Fields of spin-1/2 Free Field Thy

$$\underline{\underline{\psi}} = \left[\begin{array}{c} \psi_1 \\ \psi_2 \\ \hline \psi_3 \\ \psi_4 \end{array}\right] \begin{array}{l} \} \text{ Rot \& Boosts spin } 1/2 \\ \\ \} \text{ spin } 1/2 \end{array}$$

γ^μ $\underline{4}$ $\underline{4\times4 \text{ matrices}}$

$$\{\gamma^\mu, \gamma^\nu\} = 2g^{\mu\nu}$$

$\gamma^0\gamma^0 = \mathbb{1}$

$\gamma^1\gamma^1 = -\mathbb{1}$

$\bar{\psi} = \psi^\dagger\gamma^0$ Dirac Conj.

$\underline{\gamma^\mu}\underline{\partial_\mu}$ $\quad$ $\underline{i\gamma^0\partial^0} + i\underline{\vec{\gamma}\cdot\vec{\partial}}$

$$\mathcal{L}(\bar{\psi}, \psi) = \bar{\psi}(i\gamma^\mu\partial_\mu - m)\psi$$

$$(i\gamma^\mu\partial_\mu - m)\psi(x) = 0$$

$$(p_\mu\gamma^\mu - m)\psi(p) = 0$$

$p^0 = +E_p$

$$(p_\mu\gamma^\mu - m)\,u(p) = 0$$

$v(p) = 0$

$p^0 = -E_p$

$p^0 > 0$ $\quad$ $\psi(p) = u_+$ or $u_-(p)$ $\quad$ $\pm$ = spin

$$\sqrt{2m}\begin{bmatrix}1\\0\\1\\0\end{bmatrix} \quad \sqrt{2m}\begin{bmatrix}0\\1\\0\\1\end{bmatrix} \text{ at rest.}$$

$p^0 < 0$ $\quad$ $V_\pm$ $\quad$ -1, -1

$$\mathcal{H} = \overset{\circ}{=} i \hat{\bar{\Psi}} \vec{\gamma} \cdot \vec{\partial} \hat{\psi} + m \hat{\bar{\Psi}} \hat{\psi}$$

no time deriv's.

$$\hat{\psi}(\vec{x}) = \int \frac{d^3p}{(2\pi)^3 2E_p} e^{i\vec{p}\cdot\vec{x}} \sum_s \left[\hat{b}_{ps}\, u(p,s) + \hat{d}^{\dagger}_{-ps}\, V(-p,s) \right]$$

annih. create

$$E_p = +\sqrt{\vec{p}^2 + m^2}$$

$$\psi^{\dagger} = \quad '' \quad e^{-i\vec{p}\cdot\vec{x}} \sum_s \left[b^{\dagger}_{ps}\, u^{\dagger}(p,s) + d_{-ps}\, V^{\dagger}(-p,s) \right]$$

$$\partial \to i\vec{p}$$

$$\bar{\Psi}\big(\underbrace{+\vec{p}\cdot\vec{\gamma} + m - p^0\gamma^0}_{(\ )u=0} + p^0\gamma^0$$

$$\bar{\psi}\,\psi \qquad H = \int \frac{d^3p}{(2\pi)^3 2E_p} \sum_s \left[E_p\, b^\dagger b_{p,s} - E_p\, d\, d^\dagger_{p,s} \right]$$

If $[\psi_a(x), \psi_b^\dagger(y)] = -i\,\delta^3(x-y)\,\delta_{ab}$ As for Bosonic Op

Then $[b(p,s), b^\dagger(q,s')] = \delta_{ss'}\, 2E_p (2\pi)^3 \delta^3(\vec{p}-\vec{q})$

Oh, but spin-1/2 fields

$$\{\psi_a(x), \psi_b^\dagger(y)\} = \delta^3(x-y)\,\delta_{ab} = \psi\psi^\dagger + \psi^\dagger\psi$$

$$\{\psi_a(x), \psi_b(y)\} = 0 = \hat{\psi}_a(x)\hat{\psi}_b(y) + \hat{\psi}_b(y)\hat{\psi}_a(x)$$

- Sign each time you switch order!

