Things I forgot last time

Lecture Recordings, HW solutions will now be Password Protected. Use:

Username: QFTI2021 (I not 1)

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Remarks about books: Different conventions

Srednicki → Theory vs Phenomenology Peskin

- Order to approach material

Perturbation theory { creation & annih. ops. Poor Foundat.
Path integral theor. stronger

Relativistic Field Theory (but not yet Quantum)

- Language, Tools, concepts

Scalar - # at each point in space

$\underline{\varphi}(x)$

$$\underset{=}{x}^\mu = \begin{bmatrix} t \\ x \\ y \\ z \end{bmatrix} \text{ or } \begin{bmatrix} x^0 \\ x^1 \\ x^2 \\ x^3 \end{bmatrix}$$

$c = 1$

time, space are in same units (meters) "m/c"

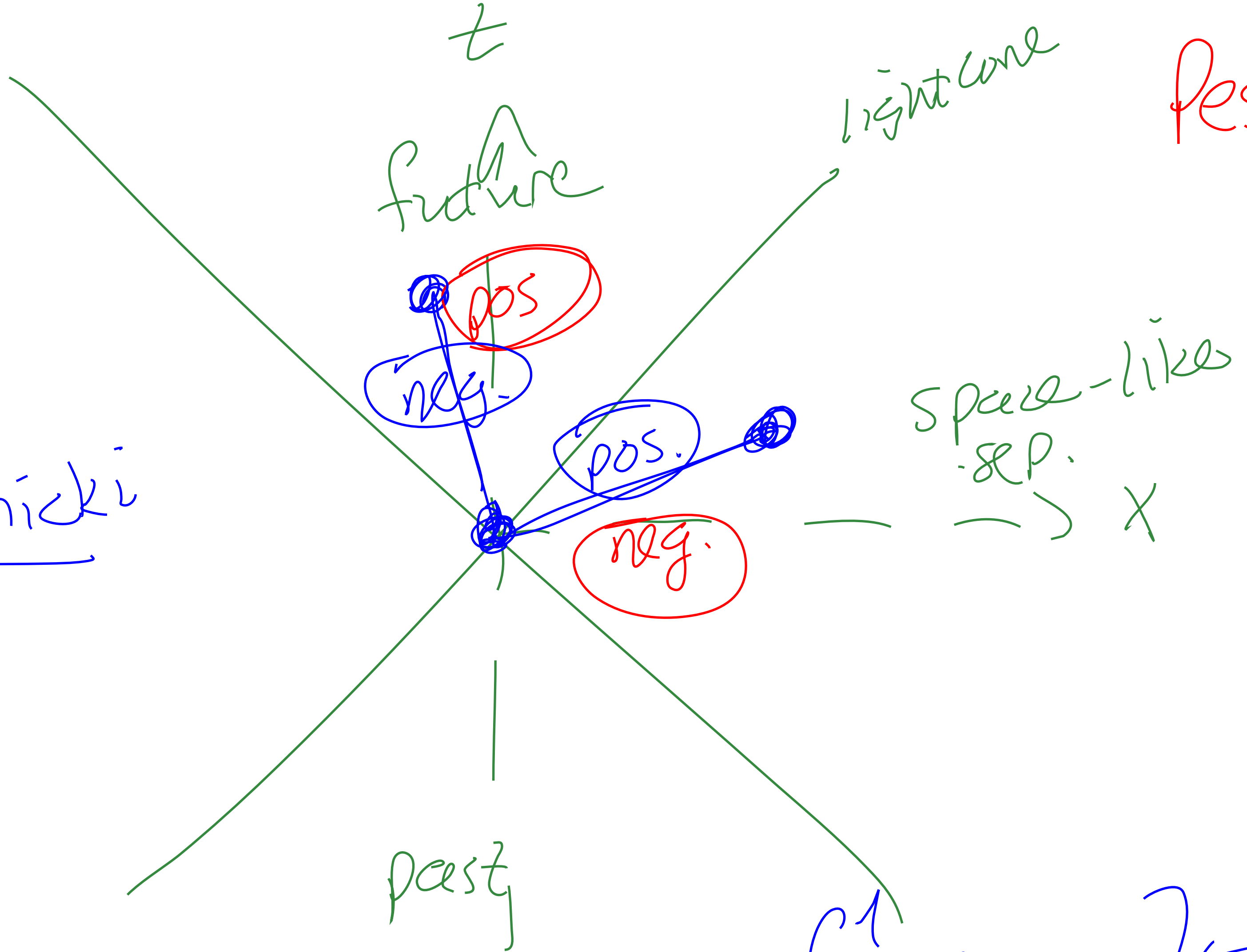
In this lecture $\hbar$ has units. $\frac{kg\, m^2}{s} \sim (\text{mass}) * (\text{dist})$

