Quantum Field Theory Homework 6

Due 9 July 2021

1 Muon production

Consider the cross-section for $e^+e^- \to \mu^+\mu^-$, which we found in class:

$$\sigma(e^-e^+ \to \mu^-\mu^+) = \frac{4\pi\alpha^2}{3s}\left(1 + \frac{2m_\mu^2}{s}\right)\sqrt{1 - \frac{4m_\mu^2}{s}} \tag{1}$$

where $s = (2E)^2$ is the square of the center-of-mass (CM) total system energy, or $s = 4E^2$ where E is the energy of *either* e^+ or e^- (in the CM frame).

For what value of s, or equivalently of CM electron energy E, is the cross-section at its maximum? Express your answer in terms of the muon mass m_μ, but then express it in MeV using $m_\mu = 105.658371$ MeV.

For obscure historical reasons, experimentalists like to work in terms of *barns*, where $1\,\mathbf{b} = [10^{-24}\,\mathrm{cm}^2]^{-1}$. Using $\hbar c = 197.3269718\,\mathrm{MeV\,fm}$ (which is useful for converting between energies and distances, and is often approximated as 197 MeV fm or 200 MeV/fm), and $\alpha = 1/137.035999074$, express the maximum value of the cross-section in units of barns (or millibarns or microbarns as appropriate).

Now consider Eq. (1) in the $m_\mu^2/s \ll 1$ regime. Find the expansion of the cross-section to first nonvanishing order in m_μ, which should be m_μ^4. Can you find a good theoretical reason why there is no term of order m_μ^1? Can you find a reason there is no m_μ^2 term? [This is hard, don't be too upset if you can't – especially for the case of the m_μ^2 term, which is very subtle!]

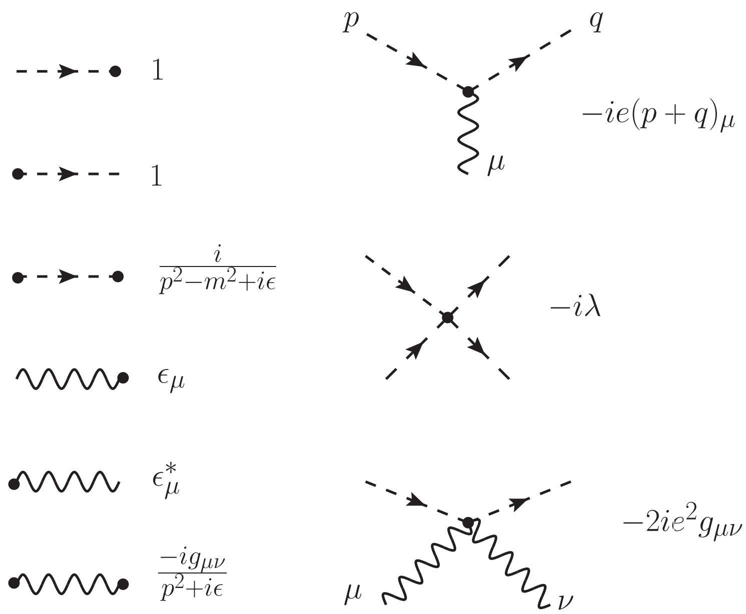
2 Complex scalar QED

Consider the theory of one complex scalar Φ and a gauge boson A^μ. Because the complex scalar is charged, we *can* associate a direction (put an arrow) on its line. The Lagrangian and Feynman[1] rules are:

$$\mathcal{L} = (D_\mu\Phi)^*(D^\mu\Phi) - m^2\Phi^*\Phi - \frac{\lambda}{4}(\Phi^*\Phi)^2 - \frac{1}{4}F_{\mu\nu}F^{\mu\nu}\,, \qquad D_\mu = \partial_\mu - ieA_\mu \tag{2}$$

and

[1] Feynperchild?



That is, incoming and outgoing scalars contribute a factor of 1 (there is no $u(p,\sigma)$ or $v(p,\sigma)$ and the vertex is $-i(p+q)_\mu$ with μ the index on the gauge field line and p, q the momenta of the scalar lines in the direction that the scalar charge is flowing.

Consider the process $\Phi\Phi \to \Phi\Phi$ (two particles in, two particles out). Draw the three diagrams which contribute to the matrix element. Evaluate $\mathcal{M}$ to $\mathcal{O}(e^2, \lambda)$:

$$|\mathcal{M}|^2 = \left(e^2 \frac{u-s}{t} + e^2 \frac{t-s}{u} + \lambda\right)^2 . \tag{3}$$

Is the interference between the three terms constructive or destructive? (Careful. Think about the relative signs of s, t, and u.)

Use crossing to determine the squared matrix element for $\Phi\Phi^* \to \Phi\Phi^*$. What about constructive/destructive interference in this case?

3 Scalars and spinors and photons

Now consider a theory with both a charged spinor (electron) and a charged scalar (selectron?) together with a photon:

$$\mathcal{L}(\psi, \Phi, A) = \bar{\psi} i\gamma^\mu(\partial_\mu + ieA_\mu)\psi + [(\partial_\mu - ieA_\mu)\Phi^*]\,(\partial^\mu + ieA^\mu)\Phi - \frac{1}{4}F_{\mu\nu}F^{\mu\nu} \tag{4}$$

To make life easier I take each particle to be massless and I ignore scalar self-interactions.

In terms of the Mandelstamm variables s, t, u, compute the (spin summed/averaged) square amplitudes for

1. the scattering $\psi\Phi \to \psi\Phi$

2. the scattering $\psi\Phi^* \to \psi\Phi^*$
3. the annihilation $\psi\bar{\psi} \to \Phi\Phi^*$
4. (extra credit) annihilation to photons $\Phi^*\Phi \to \gamma\gamma$

Hint: if you get the first one, use crossing to get the next two, which should be easy. The last one is tricky because there are three diagrams (not two, three). That's why it's extra credit.

4 Photon-electron scattering

Return to the derivation of the photon-electron scattering cross-section discussed in class. Repeat the calculation, *not* approximating $m_e \ll 1$, that is, fully including the electron mass in the calculation.

Now consider the case where the photon carries a very small energy, $p'_0 \ll m_e$. Compute, to lowest order in p'_0, the total cross-section for photon-electron scattering. See if your result reproduces the known result for *Thomson scattering*,

$$\sigma_{th} = \frac{8\pi}{3}\frac{\alpha^2}{m_e^2}\,.$$