What do b, d do?

$$\{b, d\} = 0 = \{b, b\} = \{d, d\} = \{b, d^\dagger\} = \{b^\dagger, d\}$$

$$\rightarrow \{d(p,s), d^\dagger(p',s')\} = \delta_{ss'} (2\pi)^3 2E_p \delta^3(\vec{p} - \vec{p}')$$

$$\{b \qquad b^\dagger \qquad \text{''} \quad \text{''} \quad \text{''} \quad \text{''}$$

$$H = \int \frac{d^3p}{(2\pi)^3 2E_p} \sum_s \left[E_p\, b^\dagger b(p,s) - E_p\, d\, d^\dagger(p,s) \right]$$

$$d(p,s) d^\dagger(p,s) = -d^\dagger(p,s) d(p,s) + (2\pi)^3 2E_p \delta^3(p-p)$$

When volume was finite, $\int d^3p \to \sum$ $\quad V \delta^{\text{kron.}} \quad 2E_p V_{\text{space}}$

$$H = -V_{\text{space}} \sum_s \int_p \frac{d^3p}{(2\pi)^3} E_p + \int \frac{d^3p}{(2\pi)^3 2E_p} \sum_s \left[E_p (b^\dagger b) + E_p (d^\dagger d) \right]^{1/2}$$

Happy but confused.

Weird creat $\underline{b^+}$

annih $\underline{b}$

$\underline{\{b,b\} = 0 = \{b^+,b^+\}}$

$\{b,b^+\} = 1$ for 1 mode of box

What is this weird thing?

1 Mode. Fermionic HO.

$$\underline{\underline{bb}} = \frac{1}{2}(bb+bb) = \frac{1}{2}\{b,b\} = 0$$

b is nilpotent!!

$b^2 = 0 \rightarrow b$ is nilpotent.

$b^+b^+ = 0$

$$\underline{bb^+} = -\underline{b^+b} + [bb^+ + b^+b]$$
$$\qquad\qquad\qquad\qquad = 1$$

Only op's : $1, b, b^+, b^+b$ complete

N-dim H space

Op's are $N \times N$ matrices N^2 indep.

$N^2 = 4 \quad \underline{\underline{N = 2}}$

$H = b^\dagger b$

There is a state $|0\rangle$ with $\boxed{b|0\rangle = 0}$

Proof. Assume is state. $|\psi\rangle$

either $b|\psi\rangle = 0$ $\quad |\psi\rangle = |0\rangle$

or $b|\psi\rangle \neq 0$ $\quad b|\psi\rangle$

$b\, b|\psi\rangle = 0$

Norm: $\langle 0|0\rangle = 1$

$|1\rangle = b^\dagger|0\rangle$

$\boxed{|1\rangle, |0\rangle \text{ norm'd.}}$

$\langle 1|1\rangle = \langle 0|b\ b^\dagger|0\rangle$

$= \langle 0|(-b^\dagger b + 1)|0\rangle = 1\langle 0|0\rangle = 1$

$b|1\rangle = |0\rangle \quad b^\dagger|1\rangle = 0 \quad bb^\dagger|1\rangle = 0 \quad b^\dagger b|1\rangle = 1|1\rangle$

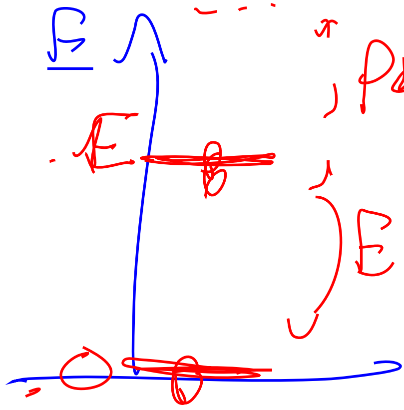
$b|0\rangle = 0 \quad b^\dagger|0\rangle = |1\rangle \quad bb^\dagger|0\rangle = |0\rangle \quad b^\dagger b|0\rangle = 0 = 0|0\rangle$

