Mechanik

$$E = T + V \qquad \vec{F}^{(ij)} = -\vec{F}^{(ji)}$$

$$T_{trans} = \frac{m}{2}\dot{\vec{x}}^2 \qquad T_{rot} = \frac{J}{2}\dot{\varphi}^2$$

$$\vec{F} = m\ddot{\vec{x}} = \dot{\vec{p}} \qquad \vec{F} = -\vec{\nabla} V$$

$$\vec{L} = \vec{R} \times \vec{P} + \vec{L}_{int} \qquad \dot{\vec{L}} = \vec{N} = \vec{x} \times \vec{F}$$

$$V = -\int_{\vec{x}_0}^{\vec{x}} d\vec{x}' \cdot \vec{F}(\vec{x}') \qquad W = \int_C d\vec{x} \cdot \vec{F}(\vec{x})$$

$$\vec{R} = \frac{\int dV \rho(\vec{x})\vec{x}}{\int dV \rho(\vec{x})} \qquad M = \int \mathrm{d}V \rho(\vec{x})$$

$$M = m_1 + m_2 \qquad \vec{r} = \vec{x}^{(1)} - \vec{x}^{(2)}$$

$$\mu = \frac{m_1 m_2}{m_1 + m_2} \qquad \mu\ddot{\vec{r}} = \vec{F}$$

$$J = \int \mathrm{d}V\ \rho(\vec{x})\,(\vec{n} \times \vec{x})^2 \qquad J = J^S + M\vec{R}^2$$

$$J_{ij} = \int \mathrm{d}V\ \rho(\vec{x})\,(|\vec{x}|^2\delta_{ij} - x_i x_j) \qquad J_{ij} = J_{ij}^S + M(\vec{R}^2\delta_{ij} - R_i R_j)$$

$$m_i\ddot{\vec{x}}^{(i)} = \vec{F}^{(i)} + \sum_{j=1}^{k} \lambda_j \vec{\nabla}^{(i)} f_j, \quad f_j(\vec{x}^{(1)}, \ldots, \vec{x}^{(N)}, t) = 0$$

$$\left(\sum_{i=1}^{N} m_i \ddot{\vec{x}}^{(i)} - \vec{F}^{(i)}\right) \delta\vec{x}^{(i)} = 0$$

$$L = T - V \qquad \frac{d}{dt}\frac{\partial L}{\partial \dot{q}_j} - \frac{\partial L}{\partial q_j} = 0 \quad \text{für} \quad j = 1, \ldots, s$$

$$L = \frac{1}{2}\left(\dot{q}^T \underline{M}\dot{q} - q^T \underline{K} q\right) \qquad M_{ij} = \left.\frac{\partial^2 T}{\partial \dot{q}_i\, \partial \dot{q}_j}\right|_{q=0} \qquad K_{ij} = \left.\frac{\partial^2 V}{\partial q_i\, \partial q_j}\right|_{q=0}$$

$$\underline{M}\ddot{q} + \underline{K}q = 0 \qquad \det(\underline{K} - \omega^2 \underline{M}) = 0$$

$$S = \int_{t_1}^{t_2} \mathrm{d}t\, L(q, \dot{q}, t)\,, \qquad \delta S = 0$$

$$H = \sum_{i=1}^{s} \dot{q}_i p_i - L(q, \dot{q}(q, p, t), t) \qquad p_i = \frac{\partial L}{\partial \dot{q}_i}$$

$$\dot{q}_i = \frac{\partial H}{\partial p_i} \qquad -\dot{p}_i = \frac{\partial H}{\partial q_i} \qquad \frac{\partial H}{\partial t} = -\frac{\partial L}{\partial t}$$

$$\{f,g\}_{q,p} = \sum_{j=1}^{s}\left(\frac{\partial f}{\partial q_j}\frac{\partial g}{\partial p_j} - \frac{\partial f}{\partial p_j}\frac{\partial g}{\partial q_j}\right) \qquad \frac{\mathrm{d}f}{\mathrm{d}t} = \{f,H\}_{q,p} + \frac{\partial f}{\partial t}$$

$$\{F,G\}_{q,p} = \{F,G\}_{Q,P} \qquad \text{Falls} \quad \frac{\mathrm{d}A}{\mathrm{d}t} = \frac{\mathrm{d}B}{\mathrm{d}t} = 0 \quad \Rightarrow \quad \{H,\{A,B\}\} = \frac{\partial}{\partial t}\{A,B\}$$

$$\dot{q}_i = \{q_i,H\}_{q,p} \qquad \{q_i,q_j\}_{q,p} = \{p_i,p_j\}_{q,p} = 0$$

$$\dot{p}_i = \{p_i,H\}_{q,p} \qquad \{q_i,p_j\}_{q,p} = -\{p_j,q_i\}_{q,p} = \delta_{ij}$$