Invariant dist^2:

$$-t^2 + x^2 = [t, x, y, z] \begin{bmatrix} t \\ x \\ y \\ z \end{bmatrix} = [t\, x\, y\, z] \begin{bmatrix} -1 & 0 & 0 & 0 \\ 0 & +1 & 0 & 0 \\ 0 & 0 & +1 & 0 \\ 0 & 0 & 0 & +1 \end{bmatrix} \begin{bmatrix} t \\ x \\ y \\ z \end{bmatrix}$$

$$= x_\mu x^\mu = x^\nu g_{\nu\mu} x^\mu$$

Peskin



Srednicki

$g^{\mu\nu}$ inverse matrix of $\underline{g_{\mu\nu}} = \begin{bmatrix} 1 & & & \\ & -1 & & \\ & & -1 & \\ & & & -1 \end{bmatrix}$ ← $\underline{g_{\mu\nu}}$ $\underline{g^{\mu\nu}}$

$$g^{\mu\nu} = \begin{bmatrix} 1 & & & \\ & -1 & & \\ & & -1 & \\ & & & -1 \end{bmatrix}$$

x^{μ} contravariant vector // index

x_{μ} covariant vector // index

$\sum_{\mu=0123} x_{\mu} x^{\mu}$ same in all frames $\rightarrow$ scalar

Index μ 4 comp.

μ μ summed

$x^{\mu} x^{\mu}$ made a mistake

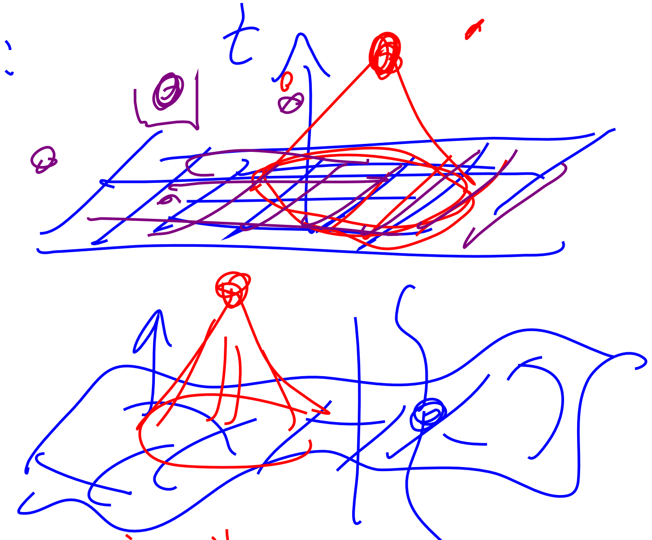
$x^{\mu}_{\nu} x^{\nu} y^{\nu}$ made a mistake

Index can appear
at most 1s Contra
at most 1 Covar.
both? Sum
not - "ext index"

$$\left(\frac{d}{dt}, \frac{d}{dx}, \frac{d}{dy}, \frac{d}{dz}\right) = \frac{d}{dx^{\mu}} = \partial_{\mu}$$ covariant

Scalar Field Theories:

$\varphi(x)$ $\hat{\varphi}(x)$



fixed time, all $\vec{x}$

surf. timelike normal

Cauchy Surface

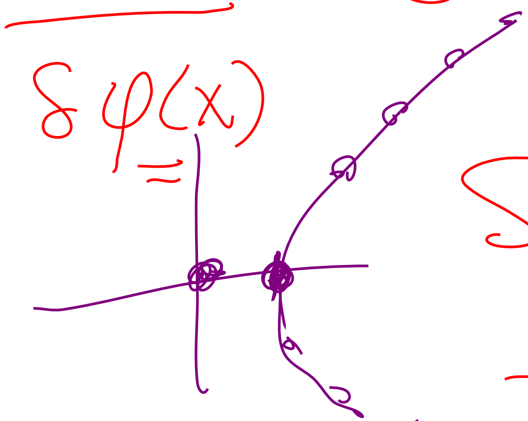
What is $\varphi(x)$ everywhere

Need Rules for "updating" φ which φ should obey

Hyperbolic partial Diff Eq.