$b^\dagger b$ # operator

$$H = E\, b^\dagger b$$

$$H|1\rangle = 1|1\rangle$$

$$H|0\rangle = 0|0\rangle$$

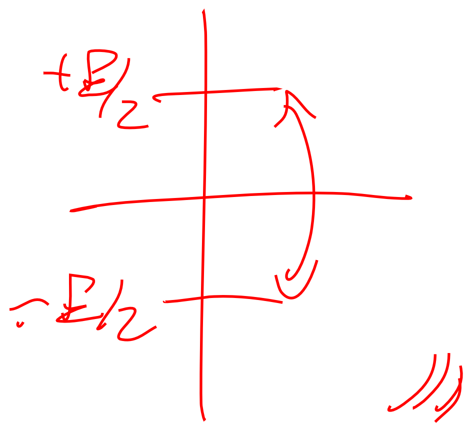


Pauli Excl.

Basis $\quad c_0|0\rangle + c_1|1\rangle \to \begin{bmatrix} c_1 \\ c_0 \end{bmatrix}$

$$b = \begin{bmatrix} 0 & 0 \\ 1 & 0 \end{bmatrix} \qquad b^\dagger b = \begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix}$$

$$b^\dagger = \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix} \qquad b b^\dagger = \begin{bmatrix} 0 & 0 \\ 0 & 1 \end{bmatrix}$$

