$$Z(J) \; ; \; \frac{1}{Z(0)} \frac{i^n \delta^n Z(J)}{\delta J(x_1) \cdots \delta J(x_n)} \Bigg|_{J=0} = \langle 0 | T(\varphi(x_1) \cdots \varphi(x_n)) | 0 \rangle$$

Thing we need.

$$Z(J) = \int \mathcal{D}\varphi(x^\mu) \, \exp\Big[i \int \underbrace{\tfrac{1}{2}\partial_\mu \varphi \partial^\mu \varphi - \tfrac{m^2}{2}\varphi^2 - \tfrac{\lambda}{24}\varphi^4}_{\mathcal{L}} - \varphi J \Big]$$

$$\frac{i\delta}{\delta J(x)} \; - - - - - - - \; \varphi(x)$$

$$e^{-\frac{i\lambda}{24}\int \varphi^4 d^4x} = e^{-\frac{i\lambda}{24}\int d^4x \left[\frac{i\delta}{\delta J(x)}\right]^4}$$

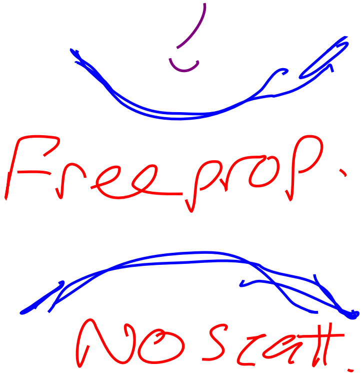
put outside $Z(J)$

$$\sum_{n=0}^{\infty} \left(-\frac{i\lambda}{24}\right)^n \frac{1}{n!} \left[\int d^4x \, \varphi(x)\right]^n \qquad \frac{i\delta}{\delta J}$$

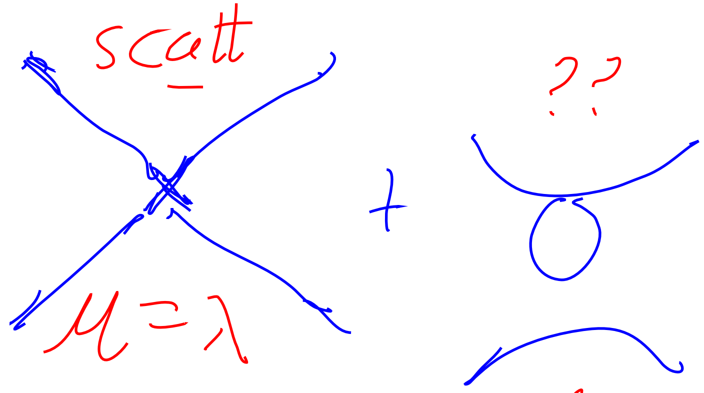
$$Z(J) = \sum_{n=0}^{\infty} \left[\frac{-i\lambda}{24}\right]^n \frac{1}{n!} \left(\int d^4x_1 \cdots d^4x_n \left(\frac{i\delta}{\delta J(x_1)}\right)^4 \cdots \left(\frac{i\delta}{\delta J(x_n)}\right)^4 \right) e^{\,i\int_{xy} J(x) G(x-y) J(y)/2}$$

$G(x-y)$ free 2-pt func.