$$\{f,g\} = -\{g,f\} \Rightarrow \{f,f\} = 0 \quad \forall f$$

$$\{c,g\} = 0 \quad \forall g \text{ und } c = \text{const.}$$

$$\{f,gh\} = g\{f,h\} + \{f,g\}h$$

$$\{c_1 f_1, +c_2 f_2, g\} = c_1\{f_1,g\} + c_2\{f_2,g\}$$

$$\{f,\{g,h\}\} + \{g,\{h,f\}\} + \{h,\{f,g\}\} = 0$$

Allgemeine mathematische Formeln

Taylorreihen:

$$f(x) = f(0) + \sum_n \frac{d^n f(x)}{dx^n}\bigg|_{x=0} \frac{x^n}{n!}$$

$$\sin x = x - \frac{x^3}{3!} + \cdots + \frac{(-1)^n x^{2n+1}}{(2n+1)!} + \ldots \qquad (x \in \mathbb{R})$$

$$\cos x = 1 - \frac{x^2}{2!} + \cdots + \frac{(-1)^n x^{2n}}{(2n)!} + \ldots \qquad (x \in \mathbb{R})$$

$$e^x = 1 + x + \frac{x^2}{2} + \cdots + \frac{x^n}{n!} + \ldots \qquad (x \in \mathbb{R})$$

$$\frac{1}{1-x} = 1 + x + x^2 + \cdots + x^n + \ldots \qquad (|x| < 1)$$

Zylinderkoordinaten:

$$\vec{x} = \begin{pmatrix} \rho\cos\varphi \\ \rho\sin\varphi \\ z \end{pmatrix}, \quad \vec{e}_\rho = \begin{pmatrix} \cos\varphi \\ \sin\varphi \\ 0 \end{pmatrix}, \quad \vec{e}_\varphi = \begin{pmatrix} -\sin\varphi \\ \cos\varphi \\ 0 \end{pmatrix}, \quad \vec{e}_z = \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix}$$

$$h_\rho = 1 \qquad h_\varphi = \rho \qquad h_z = 1$$

$$dV = \rho\, d\rho\, d\varphi\, dz$$

$$\vec{\nabla} f = \frac{\partial f}{\partial\rho}\vec{e}_\rho + \frac{1}{\rho}\frac{\partial f}{\partial\varphi}\vec{e}_\varphi + \frac{\partial f}{\partial z}\vec{e}_z$$

$$\vec{\nabla}\cdot\vec{A} = \frac{1}{\rho}\frac{\partial}{\partial\rho}\left(\rho A_\rho\right) + \frac{1}{\rho}\frac{\partial A_\varphi}{\partial\varphi} + \frac{\partial A_z}{\partial z}$$

$$\vec{\nabla}\times\vec{A} = \left(\frac{1}{\rho}\frac{\partial A_z}{\partial\varphi} - \frac{\partial A_\varphi}{\partial z}\right)\vec{e}_\rho + \left(\frac{\partial A_\rho}{\partial z} - \frac{\partial A_z}{\partial\rho}\right)\vec{e}_\varphi + \frac{1}{\rho}\left(\frac{\partial}{\partial\rho}(\rho A_\varphi) - \frac{\partial A_\rho}{\partial\varphi}\right)\vec{e}_z$$

$$\Delta f = \frac{1}{\rho}\frac{\partial}{\partial\rho}\left(\rho\frac{\partial f}{\partial\rho}\right) + \frac{1}{\rho^2}\frac{\partial^2 f}{\partial\varphi^2} + \frac{\partial^2 f}{\partial z^2}$$

Kugelkoordinaten:

$$\vec{x} = \begin{pmatrix} r\sin\theta\cos\varphi \\ r\sin\theta\sin\varphi \\ r\cos\theta \end{pmatrix}, \quad \vec{e}_r = \begin{pmatrix} \sin\theta\cos\varphi \\ \sin\theta\sin\varphi \\ \cos\theta \end{pmatrix}, \quad \vec{e}_\theta = \begin{pmatrix} \cos\theta\cos\varphi \\ \cos\theta\sin\varphi \\ -\sin\theta \end{pmatrix}, \quad \vec{e}_\varphi = \begin{pmatrix} -\sin\varphi \\ \cos\varphi \\ 0 \end{pmatrix}$$

$$h_r = 1 \qquad h_\theta = r \qquad h_\varphi = r\sin\theta$$

$$dV = r^2\sin\theta\, dr\, d\varphi\, d\theta$$

$$\vec{\nabla} f = \frac{\partial f}{\partial r}\vec{e}_r + \frac{1}{r}\frac{\partial f}{\partial\theta}\vec{e}_\theta + \frac{1}{r\sin\theta}\frac{\partial f}{\partial\varphi}\vec{e}_\varphi$$