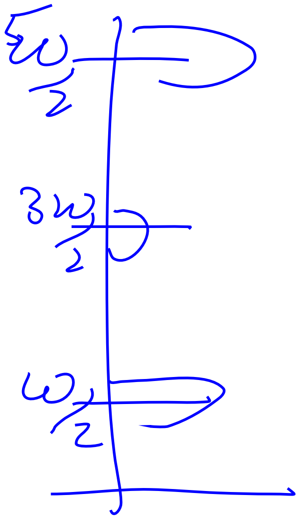


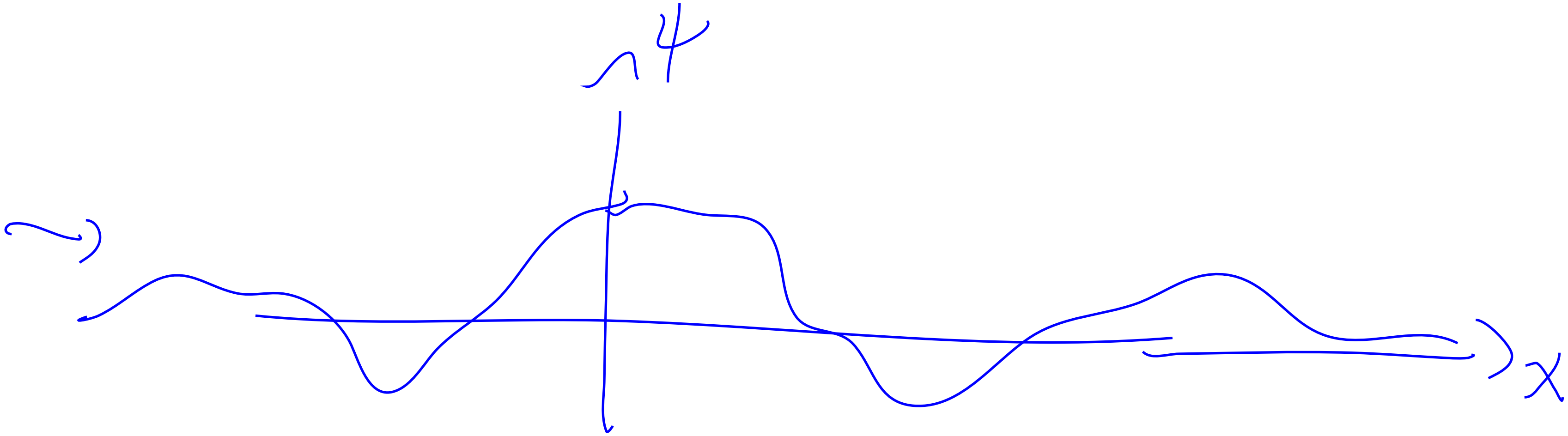
SHO $\quad a^\dagger a + \frac{1}{2}$

$$b^\dagger b - \frac{1}{2}$$

SHO

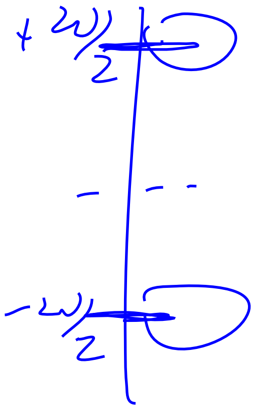
$$|\psi\rangle = \psi_0|0\rangle + \psi_1|1\rangle + \psi_2|2\rangle + \cdots$$

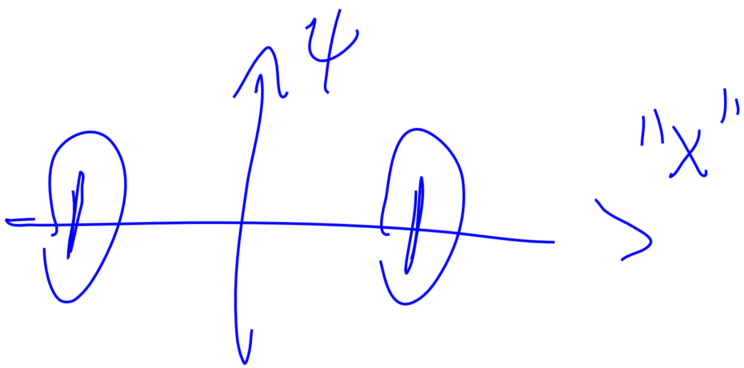




FHO

$$|\psi\rangle = \psi_0|0\rangle + \psi_1|1\rangle$$





2 FHO's

$\underline{b^+ b}$

$\underline{d^+ d}$

$\underbrace{1}_{1}, b, \underbrace{d, b^+, d^+}_{4}, \underbrace{bb^+, bd, bd^+, \ldots}_{6 \text{ choices}}$

$\underbrace{b^+bd \quad dd^+b \ldots}_{4}, \underbrace{bb^+dd^+}_{1} = \underline{\underline{16 \text{ op's}}}$