$$\frac{\delta^4 Z(J)}{\delta J^4} = \frac{i\partial}{\partial J}$$



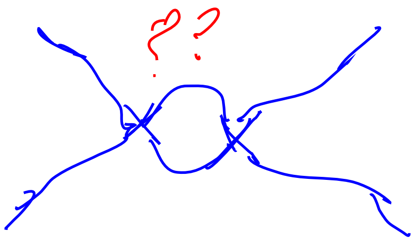
$n=0$



$+ \cdots$

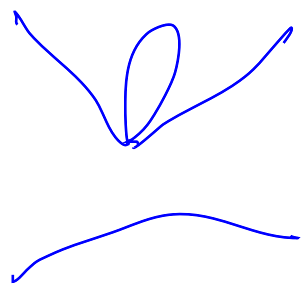
$n=1$

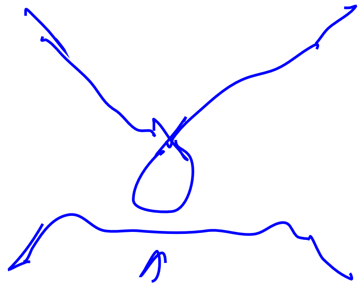
First

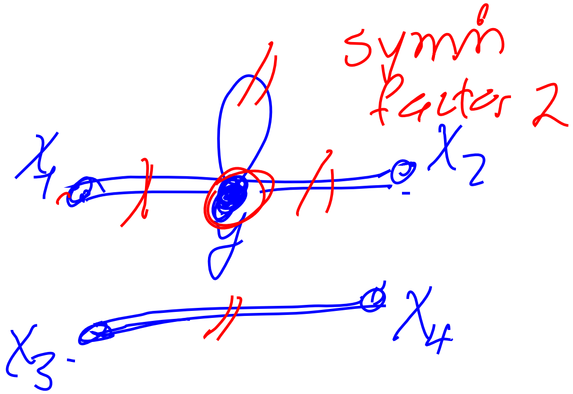


$+ \cdots$ many diagrams

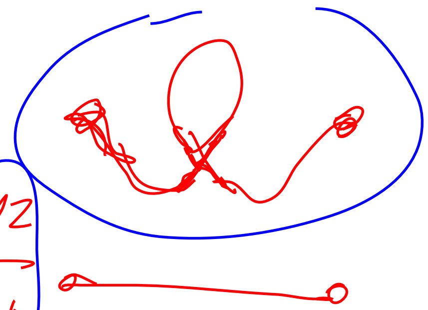
$n=2$

What Is  this?

$\frac{\delta}{\delta J}$  $\int d^4x \left(\frac{\delta}{\delta J}\right)^4$



$$\frac{i\delta}{\delta J(x_1)} \cdots \frac{i\delta}{\delta J(x_4)} \int_y \left(\frac{i\partial}{\partial J(y)}\right)^4 Z_0(J)$$



$4\cdot 3 = 12$

$\frac{12}{24}$

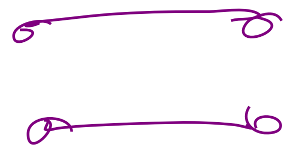
24

$$= \frac{-i\lambda}{2} \int_y G(x_1-y)\, G(y-y)\, G(y-x_2)\, G(x_3-x_4)$$

Symm factor

Correction to $G(x_1-x_2)$

$G(x_1-x_2)$



$G(x_3-x_4)$

π

2-point function: what is $\langle 0 | T(\varphi(x_1)\varphi(x_2)) | 0 \rangle$?

Isn't that just $G(x_1 - x_2)$?? NO

(yes if $\lambda = 0$

But not if $\lambda \neq 0$

---

what is

$$\frac{i\,\delta}{\delta J(x_1)} \frac{i\,\delta}{\delta J(x_2)} \left[Z(J) = \sum_{n=0}^{\infty} \frac{1}{n!} \left(\frac{-i\lambda}{24}\right)^n \int d^4y_1 \ldots d^4y_n \left(\frac{i\,\delta}{\delta J(y_1)}\right)^4 \cdots \left(\frac{i\,\delta}{\delta J(y_n)}\right)^4 \exp i \int_{z_1 z_2} J(z_1) G(z_1 - z_2) J(z_2)/2 \right]$$

$J = 0$

Lowest $\frac{\partial}{\partial J}\frac{\partial}{\partial J} e^{\frac{1}{2} J G J} \to G(x_1 - x_2)$

$+$

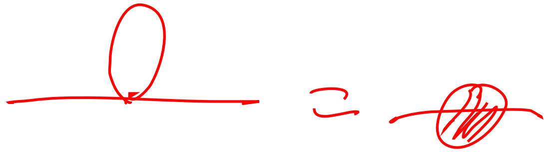
Next: $\frac{\delta}{\delta J(x_1)} \frac{\delta}{\delta J(x_2)} \frac{-i\lambda}{24} \int_y \left(\frac{\delta}{\delta J(y)}\right)^4 e^{\frac{1}{2} J G J}$

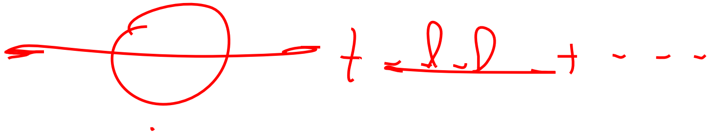
$$\frac{-i\lambda}{2} \int G(x_1 - y)\, G(y - y)\, G(y - x_2)\, d^4y$$

$$-i\lambda \frac{4 \cdot 3}{24} = \frac{-i\lambda}{2}$$

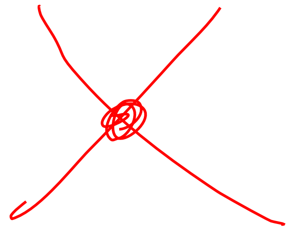
$$G_{full}(x_1-x_2) = G(x_1-x_2) + \frac{-i\lambda}{2}\int_y G(x_1-y)G(y-y)G(y-x_2)$$

