(Z\alpha)$</td>
<td>$\alpha^3 \ln \alpha$</td>
<td>$\alpha(Z\alpha)^2 \ln(Z\alpha)$</td>
</tr>
<tr>
<td>- Radiative width</td>
<td>$\alpha(Z\alpha)^2$</td>
<td>$\alpha^3$</td>
<td>$\alpha(Z\alpha)^2$</td>
</tr>
<tr>
<td>- Vacuum pol.</td>
<td>$\alpha(Z\alpha)^2$</td>
<td>$\alpha^3$</td>
<td>$\alpha \ln(Z\alpha m/m_e)$</td>
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<tr>
<td>- Annihilation</td>
<td></td>
<td></td>
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<tr>
<td>– virtual</td>
<td>–</td>
<td>$\alpha^2$</td>
<td>–</td>
</tr>
<tr>
<td>– real</td>
<td>–</td>
<td>$\alpha^3$</td>
<td>–</td>
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<tr>
<td>Nuclear effects</td>
<td></td>
<td></td>
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<tr>
<td>- Magnetic moment</td>
<td>$(Z\alpha)^2 m/M$</td>
<td>$\alpha^2$</td>
<td>$(Z\alpha)^2 m/M$</td>
</tr>
<tr>
<td></td>
<td>or $\alpha(Z\alpha) m/m_p$</td>
<td></td>
<td>or $\alpha(Z\alpha) m/m_p$</td>
</tr>
<tr>
<td>- Charge distribution</td>
<td>$(Z\alpha mcR_N/\hbar)^2$</td>
<td>–</td>
<td>$(Z\alpha mcR_N/\hbar)^2$</td>
</tr>
</tbody>
</table>
Bethe-Salpeter equation

- the theory agrees with experiment very well
- to find energy levels of any composite system
- positions of the poles of respective Greens functions

\[ \hat{G} = S_0 + S_0 K_{BS} \hat{G} \]

For Lamb shift:
Bethe-Salpeter equation

- the theory agrees with experiment very well
- to find energy levels of any composite system
- positions of the poles of respective Greens functions

\[ \hat{G} = S_0 + S_0 K_{BS} \hat{G} \]

For Lamb shift:

\[ e \]
Lamb shift from QED

\[ \begin{align*}
\text{Pre-Lamb experiment} & \quad \text{Lamb experiment} & \quad \text{Post-Lamb experiment} \\
\text{Summary} & \\
\end{align*} \]
Lamb shift from QED

\[
\Delta E = \left\{ \left[ \frac{1}{3} \ln \frac{m(Z\alpha)^{-2}}{m_r} + \frac{11}{72} \right] \delta_{l_0} - \frac{1}{3} \ln k_0(n, l) \right\} \frac{4\alpha(Z\alpha)^4}{\pi n^3} \left( \frac{m_r}{m} \right)^3 m
\]

- \(m_r\) is reduced mass
- \(\ln k_0(n, l)\) is Bethe logarithm, can be calculated to arbitrary accuracy
- normalized infinite sum of matrix elements of the coordinate operator over the Schrödinger-Coulomb wave function
Higher order contributions

- one-loop and two loop self-energy
- highest order terms important for the comparison of theory and experiment are $\alpha(Z\alpha)^7$ and $\alpha^2(Z\alpha)^6$
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- highest order terms important for the comparison of theory and experiment are \( \alpha (Z\alpha)^7 \) and \( \alpha^2 (Z\alpha)^6 \)

\[
\Delta E = -6.862(1) \frac{\alpha^2 (Z\alpha)^5}{\pi n^3} \left( \frac{m_r}{m} \right)^3 m \delta_{l0} = -296.92 \ (4 \ \text{kHz}|n=1)
\]

\[
= -37.115 \ (5 \ \text{kHz}|n=2)
\]
Higher order contributions

Figure: Six gauge invariant sets of diagrams for corrections of order $\alpha^2 (Z\alpha)^5 m$
Future

Experimental future

- The agreement between theory and experiment is extraordinary.
- Direct microwave techniques reached their peak, room for improvement: fine structure measurements.
- The real future are two photon Doppler shift experiments.
- GSI future, Lamb shift studies at the ESR electron cooler.
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Theoretical future
- obviously even higher corrections $\alpha(Z\alpha)^8$, $\alpha(Z\alpha)^9$, etc.
- however progress is mostly limited by the need to use experimental values for fundamental constants
Future at GSI

- a micro-strip Ge detector, detector 48 vertical strips, 128 horizontal strips
- Focusing Compensated Asymmetric Laue (FOCAL) geometry

View of experimental chamber
Summary

- spectrum of hydrogen has stimulated development of physics in general and quantum mechanics in particular
- many cornerstones of physics
- Lambs shift discovery revitalized development of QED
- today hydrogen and hydrogen like spectroscopy serves as a extreme precision test of QED, with unprecedented accuracy
- similar measurement still being performed
- Lamb shift remains at the forefront of physics all this time
- there are still things that can be learned
Bethe’s calculation

Self-energy of electron in quantum state $m$

\[ W = -\frac{2e^2}{3\pi\hbar c^3} \int_0^K kdk \sum_n \frac{|v_{mn}|^2}{E_n - E_m + k} \]

- where $k = \hbar\omega$ is energy of the quantum
- $v = p/m = (\hbar/im)\Delta$ is velocity

Self-energy of free electron

\[ W_0 = -\frac{2e^2}{3\pi\hbar c^3} \int kdk \frac{v^2}{k} \]