4x4 space

$|00\rangle$

$|01\rangle$

$|10\rangle$

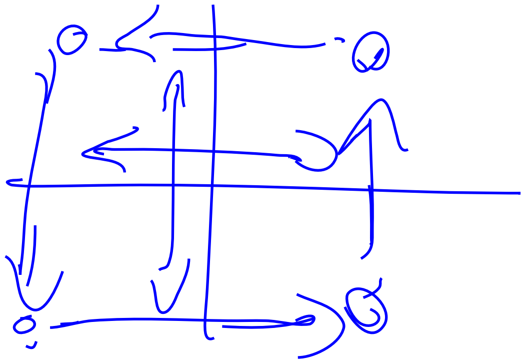
$|11\rangle$

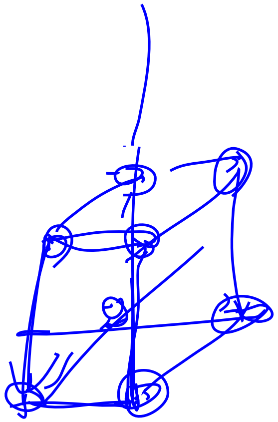
$|10\rangle = b^+|00\rangle$

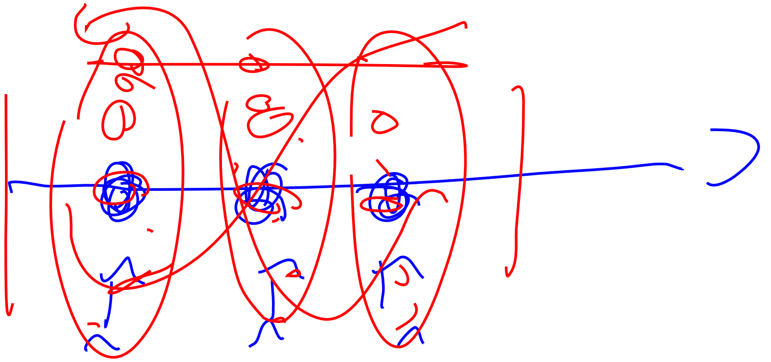
$|01\rangle = d^+|00\rangle$

$|11\rangle = d^+|10\rangle = d^+b^+|00\rangle$

or $b^+|01\rangle = b^+d^+|00\rangle$







QM $\psi(x)$ is 3 c numbers

N values

$\varphi_1 \quad \varphi_2 \quad \varphi_3$

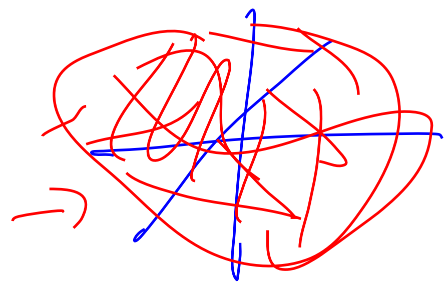
$\hat{\psi}_1 \quad \hat{\psi}_2 \quad \hat{\psi}_3$

N pts

Boson in QFT

$\Psi(\varphi_1, \varphi_2, \varphi_3)$ $\mathbb{R}^N$

$\varphi \partial \varphi$

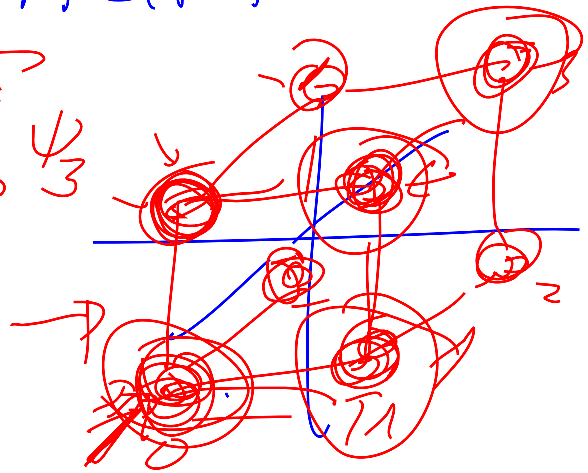


Fermion in QFT 2^N

$H = i\bar{\psi}_1\psi_1 + i\bar{\psi}_2\psi_2 + i\bar{\psi}_3\psi_3$

$i(\bar{\psi}_1\psi_2 + \bar{\psi}_2\psi_1)$

$\bar{\psi}\partial\psi$



Next: Add scalars

$$\mathcal{L}(\varphi, \bar\psi, \psi) = \frac{1}{2}\partial_\mu\varphi\partial^\mu\varphi - \frac{m_\varphi^2}{2}\varphi^2 - \frac{\lambda}{24}\varphi^4 \quad -\bar\eta\psi - \bar\psi\eta$$

int. $= -\varphi J(x)$

$$+ \bar\psi(i\partial_\mu\gamma^\mu - M_\psi)\psi + g\varphi\bar\psi\psi$$

const = int.

$$Z(J,\eta,\bar\eta) = \int \mathcal{D}\varphi\, \mathcal{D}\bar\psi, \psi \; \exp i\int d^4x\, \mathcal{L}(\varphi \quad)$$

$-J\varphi - \bar\psi\eta$
$-\bar\eta\psi$

$-\bar\eta\psi = +\psi\bar\eta$

Grassmann #s

$\eta\check\eta = -\check\eta\eta$

class. var.

integrating over

Anticommuting integration Hermann variables. Grassmann Numbers

Totally anticommuting $\{\psi,\psi\} = 0 = \{\psi,\bar\psi\} = 0$

Normal #s

$$\frac{\partial}{\partial \bar{J}} \bar{J} = 1 \qquad \frac{\partial}{\partial \bar{J}} e^{A\bar{J}} = \left(\frac{\partial}{\partial \bar{J}} A\bar{J}\right) e^{A\bar{J}}$$

$$\frac{\partial}{\partial J} k = 0$$

Grassmann

$$\frac{\partial}{\partial \eta} \eta = 1 \qquad \frac{\partial}{\partial \eta} e^{\bar{\psi}\eta} = \left(\frac{\partial}{\partial \eta} \bar{\psi}\eta\right) e^{\bar{\psi}\eta}$$