$$+ \lambda^2 \cdots$$



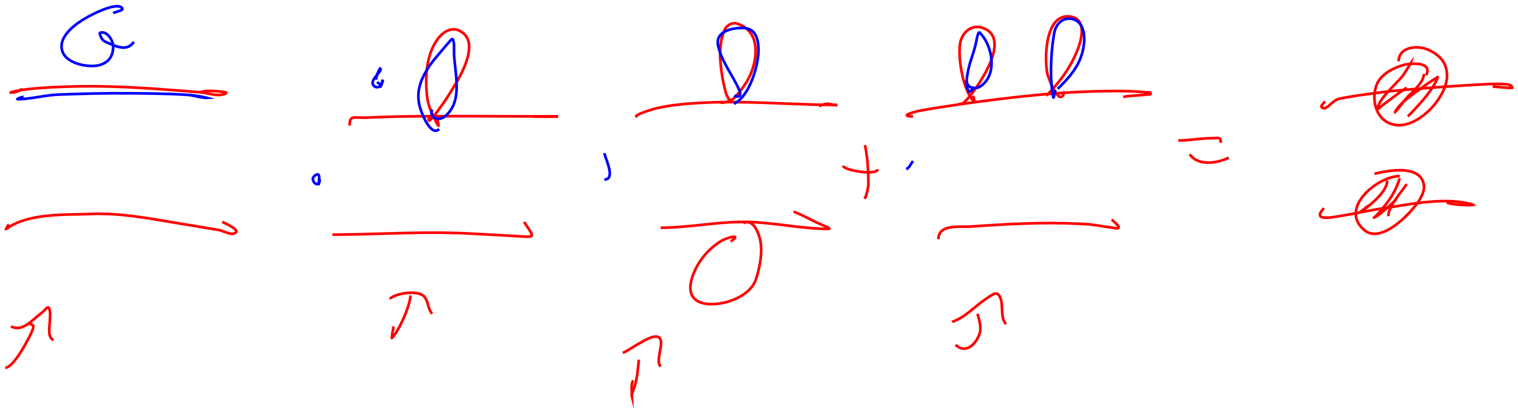


Every time I calc. something else

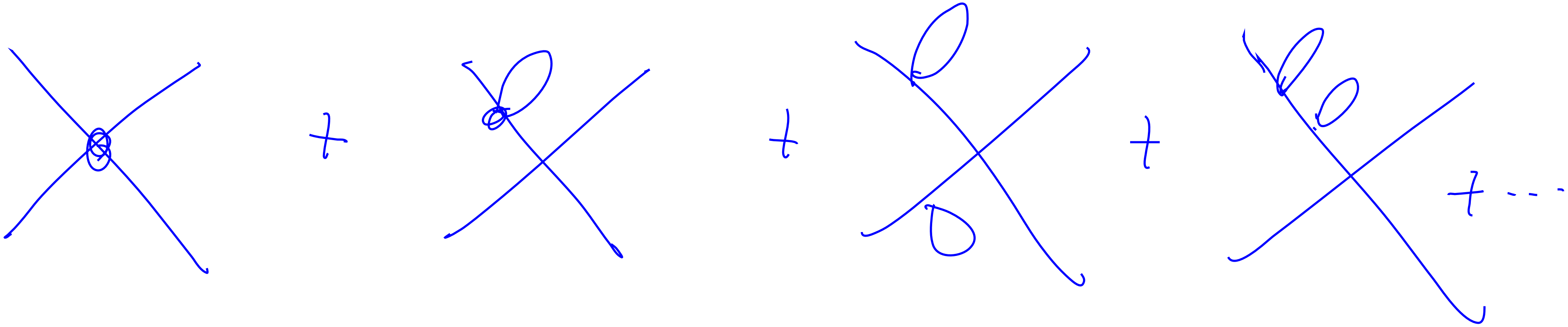


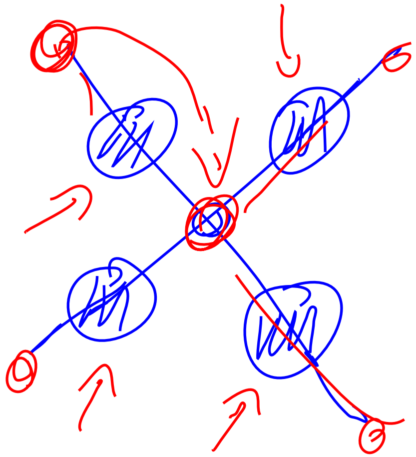
Each prop. is first term in a series,

Same series as

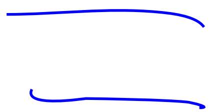


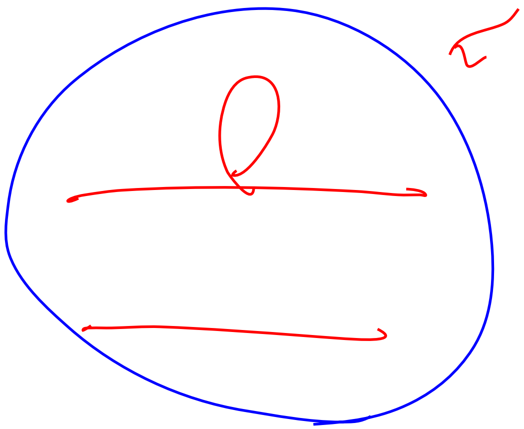
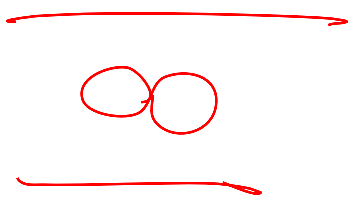
$$= G_{full}(x_1-x_2) \times G_{full}(x_3-x_4)$$



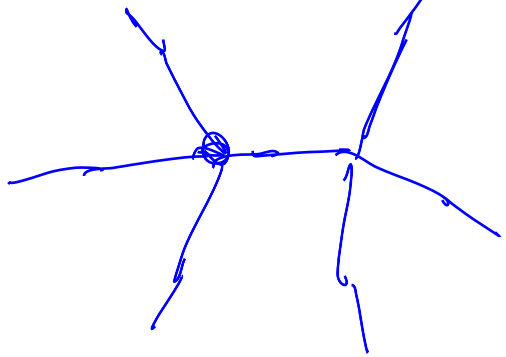
=  $\boxed{G_{\text{full}}}$ on each outside line, not just G.

Propagator