$$\vec{\nabla}\cdot\vec{A} = \frac{1}{r^2}\frac{\partial}{\partial r}(r^2 A_r) + \frac{1}{r\sin\theta}\left(\frac{\partial}{\partial\theta}(\sin\theta\, A_\theta) + \frac{\partial A_\varphi}{\partial\varphi}\right)$$

$$\begin{aligned}\vec{\nabla}\times\vec{A} = {}& \frac{1}{r\sin\theta}\left(\frac{\partial}{\partial\theta}(A_\varphi\sin\theta) - \frac{\partial A_\theta}{\partial\varphi}\right)\vec{e}_r + \frac{1}{r}\left(\frac{1}{\sin\theta}\frac{\partial A_r}{\partial\varphi} - \frac{\partial}{\partial r}(rA_\varphi)\right)\vec{e}_\theta + \\ & + \frac{1}{r}\left(\frac{\partial}{\partial r}(rA_\theta) - \frac{\partial A_r}{\partial\theta}\right)\vec{e}_\varphi\end{aligned}$$

$$\Delta f = \frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2\frac{\partial f}{\partial r}\right) + \frac{1}{r^2\sin\theta}\left(\frac{\partial}{\partial\theta}\left(\sin\theta\frac{\partial f}{\partial\theta}\right) + \frac{1}{\sin\theta}\frac{\partial^2 f}{\partial\varphi^2}\right)$$

Vektoridentitäten:

$$\vec{a}\times\vec{b}=\begin{pmatrix}a_1\\a_2\\a_3\end{pmatrix}\times\begin{pmatrix}b_1\\b_2\\b_3\end{pmatrix}=\begin{pmatrix}a_2b_3-a_3b_2\\a_3b_1-a_1b_3\\a_1b_2-a_2b_1\end{pmatrix}$$

$$\vec{a}\times(\vec{b}\times\vec{c})=(\vec{a}\cdot\vec{c})\,\vec{b}-(\vec{a}\cdot\vec{b})\,\vec{c}$$

$$\int_V d^3r\;\vec{\nabla}\cdot\vec{A}=\int_{\partial V}d\vec{\sigma}\cdot\vec{A}\qquad\qquad \int_F d\vec{\sigma}\cdot(\vec{\nabla}\times\vec{A})=\oint_{\partial F}d\vec{s}\cdot\vec{A}$$

Integrale:

$$\int_0^{2\pi}\mathrm{d}x\,\sin^2 x=\int_0^{2\pi}\mathrm{d}x\,\cos^2 x=\pi\qquad \int_0^{2\pi}\mathrm{d}x\,\sin x\,\cos x=0\qquad \int_0^{\pi}\mathrm{d}x\,\sin^3 x=\frac{4}{3}$$

$$\int\mathrm{d}x\,\frac{1}{\sqrt{x^2+a^2}}=\ln\left(x+\sqrt{x^2+a^2}\right)+C\qquad \int\mathrm{d}x\,\frac{1}{(x^2+a^2)^{3/2}}=\frac{x}{a^2\sqrt{x^2+a^2}}+C$$

$$\int\mathrm{d}x\,\frac{x^3}{(x^2+a^2)^{3/2}}=\frac{x^2+2a^2}{\sqrt{x^2+a^2}}+C\qquad \int\mathrm{d}x\,\frac{1}{x}=\ln x+C$$

$$\int_a^{\infty}\mathrm{d}x\,\frac{b}{x\sqrt{x^2-b^2}}=\arcsin\left(\frac{b}{a}\right)\qquad \text{falls }\; a>b>0$$

$$\int_{-\infty}^{\infty}\exp(-x^2)dx=\sqrt{\pi}\qquad \int_0^{\infty}x\exp(-x^2)dx=\frac{1}{2}$$

$$\int x^n dx=\frac{x^{n+1}}{n+1}+C\qquad \int a^x dx=\frac{a^x}{\ln a}+C$$

Sonstiges:

$$\Theta(x)=\begin{cases}0, & x<0\\ 1, & x\geq 0\end{cases}$$

$$\int_\alpha^\beta dx f(x)\delta(x-x_0)=\begin{cases}f(x_0) & \text{falls } x_0\in]\alpha,\beta[,\\ 0 & \text{sonst.}\end{cases}$$

$$\frac{\mathrm{d}}{\mathrm{d}x}\sin x=\cos x\qquad \frac{\mathrm{d}}{\mathrm{d}x}\cos x=-\sin x\qquad \sin^2 x+\cos^2 x=1$$

$$\sin x=\frac{1}{2i}\left(e^{ix}-e^{-ix}\right)\qquad \cos x=\frac{1}{2}\left(e^{ix}+e^{-ix}\right)$$

$$\sinh x=\frac{1}{2}\left(e^{x}-e^{-x}\right)\qquad \cosh x=\frac{1}{2}\left(e^{x}+e^{-x}\right)$$