Write Action $S[\varphi(x)]$ depends on φ throughout Spacetime

$$\frac{\delta S}{\delta \varphi(x)} = 0 \longrightarrow \text{(HPD) Eq.}$$ which φ must obey



S must be $\int d^4x \, \mathcal{L}$ $\rightarrow$ Diff. Eq.

not $\int d^4x \, d^4y \, F(\varphi(x), \varphi(y))$ nonlocal relations

$$S[\varphi] = \underbrace{\int d^3x\, dt}_{\int d^4x}\ \mathcal{L}\left[\varphi(x^\mu);\ \partial_\mu \varphi(x^\mu)\right]$$

$\mathcal{L}$: Lagrange density

$\int$ mass density d^3x = mass
En. " " Energy
Lag. " " Lagrangian

$$\mathcal{L}(\varphi, \partial_\nu \varphi) = (\text{terms w/o deriv's}) + (\text{terms with deriv's})$$

$$= -V + T$$

w/o deriv's: Taylor

$$V(\varphi) = c_0 + c_1\varphi + c_2\varphi^2 + c_3\varphi^3 + c_4\varphi^4 + c_5\varphi^5 + \ldots$$

c_0: boring
$c_1\varphi$: can be eliminated
$c_3\varphi^3$: sometimes missing
$c_5\varphi^5 + \ldots$: don't need

With Derivatives:

$\cancel{\partial_\mu \varphi}$ cov. vect. $\cancel{\varphi \, \partial_\mu \varphi}$ cov. vect. NO. $\mathcal{L}$ should be a Scalar.

$$S = \int d^4x \; \mathcal{L}(\varphi, \partial_\nu \varphi)$$

S in another reference frame?

↳ laws S gives me should be same

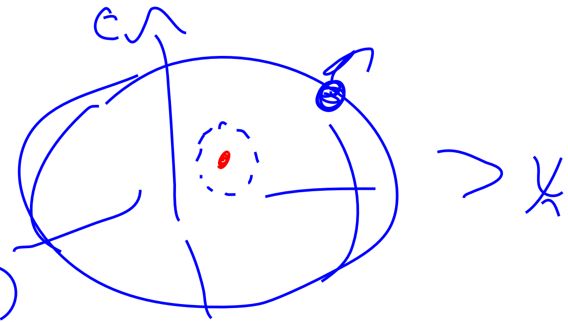
$\int d^4x$ same in all frames

$\mathcal{L}(\varphi, \partial_\nu \varphi)$ same in all frames. S same in all frames

$\mathcal{L}$ should be a scalar $A_\mu B^\mu$ indices in pairs

$$\partial_\mu \partial^\mu \varphi \qquad \partial_\mu \varphi \, \partial^\mu \varphi \qquad \varphi \, \partial_\mu \partial^\mu \varphi \quad + \ldots$$

$$S = \int \partial_\mu \partial^\mu \varphi \, d^4x + \dots$$



$$\frac{\delta S}{\delta \varphi(y)} = ? \quad \int d^4x \, \partial_\mu[\partial^\mu \varphi] = \int_\Sigma (\partial^\mu \varphi) \cdot \Sigma_\mu$$

$$\frac{\delta}{\delta \varphi(y)} \int \partial_\mu(\partial^\mu \varphi) \, d^4x = 0$$

$\mathcal{L}$ and $\mathcal{L}'$ are equiv. if they differ by $\partial_\mu[V^\mu]$

$$\mathcal{L} = -V + \cancel{c\,\partial_\mu \partial^\mu \varphi} + \dots$$

$$\mathcal{L}' = -V + 0 + \dots$$

$\varphi\,\partial_\mu\partial^\mu\varphi$ and $(\partial_\mu\varphi)(\partial^\mu\varphi) \equiv g^{\mu\nu}(\partial_\mu\varphi)(\partial_\nu\varphi)$

$$\partial_\mu\left[\varphi\,\partial^\mu\varphi\right] = (\partial_\mu\varphi)(\partial^\mu\varphi) + \varphi\,\partial_\mu\partial^\mu\varphi$$