=  Boring = Not scattering

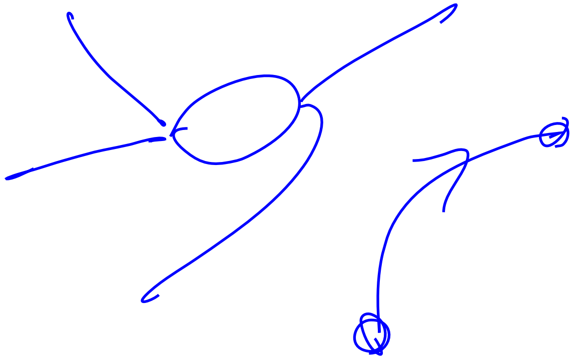
  $\frac{1}{Z(0)}$ kills this

$G\varphi$'s



2 part → 4 part.

Interesting



2 part - 2 part

1 - flying unscatt not particip.

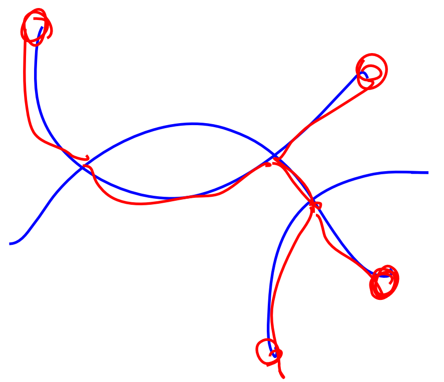
Boring

Avoid boring things: Calc. where "these things" don't show up?

Disconnected Diagrams/Graphs

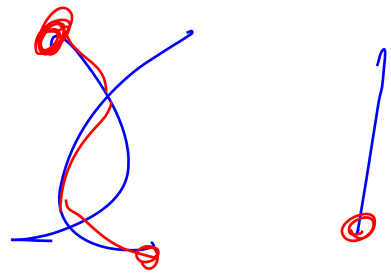
Graphs are in 2 types:

Connected



Follow lines from any pt to any other

Disconnected



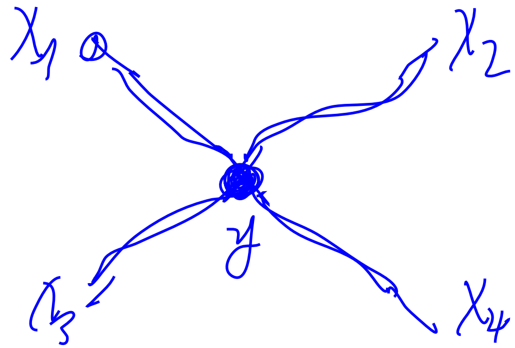
2 components

Some pts cannot be conn.