$$\frac{\partial}{\partial \eta} \bar{\psi}\eta = \left(\frac{\partial}{\partial \eta}\bar{\psi}\right)\eta - \bar{\psi}\frac{\partial}{\partial \eta}\eta$$

$$\frac{\partial}{\partial \eta} \bar{\psi} = 0$$

$$- \bar{\psi} e^{\bar{\psi}\eta}$$

$\bar{\psi}\eta$ even in anti' things

$$\bar{\psi} e^{\bar{\psi}\eta} = e^{\bar{\psi}\eta}\bar{\psi} \quad \text{good!}$$

$$\frac{i\delta}{\delta J(x)} e^{i\int d^4y\,(-J\varphi(y))} = \Big[\underbrace{\frac{i\delta}{\delta J(x)} \int_y i\, d^4y\,(-J(y))\,\varphi(y)}_{\delta^4(x-y)\ \rightarrow\ +\varphi(x)}\Big] e^{i(\dots)} = \varphi(x)\, e^{(\dots)}$$

$$\frac{i\delta}{\delta \bar\eta(x)} e^{i\int d^4y\,(-\bar\eta(y)\psi(y))} = \Big(\underbrace{\frac{i\delta}{\delta \bar\eta(x)} \int_y (-i)\,\bar\eta(y)\,\psi(y)}_{\delta^4(x-y)}\Big) e^{i(\dots)} = \psi(x)\, e^{i(\dots)}$$

$$-i\frac{\delta}{\delta \eta(x)} e^{i\int d^4y\,(-\bar\psi(y)\eta(y))} = \Big(-i\frac{\delta}{\delta \eta(x)} \int_y (-i)\,\bar\psi\,\eta(y)\Big) e^{i(\dots)} = +\bar\psi\, e^{i(\dots)}$$

$$\langle 0|\bar\psi(x)\,\psi(y)\,\varphi(z)|0\rangle = -i\frac{\delta}{\delta\eta(x)}\;+i\frac{\delta}{\delta\bar\eta(y)}\;i\frac{\delta}{\delta J(z)}\,Z(J,\bar\eta,\eta)\Big|_{J,\bar\eta,\eta=0}$$

$\psi\;\bar\psi$ should have $-$ sign?

$$\frac{\delta}{\delta\eta}\frac{\delta}{\delta\bar\eta}\bar\eta\,\eta = \frac{\delta}{\delta\eta}\eta = 1$$

$\psi\bar\psi$

$$\frac{\partial}{\partial\eta}\bar\eta \to -\bar\eta\frac{\partial}{\partial\eta}$$

$$\frac{\delta}{\delta\bar\eta}\frac{\delta}{\delta\eta}\bar\eta\,\eta = -\frac{\delta}{\delta\bar\eta}\bar\eta\,\frac{\delta}{\delta\eta}\eta = -1$$

$(-\bar\eta)$

$$\frac{\partial}{\partial\eta}\frac{\partial}{\partial\bar\eta} \to \frac{\partial}{\partial\bar\eta}\frac{\partial}{\partial\eta}$$

all $-$ signs

when we have

$$\int \mathcal{D}\phi \; e^{i\int\left(\phi \frac{M}{2}\phi - \phi J\right) - \cancel{\frac{\lambda}{4!}\phi^4}}$$

Replace w/ $\sum_n \left(\frac{\delta}{\delta J}\right)^4$

complete square

$$e^{i\int_{xy} J(x)\frac{M^{-1}}{2}J(y)} \left(\text{Det}\, M\right)^{-1/2}$$

$$\int \mathcal{D}\bar{\psi}\psi \; e^{i\int\left(\bar{\psi}(\not{\partial}-m)\psi - \bar{\eta}\psi - \bar{\psi}\eta\right)}$$

$$= e^{i\int \bar{\eta}\left(\frac{1}{\not{\partial}-m}\right)\eta} \; \text{Det}(\not{\partial}-m)^{+1}$$