Add with any coeff.

$$\mathcal{L} = -V + c_1(\partial_\mu\varphi)(\partial^\mu\varphi) + \cancel{c_2\,\varphi\,\partial_\mu\partial^\mu\varphi}$$

$$- c_2\,\partial_\mu\left[\varphi\,\partial^\mu\varphi\right]$$

$$- c_2(\partial_\mu\varphi)\partial^\mu\varphi \quad \cancel{- c_2\,\varphi\,\partial_\mu\partial^\mu\varphi}$$

$$\mathcal{L} = -V + (c_1 - c_2)(\partial_\mu\varphi)(\partial^\mu\varphi)$$

Consider

$S = \int d^4x \, \mathcal{L}$

$(\varphi \to -\varphi$ Symmetric$)$

$$\mathcal{L}[\varphi, \partial_\mu \varphi] = \underset{\substack{\Downarrow \\ 1/2}}{K} \, \partial_\mu \varphi \, \partial^\mu \varphi + \underbrace{C_0 + \overset{\text{shift}}{\cancel{C_1 \varphi}} + C_2 \varphi^2 + C_4 \varphi^4 + C_6 \varphi^6 \cdots}_{-V(\varphi)}$$

$\varphi \to A\varphi + B$

choosing diff't units

"A" freedom

$K = 1/2$

$\partial_\varphi V = 0$ at $\varphi = 0$

V must be bounded from below

$$\mathcal{L} = \frac{1}{2} \partial_\mu \varphi \, \partial^\mu \varphi + C_0 + C_2 \varphi^2 + C_4 \varphi^4 + C_6 \varphi^6 \cdots$$

S units of action: $E \cdot \ell = \int d^4x \, \mathcal{L}$

$\int d^4x \sim \ell^4$

$\boxed{\mathcal{L} : E \cdot \ell^{-3}}$ energy density

$[\varphi]$ units of

$[S] = \varepsilon \cdot \ell$ $\quad [\partial_\mu] = \ell^{-1}$

$[dx] = \ell$ $\quad [\mathcal{L}] = \varepsilon \ell^{-3}$

$$[\partial_\mu \varphi \partial^\mu \varphi] = \varepsilon \ell^{-3} = [\varphi]^2 [\partial^\mu]^2$$

$$\varepsilon \ell^{-1} \qquad \ell^{-2}$$

$$\boxed{[\varphi] = \varepsilon^{1/2} \ell^{-1/2}}$$

$$\mathcal{L} = \frac{1}{2} \partial_\mu \varphi \partial^\mu \varphi + c_0 + c_2 \varphi^2 + c_4 \varphi^4 + \cancel{c_6 \varphi^6} + \cancel{c_8 \varphi^8} \ldots$$

$[c_0] = [\mathcal{L}] = \varepsilon \ell^{-3}$. Interp? If $\varphi = 0$, ε-density of empty space $(2.4\,\mathrm{meV})^4$

$[c_2] = \ell^{-2}$ or freq^2. $-c_2 = \frac{\omega_0^2}{2} = \frac{m^2 \hbar^2}{2}$ $\quad \omega$ osc. freq. m mass. order-1

$[c_4]$: $[\mathcal{L}] = [c_4][\varphi^4]$

$$\varepsilon \ell^{-3} \qquad \frac{1}{\varepsilon \cdot \ell} \qquad \varepsilon^2 \ell^{-2}$$

c_4 same as $\hbar^{-1}$. Expect $c_4 \sim \frac{(\lambda)}{\hbar}$

$$[c_6] \ldots \to \frac{1}{\varepsilon^2} = \frac{\ell^2}{\varepsilon^2 \ell^2} = \frac{\ell^2}{\hbar^2} \leftarrow$$

Fund. length scale
shortest scale where descrip is valid

c_6 suppressed.