Avoid Disconnected: Define $W(J) = \log Z(J)$

$$\frac{i^n \delta^n}{\delta J(x_1) \cdots \delta J(x_n)} W(J) = \text{connected correl. func.}$$

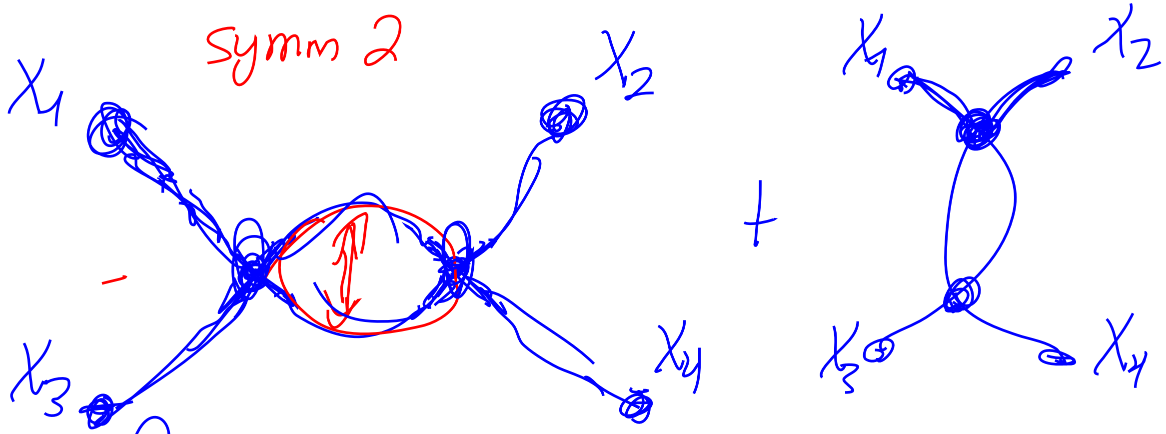
4 external lines, 1 vertex



$$\int d^4y \; \frac{-i\lambda}{\cancel{24}} \; \cancel{4\cdot 3\cdot 2\cdot 1} \; G(x_1-y)\,G(x_2-y)\,G(x_3-y)\,G(x_4-y)$$

4 lines 2 vertices

symm 2



$$\underbrace{\frac{-i\lambda}{\cancel{24}} \; \frac{-i\lambda}{\cancel{24}}}_{1 \text{ per vertex}} \; \underbrace{\frac{1}{2}}_{1/n!} \; \cancel{8\cdot 3\cdot 4\cdot 3\cdot 2} \int d^4y_1\, d^4y_2 \; G(x_1-y_1)\,G(x_3-y_1)\,G(x_2-y_2)\,G(x_4-y_2)\,G^2(y_1-y_2)$$

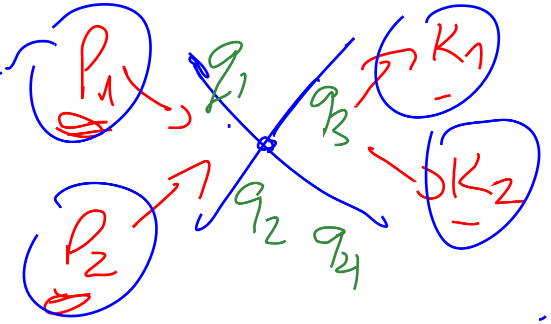
symm of vert's.

$$-\frac{\lambda^2}{2}$$

Scatt calc?

Fourier to p-space

Consider



Scatt?

$$G(x_1\, x_2\, x_3\, x_4) = -i\lambda \int d^4y\; G(x_1-y)\, G(x_2-y)\, G(x_3-y)\, G(x_4-y)$$

$$\int d^4x_1 \cdots d^4x_4\; e^{ip_1\cdot x_1}\, e^{ip_2\cdot x_2}\, e^{-ik_1\cdot x_3}\, e^{-ik_2\cdot x_4}$$

$$G(x_1-y) \longrightarrow \int \frac{d^4q_1}{(2\pi)^4}\, G(q_1)\, e^{-iq_1\cdot(x_1-y)}$$

free: $\dfrac{i}{q^2-m^2+i\epsilon}$

All word's are $\underbrace{\int d^4x_1\, e^{ip_1\cdot x_1}\, e^{-iq_1\cdot x_1}}_{q_1=p_1}$

Every external momentum $\rightarrow$ mom. on that propagator

$$\int_y \int_{x_1 x_2 x_3 x_4} \int_{q_1 q_2 q_3 q_4} e^{i p_1 \cdot x_1 + p_2 \cdot x_2 - k_1 x_3 - k_2 x_4} \, e^{-i q_1 (x_1 - y) - i q_2 (x_2 - y) - i q_3 (x_3 - y) - i q_4 (x_4 - y)} \, G(q_1) \cdots G(q_4)$$

$q_1 = p_1$

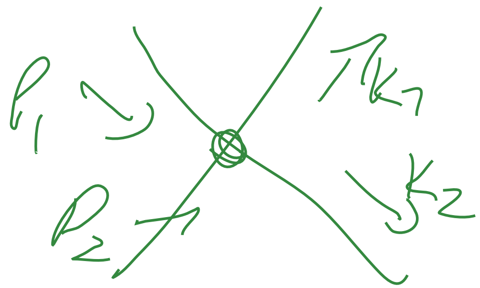
$q_1 = p_1 \quad \delta^4(p_1 - q_1)(2\pi)^4$

$\delta^4(p_2 - q_2)(2\pi)^4$

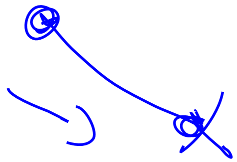
$\delta^4(k_1 + q_3)(2\pi)^4$

$\delta^4(k_2 + q_4)(2\pi)^4$

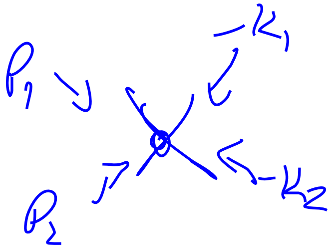
$$(-i\lambda)\, G(p_1) G(p_2) G(-k_1) G(-k_2) \underbrace{\int d^4y \, e^{iy(p_1 + p_2 - k_1 - k_2)}}_{(2\pi)^4 \delta^4(p_1 + p_2 - k_1 - k_2)}$$



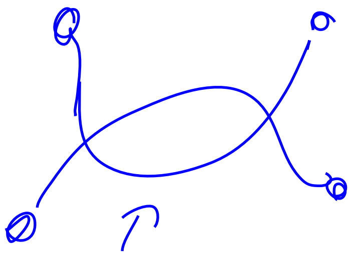
Rule:



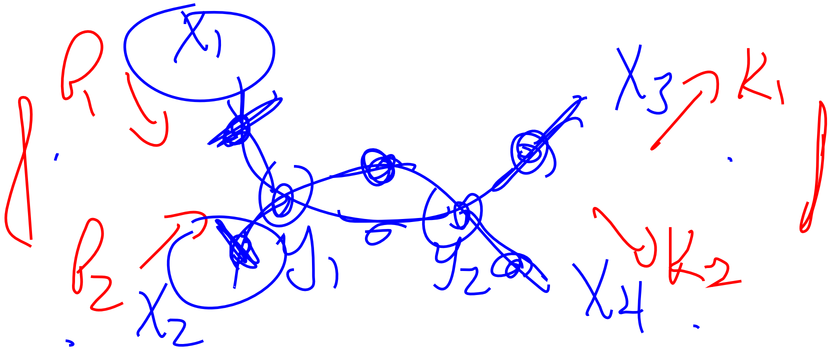
Outside line has mom. $\underline{p}$ // $-\underline{k}$



4 mom. going in had $\int_y \to (2\pi)^4 \delta^4(p_1+p_2-k_1-k_2)$



$\underline{\text{still has}}$ $\int d^4 q$



$$\left(\frac{-i\lambda}{2}\right)^3$$

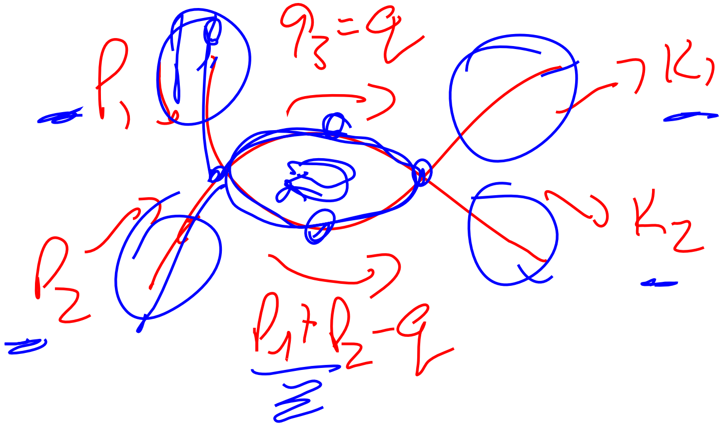
$$\int_{x_1x_2x_3x_4} e^{i(p_1x_1+p_2x_2-k_1x_3-k_2x_4)} \int_{q_1\cdots q_6} G(q_1)\cdots G(q_6) \int_{y_1y_2} e^{i\left(q_1\cdot(x_1-y_1)+q_2\cdot(x_2-y_1)+q_3\cdot(y_1-y_2)+q_4\cdot(y_1-y_2)+q_5(y_2-x_3)+q_6(y_2-x_4)\right)}$$

$$\int d^4x\, e^{i(p-q)x} = \delta^4(p-q)(2\pi)^4$$

$$(2\pi)^4 \delta^4(p_1-q_1)$$
$$\delta^4(p_2-q_2)$$
$$\delta^4(k_1-q_5)$$
$$\delta^4(k_2-q_6)$$
$$\delta^4(p_1+p_2-q_3-q_4)$$
$$\delta^4(q_3+q_4-k_1-k_2)$$