$Z_2\,\underbrace{\varphi^2(\partial_\mu\varphi\,\partial^\mu\varphi)}$?? How big is $\underline{\underline{Z_2}}$??

$$\underline{\varepsilon \ell^{-3}} = [Z_2][\underline{\partial}]^2[\underline{\varphi}]^4 = [Z_2]\,\ell^{-2}\,\varepsilon^2\ell^{-2}$$

$$[Z_2] = \frac{\ell}{\varepsilon} = \frac{\ell^3}{\varepsilon_0 \ell} \sim \frac{\ell_{fund}^2}{\hbar}$$

positive powers of fund. length.

Z_2 is $\underline{\underline{\text{tiny}}}$

Assume $\underline{\ell_{fund}}$ $\underline{\text{tiny}}$ —

Anything w. $\ell_{fund}^{\#}$ positive power $\longrightarrow$ tiny, ignored.

"$\underline{\text{Renormalizability}}$"

$$\mathcal{L} = \frac{1}{2}\partial_\mu\varphi\,\partial^\mu\varphi - \frac{\omega_0^2}{2}\varphi^2 - \frac{\lambda}{24\hbar}\varphi^4$$

$$\frac{\delta S}{\delta\varphi(y)} = \frac{\delta}{\delta\varphi(y)}\int d^4x\left[\frac{1}{2}\partial_\mu\varphi\,\partial^\mu\varphi(x) - \frac{m^2}{2}\varphi^2(x) - \frac{\lambda}{24\hbar}\varphi^4(x)\right]$$

$$\frac{\delta\varphi(x)}{\delta\varphi(y)} = \delta^4(x-y).\qquad \frac{\delta\,\partial_\mu\varphi(x)}{\delta\varphi(y)} = \partial_\mu\,\delta^4(x-y)$$

That means, Int. by parts.

$$-\frac{\lambda}{24\hbar}\int d^4x\,\frac{\delta\varphi^4(x)}{\delta\varphi(y)} = -\frac{\lambda}{24\hbar}\int d^4x\,4\varphi^3(x)\,\frac{\delta\varphi(x)}{\delta\varphi(y)} = -\frac{\lambda}{6\hbar}\varphi^3(y)$$

$$-\frac{m^2}{2}\int d^4x\,\frac{\delta\varphi^2(x)}{\delta\varphi(y)} = -m^2\varphi(y)$$

$$\int d^4x\,\frac{\delta}{\delta\varphi(y)}\left[\frac{1}{2}\partial_\mu\varphi(x)\,\partial^\mu\varphi(x)\right] = \int d^4x\,\partial_\mu\varphi(x)\,\frac{\delta\,\partial^\mu\varphi(x)}{\delta\varphi(y)} = \int d^4x\,\partial_\mu\varphi(x)\,\partial^\mu\delta^4(x-y)$$

$$= -\int d^4x\left[\partial^\mu\partial_\mu\varphi(x)\right]\delta^4(x-y)$$

$$= -\partial^\mu\partial_\mu\varphi(x)$$

$$\frac{\delta S}{\delta \varphi(y)} = 0 = -\frac{\lambda}{6\hbar}\varphi^3(y) - \omega^2\varphi(y) - \partial_\mu\partial^\mu\varphi(y)$$

Note: $\partial_\mu\partial^\mu\varphi(y) = \partial_t^2\varphi(y) - \vec{\nabla}^2\varphi(y)$

$$\parallel$$

$$[\partial_t\ \partial_x\ \partial_y\ \partial_z]\begin{bmatrix}1 & & & \\ & -1 & & \\ & & -1 & \\ & & & -1\end{bmatrix}\begin{bmatrix}\partial_t\\ \partial_x\\ \partial_y\\ \partial_z\end{bmatrix}$$

$$\underbrace{\partial_t^2\varphi(y) = +\vec{\nabla}^2\varphi(y) \underbrace{- \omega^2\varphi(y) - \frac{\lambda}{6\hbar}\varphi^3(y)}_{\text{zero?}}}$$

~~wave eq~~

speed of light prop.

consider $\varphi(y,t) = e^{i\vec{k}\cdot\vec{y}}\varphi_k(t)$.

$\partial_t^2\varphi = (-k^2 - \omega^2)\varphi$

freq. of osc. $\sqrt{k^2+\omega^2}$

$m = \hbar\omega$.

$\longrightarrow$ $E = \hbar\omega_{osc} = \frac{1}{\hbar}\sqrt{p^2+m^2}$