$$\delta^4(p_1+p_2-q_3-q_4)\ \boxed{\delta^4(p_1+p_2-k_1-k_2)}$$

$$q_4 = p_1+p_2-q_3$$

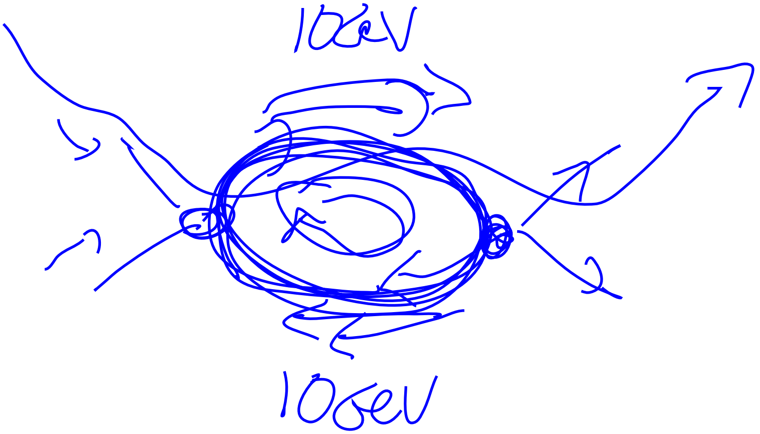


$$G \to G_{Amp} \to M$$

$$\frac{(-i\lambda)^2}{2} G(p_1)G(p_2)G(k_1)G(k_2)(2\pi)^4 \delta^4(p_1+p_2-k_1-k_2)$$

$$\times \int \frac{d^4q}{(2\pi)^4} G(q)\, G(p_1+p_2-q) = iM$$

$$\frac{1}{q^2} \qquad \frac{1}{(q-p_1-p_2)^2}$$



How much does this happen??

Each External line — $\longrightarrow$ d^4x Removes a $\int$
Vertex d^4y

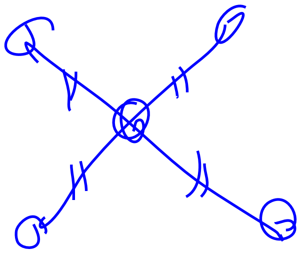
Each propagator $\longrightarrow$ $\int d^4q$

propagators: N N – q-ints

Ext's & vertices V V – δ^4's. But one is $\delta^4(\Sigma p_{in} - \Sigma k_{out})$

V−1 δ's remove q-ints.

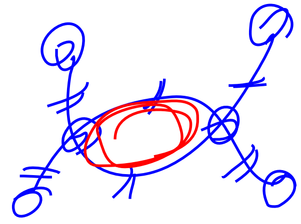
q int's left to perform = $\boxed{N - V + 1 \equiv L}$ # of loops



$V = 5$

$N = 4$

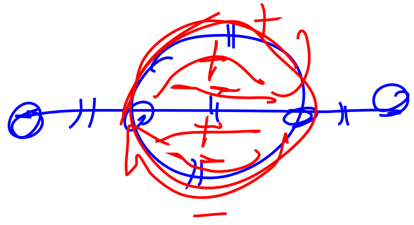
$L = 4 - 5 + 1 = 0$



$V = 6$

$N = 6$

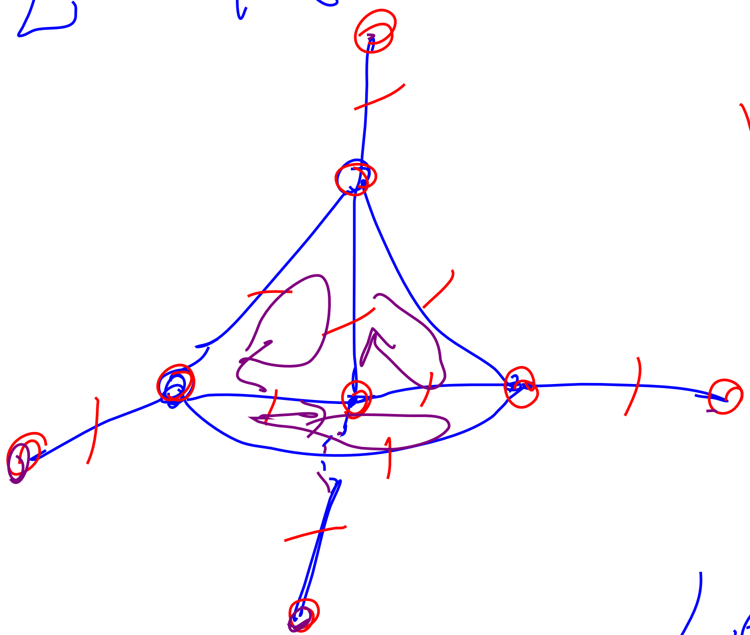
$L = 6 - 6 + 1 = 1$



$V = 4$

$N = 5$

$L = 5 - 4 + 1 = 2$



$V = 8$

$N = 10$

$L = 10 - 8 + 1 = 3$

Loop # etc will be important in future.

2 scalars φ Throw them p_1 p_2

$s = \underline{(p_1+p_2)^2} = (\underline{\text{CM energy}})^2$

Total over all (k_1, k_2) final mom, that they scatter?
cross section σ

$\Rightarrow k_1 \neq p_1$
$k_2 \neq p_1$

$\boxed{\mathcal{M} = \lambda}$ $\cdot$ $\times$

$\propto \ (+ \underline{\lambda^2/16\pi^2}$ or something$)$
$\lambda \ll 16\pi^2$

$\underline{\text{tiny}}$

$$\sigma = \frac{1}{2p_1^0 2p_2^0 |v_1 - v_2|} \int \frac{d^3k_1 d^3k_2}{(2\pi)^6 2k_1^0 2k_2^0} \lambda^2 (2\pi)^4 \delta^4(p_1+p_2-k_1-k_2)$$

$|\mathcal{M}|^2$

CM frame: $\vec{p}_1 = -\vec{p}_2$

$p_1^0 = p_2^0 = k_1^0 = k_2^0 = \frac{1}{2}\sqrt{s}$

k_2 k_1 or k_2 ??

k_1 k_2

$s = (p_1^0 + p_2^0)^2 = (2p^0)^2 - \cancel{(\vec{p}_1 + \vec{p}_2)^2}$

0

$k_2 = -k_1$

$k_{2}^0 = k_1^0$

$$\sigma = \frac{\lambda^2}{s|v_1 - v_2|} \int \frac{d^3k_1 \cancel{d^3k_2}}{(2\pi)^6\, 2k_1^0 2k_2^0} 2\pi\delta(k_1^0 + k_2^0 - \sqrt{s})(2\pi)^3\delta^3(\vec{k}_1 + \vec{k}_2 \cancel{- \vec{p}_1 - \vec{p}_2})$$

$||$

$2p_1^0 2p_2^0$

$\delta(2k_1^0 - \sqrt{s})$

$= \frac{1}{2}\delta(k_1^0 - \sqrt{s}/2)$

final state symm

$\frac{1}{2}\int k^2 dk\, d\Omega_k$

4π

$\int k^2 dk\, \delta(k^0 - \sqrt{s}/2)$

$= \int \sqrt{k^{02} - m^2}\, k^0 dk^0 \delta(\ )$

$$= \frac{\lambda^2}{2s(2\cancel{p}\cancel{\,2})} \frac{\cancel{p}\,\cancel{4}\,\frac{1}{2}}{\pi \cdot 2 \cdot 2 \cancel{4} \cdot \cancel{4}} = \frac{\lambda^2}{2 \cdot 16\pi s} = \frac{\lambda^2}{32\pi s}$$

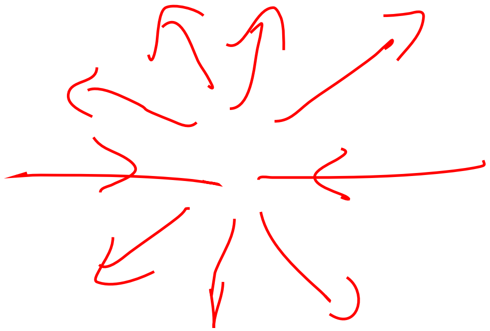
$$\boxed{\sigma = \frac{\lambda^2}{32\pi s}}$$

λ dimensionless #

$s \to \text{energy}^2 \quad (\text{cm energy})^2$

$\sigma \to \text{energy}^{-2} \to \text{length}^2$ Area

total. How much in which direction?



all same

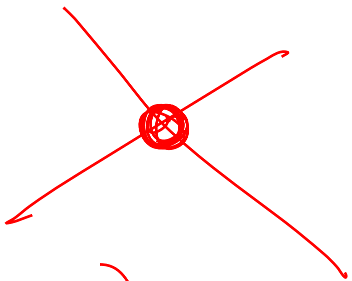
Max σ can be in S-wave is if phase-shift π

$$\boxed{\sigma = \frac{8\pi}{s}}$$

$\lambda < 16\pi$ or

small λ expansion fails.

unitarity bound.



part's meeting at a pt.

S-wave

No ang momentum

$M = \lambda$